



Cambridge International A Level

MATHEMATICS

9709/32

Paper 3 Pure Mathematics

March 2020

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **13** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Make a recognisable sketch graph of $y = x - 2 $	B1	
		1	
1(b)	Find x -coordinate of intersection with $y = 3x - 4$	M1	
	Obtain $x = \frac{3}{2}$	A1	
	State final answer $x > \frac{3}{2}$ only	A1	
	Alternative method for question 1(b)		
	Solve the linear inequality $3x - 4 > 2 - x$, or corresponding equation	M1	
	Obtain critical value $x = \frac{3}{2}$	A1	
	State final answer $x > \frac{3}{2}$ only	A1	
	Alternative method for question 1(b)		
	Solve the quadratic inequality $(x - 2)^2 < (3x - 4)^2$, or corresponding equation	M1	
	Obtain critical value $x = \frac{3}{2}$	A1	
	State final answer $x > \frac{3}{2}$ only	A1	
		3	

Question	Answer	Marks	Guidance
2	Use law of logarithm of a power and sum and remove logarithms	M1	
	Obtain a correct equation in any form, e.g. $3(2x + 5) = (x + 2)^2$	A1	
	Use correct method to solve a 3-term quadratic, obtaining at least one root	M1	
	Obtain final answer $x = 1 + 2\sqrt{3}$ or $1 + \sqrt{12}$ only	A1	
		4	

Question	Answer	Marks	Guidance
3(a)	Sketch the graph $y = \sec x$	M1	
	Sketch the graph $y = 2 - \frac{1}{2}x$, and justify the given statement	A1	
		2	
3(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.8$ and $x = 1$	M1	
	Complete the argument correctly with correct calculated values	A1	
		2	
3(c)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.88	A1	
	Show sufficient iterations to 4 d.p. to justify 0.88 to 2 d.p., or show there is a sign change in the interval (0.875, 0.885)	A1	
		3	

Question	Answer	Marks	Guidance
4	Integrate by parts and reach $ax \tan x + b \int \tan x dx$	M1*	
	Obtain $x \tan x - \int \tan x dx$	A1	
	Complete the integration, obtaining a term $\pm \ln \cos x$, or equivalent	M1	
	Obtain integral $x \tan x + \ln \cos x$, or equivalent	A1	
	Substitute limits correctly, having integrated twice	DM1	
	Use a law of logarithms	M1	
	Obtain answer $\frac{5}{18}\sqrt{3}\pi - \frac{1}{2}\ln 3$, or exact simplified equivalent	A1	
		7	

Question	Answer	Marks	Guidance
5(a)	Express LHS correctly as a single fraction	B1	
	Use $\cos(A \pm B)$ formula to simplify the numerator	M1	
	Use $\sin 2A$ formula to simplify the denominator	M1	
	Obtain the given result.	A1	
		4	

Question	Answer	Marks	Guidance
5(b)	Obtain an equation in $\tan 2x$ and use correct method to solve for x	M1	
	Obtain answer, e.g. 0.232	A1	
	Obtain second answer, e.g. 1.80	A1	Ignore answers outside the given interval.
		3	

Question	Answer	Marks	Guidance
6(a)	Separate variables correctly and attempt integration of at least one side	B1	
	Obtain term of the form $a \tan^{-1}(2y)$	M1	
	Obtain term $\frac{1}{2} \tan^{-1}(2y)$	A1	
	Obtain term $-e^{-x}$	B1	
	Use $x = 1, y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a \tan^{-1}(by)$ and $ce^{\pm x}$	M1	
	Obtain correct answer in any form	A1	
	Obtain final answer $y = \frac{1}{2} \tan(2e^{-1} - 2e^{-x})$, or equivalent	A1	
		7	

Question	Answer	Marks	Guidance
6(b)	State that y approaches $\frac{1}{2} \tan(2e^{-1})$, or equivalent	B1FT	The FT is on correct work on a solution containing e^{-x} .
		1	

Question	Answer	Marks	Guidance
7(a)	State or imply $3y^2 + 6xy \frac{dy}{dx}$ as derivative of $3xy^2$	B1	
	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1	
	Equate attempted derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	Need to see $\frac{dy}{dx}$ factorised out prior to AG
	Obtain the given answer correctly	A1	AG
		4	
7(b)	Equate denominator to zero	*M1	
	Obtain $y = 2x$, or equivalent	A1	
	Obtain an equation in x or y	DM1	
	Obtain the point (1, 2)	A1	
	State the point $(\sqrt[3]{5}, 0)$	B1	Alternatively (1.71, 0).
		5	

Question	Answer	Marks	Guidance
8(a)	Obtain $\overrightarrow{OM} = 2\mathbf{i} + \mathbf{j}$	B1	
	Use a correct method to find $\overrightarrow{MN}$	M1	
	Obtain $\overrightarrow{MN} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	A1	
		3	
8(b)	Use a correct method to form an equation for MN	M1	
	Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent	A1	
		2	
8(c)	Find $\overrightarrow{DP}$ for a point P on MN with parameter λ , e.g. $(2 - \lambda, 1 + 2\lambda, -2 + 2\lambda)$	B1	
	Equate scalar product of $\overrightarrow{DP}$ and a direction vector for MN to zero and solve for λ	M1	
	Obtain $\lambda = \frac{4}{9}$	A1	
	State that the position vector of P is $\frac{14}{9}\mathbf{i} + \frac{17}{9}\mathbf{j} + \frac{8}{9}\mathbf{k}$	A1	
		4	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\frac{A}{1+2x} + \frac{B}{1-2x} + \frac{C}{2+x}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A = -2$, $B = 1$ and $C = 4$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
9(b)	Use correct method to find the first two terms of the expansion of $(1+2x)^{-1}$, $(1-2x)^{-1}$, $(2+x)^{-1}$ or $\left(1+\frac{1}{2}x\right)^{-1}$	M1	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1FT + A1FT + A1FT	The FT is on A , B and C .
	Obtain final answer $1+5x-\frac{7}{2}x^2$	A1	
		5	

Question	Answer	Marks	Guidance
10(a)	Solve for v or w	M1	
	Use $i^2 = -1$	M1	
	Obtain $v = -\frac{2i}{1+i}$ or $w = \frac{5+7i}{-1+i}$	A1	
	Multiply numerator and denominator by the conjugate of the denominator	M1	
	Obtain $v = -1 - i$	A1	
	Obtain $w = 1 - 6i$	A1	
		6	
10(b)(i)	Show a circle with centre $2 + 3i$	B1	
	Show a circle with radius 1 and centre not at the origin	B1	
		2	
10(b)(ii)	Carry out a complete method for finding the least value of $\arg z$	M1	
	Obtain answer 40.2° or 0.702 radians	A1	
		2	