



THINKING SKILLS

Paper 3 Problem Analysis and Solution

2 hours

You will need: Answer booklet (enclosed)
Calculator

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.
 - Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
 - In some questions, if you do not show your working, full marks will not be awarded.

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

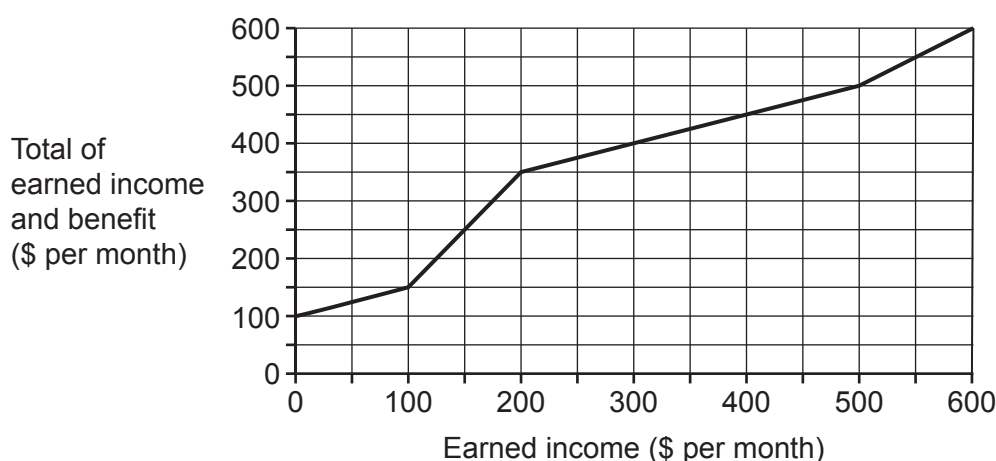
[Turn over

- 1 A government scheme for supporting the poor was badly designed. The idea was to make a payment, called a 'benefit', each (calendar) month to people whose income is below a certain level (threshold). The amount someone would receive was calculated so that, in order to 'make work pay', at every level of income, an increase in earned income would always result in an increase in the total of earned income and benefit.

Anyone whose income went above the threshold in any month would automatically be removed from the scheme on the assumption that they had now found higher-paid employment. This policy assumed that everyone receives their pay just once each month, and was problematic for people whose pay comes at some other interval.

- (a) Consider people who are paid three-quarters of the threshold by their employer every four weeks.
- Which is the only month when there could not be any of these people removed from the scheme? [1]
 - Explain how it can be deduced that such people would be removed from the scheme at least once per year (52 weeks). [1]
- (b) Consider people who are paid a fixed amount by their employer every Friday.
- What percentage pay rise could they appear to get from one month to the next when there was no actual change to their income? [1]
 - What is the maximum number of times in a year with 53 Fridays that such people could be removed from the scheme? [1]

The graph shows the relationship between earned income and benefit.



- (c) Boris is paid a fixed amount each month, and his total of earned income and benefit for the year is \$5400. How much of this is benefit? [2]
- (d) How much is the threshold? [1]
- (e) Consider people on a high annual income, well above the threshold, who are able to control when they receive their pay and how much each payment is.

Explain how such people could arrange to receive the most total money in a year from earned income and benefit, and calculate their total benefit for the year. [3]

[Turn over for Question 2]

- 2 In the Double Triple Quiz there are 4 rounds of questions. In each round each contestant is asked 5 questions. Points are awarded for correct answers. In round 1, there is no penalty for an incorrect answer or a 'pass' (no answer given). In subsequent rounds, points are deducted for incorrect answers or passes. The following table shows the points that are awarded and deducted, where for example '-10' means that 10 points are deducted.

<i>Round</i>	<i>Correct answer</i>	<i>Incorrect answer</i>	<i>Pass</i>
1	10	0	0
2	20	-5	-15
3	30	-10	-25
4	50	-20	-35

It is possible for a contestant's score (total number of points) to be negative (less than zero).

Fred answered 3 questions correctly in each round.

- (a) Show that his least possible total number of points is 180. [2]

Leah scored 15 points in the second round.

- (b) How many questions did Leah answer correctly, how many did she answer incorrectly and how many did she pass? [1]

The notation (1, 4, 0) is used to denote that a contestant has answered 1 question correctly, answered 4 questions incorrectly and passed on 0 questions.

Henry's score in the second round was 40 points greater than Isaac's score in the second round. Both of them answered at least one question correctly in the second round.

- (c) Find the four possible pairs of scores for Henry and Isaac with which this could have been achieved. [3]

Four contestants took part in last night's Double Triple Quiz. Their scores in each round and their total scores are shown in the following table.

	<i>Round 1</i>	<i>Round 2</i>	<i>Round 3</i>	<i>Round 4</i>	<i>Total score</i>
<i>Alexa</i>	30	75	70	110	285
<i>Betty</i>	10	30	110	110	260
<i>Charlie</i>	50	100	40	80	270
<i>Damon</i>	40	50	15	180	285

- (d) (i) Using the notation described above, state how many questions each contestant answered correctly, answered incorrectly, and passed in Round 4. [2]
- (ii) Charlie realises that he could have had the highest total score without answering any more questions correctly in round 4. How could he have achieved this? [1]

Alexa and Damon progressed to the final. In the final, each contestant is asked 8 questions. For each question they can choose whether it is Easy or Hard. An Easy question scores 1 point for the correct answer and a Hard question scores 2 points for the correct answer. There are no deductions for incorrect answers or passes.

Each contestant has a 'Double' which doubles the points for that question and a 'Triple' which triples the points for that question. They must use their Double and Triple once each, but not on the same question. They must choose which question they want to use each one on **before** they hear the question.

In the event of a tie, the contestant who has answered the most questions correctly in the final will be the winner.

(e) Show that the greatest number of points that a contestant can score in the final is 22. [1]

Alexa chose to attempt Easy and Hard questions alternately, beginning with an Easy one.

(f) Suppose she had scored 7 points after 5 questions and then answered the remaining 3 questions correctly. What would be her greatest and least possible total scores? Give an example of how each of these could be achieved. [2]

Damon chose to attempt Hard questions for all 8 of his questions.

After 5 questions, Alexa had 7 points and she had (in fact) already used her Double.

After 5 questions, Damon had 6 points and he had not yet used his Double.

After 8 questions, Alexa and Damon each had 16 points.

(g) Explain why Alexa was declared as the winner of the final. [3]

- 3** A satellite navigation system (SatNav) for cars requires the user to enter a destination, and then gives an Estimated Time of Arrival (ETA) if the user leaves immediately. The SatNav receives continual information about where the car is situated. The SatNav updates the ETA every 10 minutes during the journey.

The SatNav has two modes, which use different estimates of the speed that the driver will travel at, in order to calculate the time a journey will take. The first mode is called 'Speed Limit' (SL) mode.

SL mode: in this mode, the SatNav assumes that the user will travel at the legal speed limit on any roads used.

Mike is about to set off on a journey from Alphaville to Bravoburgh with the SatNav in SL mode. One road connects the two towns and it has a speed limit of 30 km/h.

- (a)** Mike enters the destination into the SatNav at 10:00, and it tells him that his ETA is 10:50.

What must the distance be between Alphaville and Bravoburgh? [1]

- (b)** After 10 minutes of driving, the SatNav updates Mike's ETA to 10:55.

(i) At what speed had Mike been driving during those 10 minutes, on average? [2]

(ii) When would he reach Bravoburgh, if he continued at this speed? [1]

The other mode the SatNav has is called 'Historical Data' (HD) mode.

HD mode: in this mode, the SatNav calculates the average time taken by all the cars in its database to get from wherever it is now to the destination. Any journey where the car exceeds the speed limit is removed from the database.

For a journey from Bravoburgh to Charliechester, Mike puts the SatNav in HD mode. One road connects the two towns, and it has a speed limit of 60 km/h. He leaves at 13:30 and the SatNav gives an ETA of 14:45.

Mike drives at precisely 60 km/h, and after 10 minutes the SatNav updates his ETA to 14:43.

- (c)** At what speed does the historical data suggest that cars usually travel on this section of road? [2]

- (d)** What is the maximum that the distance between Bravoburgh and Charliechester can be? [2]

For a journey from Charliechester to Deltabury, Mike decides to compare the two modes. When he starts, the SatNav predicts 90 minutes in SL mode and 115 minutes in HD mode. After 10 minutes of driving, the SatNav predicts 81 minutes in SL mode and 100 minutes in HD mode.

- (e)** From this information, Mike deduces what fraction of the legal speed limit the historical average speed of cars travelling on this initial section of road is.

What is this fraction? [2]

[Turn over for Question 4]

- 4 George and Rachel are playing a game of *CounterBid*. The equipment consists of a bag of counters, two small trays and two sets of the three cards shown below.

Double the number of counters in your tray	Add 5 counters to your tray	Your opponent loses 5 counters from their tray
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At the start of the first round of the game, each player takes one set of the cards and one tray, into which they place 10 counters from the bag. For subsequent rounds, each player begins the round with the set of cards and the counters in their tray that they finished the previous round with. (They do **not** take another 10 counters from the bag at the start of the round.)

In each round of the game the two players place their cards face down on the table in the order that they wish to play them. They then take up to 5 counters from their tray, which they place underneath their three cards such that their opponent cannot see how many have been assigned to each card. It is possible for one or more cards to have no counters assigned to it.

The two players then reveal their first cards and the numbers of counters assigned to them, and the game progresses as follows:

- The player who placed more counters under their card returns those counters to the bag and applies the effect of their card. The other player returns their counters to their tray and ignores the instruction on their own card.
- If the two players placed the same number of counters, they both return their counters to the bag and apply the effects of their own cards.
- If neither player placed any counters then both cards are ignored.

The same procedure is then applied to the cards placed second, and then to the cards placed third. Counters added to or removed from players' trays as a result of the cards are taken from or returned to the bag.

If either player needs to remove more counters from their tray than they have available then they immediately lose the game.

In the first round of the game, George and Rachel placed their cards and assigned counters to them as shown below.

Position	George		Rachel	
	Card played	Counters	Card played	Counters
1	Add 5 counters to your tray	2	Double the number of counters in your tray	2
2	Double the number of counters in your tray	2	Your opponent loses 5 counters from their tray	0
3	Your opponent loses 5 counters from their tray	1	Add 5 counters to your tray	2

- (a) Show that, at the end of this round, George had 21 counters in his tray. [2]
- (b) How many counters did Rachel have in her tray at the end of this round? [1]

(c) How would the outcome of this round have changed if

(i) George had placed just 1 counter on his second card, rather than 2? [1]

(ii) Rachel had placed 3 counters on her first card, rather than 2? [2]

The positions in which the cards were placed in the second round were as shown below.

<i>Position</i>	<i>George</i>		<i>Rachel</i>	
	<i>Card played</i>	<i>Counters</i>	<i>Card played</i>	<i>Counters</i>
1	Double the number of counters in your tray		Add 5 counters to your tray	
2	Your opponent loses 5 counters from their tray		Double the number of counters in your tray	
3	Add 5 counters to your tray		Your opponent loses 5 counters from their tray	

In this round, George placed 1 more counter than Rachel in two of the positions, but 2 fewer counters in the other position. He had a total of 24 counters in his tray at the end of the round.

(d) (i) In which position did George allocate 2 fewer counters than Rachel? Explain your reasoning. [1]

(ii) How many counters did George assign to each of the other two positions? [2]

Simon is thinking about the best and worst possible starts to a game of *CounterBid*.

(e) (i) What is the greatest number of counters that a player could have in their tray at the end of the first round? Give an example of such a round. [3]

(ii) Give an example of a first round in which one of the players finishes the round with no counters in their tray. [3]

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