

# Cambridge International AS & A Level

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## MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

February/March 2021

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

**BLANK PAGE**

- 1** Solve the equation  $\ln(x^3 - 3) = 3 \ln x - \ln 3$ . Give your answer correct to 3 significant figures. [3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- 2** The polynomial  $ax^3 + 5x^2 - 4x + b$ , where  $a$  and  $b$  are constants, is denoted by  $p(x)$ . It is given that  $(x + 2)$  is a factor of  $p(x)$  and that when  $p(x)$  is divided by  $(x + 1)$  the remainder is 2.

Find the values of  $a$  and  $b$ .

[5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 3** By first expressing the equation  $\tan(x + 45^\circ) = 2 \cot x + 1$  as a quadratic equation in  $\tan x$ , solve the equation for  $0^\circ < x < 180^\circ$ . [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 4** The variables  $x$  and  $y$  satisfy the differential equation

$$(1 - \cos x) \frac{dy}{dx} = y \sin x.$$

It is given that  $y = 4$  when  $x = \pi$ .

- (a) Solve the differential equation, obtaining an expression for  $y$  in terms of  $x$ . [6]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

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(b) Sketch the graph of  $y$  against  $x$  for  $0 < x < 2\pi$ .

[1]

- 5 (a)** Express  $\sqrt{7} \sin x + 2 \cos x$  in the form  $R \sin(x + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . State the exact value of  $R$  and give  $\alpha$  correct to 2 decimal places. [3]

[illegible]

**[Turn over**

6 Let  $f(x) = \frac{5a}{(2x-a)(3a-x)}$ , where  $a$  is a positive constant.

(a) Express  $f(x)$  in partial fractions.

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

**(b)** Hence show that  $\int_a^{2a} f(x) \, dx = \ln 6$ .

[4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 7** Two lines have equations  $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$  and  $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ .

(a) Show that the lines are skew.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b)** Find the acute angle between the directions of the two lines.

[3]

[illegible]

8 The complex numbers  $u$  and  $v$  are defined by  $u = -4 + 2i$  and  $v = 3 + i$ .

- (a) Find  $\frac{u}{v}$  in the form  $x + iy$ , where  $x$  and  $y$  are real. [3]

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- (b) Hence express  $\frac{u}{v}$  in the form  $re^{i\theta}$ , where  $r$  and  $\theta$  are exact. [2]

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In an Argand diagram, with origin  $O$ , the points  $A$ ,  $B$  and  $C$  represent the complex numbers  $u$ ,  $v$  and  $2u + v$  respectively.

- (c) State fully the geometrical relationship between  $OA$  and  $BC$ . [2]

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- (d) Prove that angle  $AOB = \frac{3}{4}\pi$ . [2]

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9 Let  $f(x) = \frac{e^{2x} + 1}{e^{2x} - 1}$ , for  $x > 0$ .

- (a) The equation  $x = f(x)$  has one root, denoted by  $a$ .

Verify by calculation that  $a$  lies between 1 and 1.5.

[2]

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- (b) Use an iterative formula based on the equation in part (a) to determine  $a$  correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

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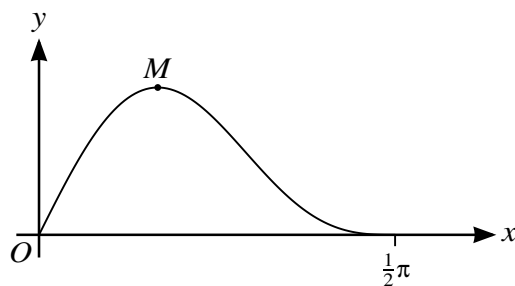
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- (c) Find  $f'(x)$ . Hence find the exact value of  $x$  for which  $f'(x) = -8$ . [6]

[illegible]



(a) Using the substitution  $u = \sin x$ , find the exact area of the region bounded by the curve and the  $x$ -axis. [5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

(b) Find the exact  $x$ -coordinate of  $M$ .

[6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]

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