Principal examiner reports for teachers

Principal examiner reports for teachers summarise what candidates did well and what candidates needed to improve for this examination series only.

This report is to be used in conjunction with the question paper and mark scheme.

For guidance on teaching and learning we provide resources for many of our syllabuses.

Resource	Where to find it
Grade descriptions – Grade descriptions describe the level of performance typically demonstrated by candidates achieving the different grades awarded for a qualification.	School support hub Select: Subject > Resource finder > Syllabus and specimen materials
Schemes of work – Schemes of work are designed to support you in your teaching and lesson planning.	School support hub Select: Subject > Resource finder > Teaching and learning
Lesson planning – Guidance on how to plan a lesson.	Lesson planning
Resource plus – There may be additional resources such as teaching packs containing lesson plans, resources and worksheets and videos for aspects of your syllabus.	School support hub Select: Subject > Resource finder > Teaching and learning
Teaching tools	Teaching Tools
Exemplar candidate responses	School support hub Select: Subject > Resource finder > Teaching and learning
Learner guide – A guide for students to help them understand what the course involves.	School support hub Select: Subject > Resource finder > Teaching and learning
Community forums – Do you have a question or something to share? Our online forums are great way to keep up to date with your subject and the global Cambridge community.	Community forum on the school support hub
Access to scripts for this series.	Access to scripts service
Results analysis is available for some syllabuses.	Results analysis
Test maker is available for some subjects.	Test maker

Other services from Cambridge International Education

- **Digital mocks service**
- **Insight assessments**
- **Professional development**
  - Focus on assessment
  - Online marking workshops

MATHEMATICS

Paper 9709/12
Pure Mathematics 1

Key messages

What candidates did well
- Solutions were well presented and contained suitable levels of detail.
- Algebraic accuracy was generally good and the overall standard of response high.
- Candidates seemed well prepared for the majority of questions and were able to apply their knowledge.

What candidates needed to improve
- Candidates must show complete working, particularly in cases when the required answer is given.
- Candidates must ensure they consider how successive parts of the same question are related, particularly when directed through the 'hence' command word or explicitly.
- Candidates are reminded of the need for rigour and clarity when presented their final answer (e.g. $y = f(x)$ and not just $f(x)$ when stating the equation of a graph).

Comments on specific questions

Question 1

Comprehensive Responses
- This was a standard question and a relatively straightforward introduction to the paper. Most candidates demonstrated a clear understanding of the required concepts. Comprehensive responses showed the required skill in solving a quadratic with no constant term and evaluated the derivative clearly whilst establishing the nature of the stationary points.

Limited Responses
- Common mistakes seen were discarding the $x = 0$ solution or the absence of the $y-$ coordinates of the stationary points.

Question 2

(a) **Comprehensive Responses**
- Use of the cosine rule was the most common (and expected) approach, although some candidates divided the bigger triangle into two unequal right angle triangles, achieving the correct result.

Limited Responses
- A minority of candidates incorrectly assumed that angle ACB was a right angle or that the triangle given was isosceles. This leads to incorrect outcomes in both part **(a)** and **(b)**.

(b) **Comprehensive Responses**
- This was answered well by most candidates. Most took the approach of $\frac{1}{2}ab\sin C$ to find the area of the triangle, which gave the result in the fewest number of steps.

Limited Responses
- Some candidates did not work to the required accuracy in both **(a)** and **(b)**. The accuracy of three significant figures stressed in both parts of the question serves as a reminder to use calculator accuracy or at least 4sf in their working.

Question 3

(a) **Comprehensive Responses**

- The majority of responses took one of two approaches. The minority who used general vertex form of a quadratic where able to provide a brief complete solution. The more common approach was to begin by differentiating and forming two equations to find the constants a and b.

Limited Responses

- Whilst a number of fully correct solutions was seen, the required algebraic accuracy meant correct solutions were not found from the correct process.

(b) **Comprehensive Responses**

- Comparatively few fully correct solutions were seen in this question. Most successful attempts came from using their exact answer to **Question 3(a)** and applying the appropriate transformation.

Limited Responses

- A significant number of candidates were unable to attempt the question and all candidates are reminded that the equation of a graph requires a '$y =$' in their final response.

Question 4

(a) **Comprehensive Responses**

- This was a routine question, candidates knew the process for finding an inverse function, including the required factorisation.

Limited Responses

- Candidates often could not proceed beyond the point where the required subject is on both sides of the equation. A minority left their response in the form '$y =$' and are reminded to show that the right hand side is the inverse function.

(b) **Comprehensive Responses**

- Almost all candidates were able to form the initial composite function. The algebraic accuracy required to deal with the embedded fractions was present. Several candidates produced fully correct solutions, demonstrating clear, logical working and efficient use of algebraic techniques.

Limited Responses

- Candidates were unable to deal with the embedded fractions and how to simplify to the required form.

Question 5

(a) **Comprehensive Responses**

- The simplicity of this trigonometric identity and lead to mostly excellent responses.

Limited Responses

- Candidates are reminded that in the case of answer given, full and complete working must be shown to get to the required result. In this case the step of simplifying the numerators needed to be shown when forming a common denominator.

(b) **Comprehensive Responses**

- Candidates understood the connection (and differences) between **Question 5(a)** and **Question 5(b)**. This then often led to 4 solutions, two from each of the positive and negative root.

Limited Responses

- A significant number of candidates did not recognise the different numerator when compared to part **(a)** and hence lost accuracy marks. Candidates are also reminded to take both the

positive and negative root when solving $\cos^2\theta = k$. Some candidates needed to respond more closely to the 'Hence' command word and start solving the equation from scratch.

Question 6

(a) All responses

- Most candidates were able to answer this question correctly. The only error observed was the incorrect square root of a fractional number.

(b) Comprehensive Responses

- Candidates demonstrated accurate use of simultaneous equations resulting in a high proportion of correct responses.

Limited Responses

- Some candidates did not show their method to solve a quadratic equation. In particular when solving by factorisation their terms should be consistent with their equation. Others stated a rounded decimal answer when an exact distance was required.

Question 7

(a) Comprehensive Responses

- Those who recognised the required technique to convert the expression into three terms of index form were very successful and most of these candidates were able to score well on this part.

Limited Responses

- Few candidates were able to provide a complete response to this question. It appeared that the concept being tested was very unfamiliar to many. Most integrated the numerator and denominator separately before attempting to combine, this resulted in no credit.

(b) Comprehensive Responses

- Candidates recognised the need for a vertical translation and a smaller number also identified the required 'k–7'.

Limited Responses

- Of those that recognised the vertical translation, some vectors had no k in them despite the question stating 'in terms of k'. The majority here either gave no response or produced unnecessary algebra.

Question 8

(a) Comprehensive Responses

- This was a familiar question on the binomial theorem produced a high proportion of fully correct answers.

Limited Responses

- Candidates sometimes did not recognise the negative term in the question, which hindered progress.

(b) Comprehensive Responses

- Candidates understood to substitute $\sqrt{2}$ in their expansion and were able to evaluate powers of $\sqrt{2}$ successfully.

Limited Responses

- Some candidates did not refer to the demand in the question of 'use your answer to **8(a)**' in some response and others did not state their answer in the given form. Of those who did substitute $\sqrt{2}$, there were errors seen when evaluating higher powers.

Question 9

Comprehensive Responses

- Candidates always recognised that $(8x)^{\frac{1}{3}} = 2x^{\frac{1}{3}}$ and went on to successfully solve the disguised quadratic.

Limited Responses

- Candidates often tried to cube both sides as their first step. Of those who recognised the disguised quadratic, some candidates cube-rooted rather than cubing at the end or concluded incorrectly that a negative answer is inappropriate for a cube root.

Question 10

(a) **Comprehensive Responses**

- This was a familiar question on an arithmetic series and many candidates were able to gain full marks as a result.

Limited Responses

- Candidates were sometimes unaware of the summation formulae for arithmetic series. Some candidates also confused the 20^{th} term and the sum of the first 20 terms.

(b) **Comprehensive Responses**

- The majority of correct responses used an equation (rather than inequality) and then interpreted the solutions correctly.

Limited Responses

- Some candidates lost sight of the context of the question, stating the meaningless answer of N=0. In the majority of cases candidates achieved a correct inequality/equation and chose the integer value of N as their final answer. Some candidates used a trial-and-error approach which did not explain their reasoning.

Question 11

(a) **Comprehensive Responses**

- This was another familiar question, almost all attempts found dy/dx correctly and this enabled the resulting quadratic to be solved and the two stationary points to be found. Using the points to find the gradient and hence the required equation with evidence is essential.

Limited Responses

- Most candidates successfully found the stationary points. After this point, candidates are reminded that this is a 'show that' question and hence assuming the required result and verifying by substitution is insufficient.

(b) **All Responses**

- Most candidates were able to use substitution successfully.

(c) **Comprehensive Responses**

- The curve and line could be combined initially or worked separately; the first approach is the more efficient and the second approach is the safer of the two. A complete response to this type of question should include both correct working and a correct final answer.

Limited Responses

- Of those who attempted the question, the most common errors observed were sign errors at the subtraction stage or use of the incorrect limits at the integration stage.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2

There were too few candidates for a meaningful report to be produced.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3

Key messages

What candidates did well
- Coefficients of the terms in the Binomial expansion.
- Use of iterative formula to find root of equation.

What candidates needed to improve
- Solution of equation requiring logarithms.
- Sketching standard trig functions and log.
- Long division of quartic polynomial by quadratic polynomial.
- Integration of trig functions such as $\sin^2 x.\cos x$, $\tan x$, and $\tan x.\sec^2 x$.

Comments on specific questions

Question 1

Comprehensive responses
- Candidates who recognised that this was a disguised quadratic question tended to produce a virtually correct solution.

Limited responses
- Candidates needed to show an intermediate step in their working, such as $x = \log_2 1.2157$.
- Rounded too early.
- Most candidates simply took logs of each individual term. Candidates need to show correct knowledge of log work.

Question 2

(a) **Comprehensive responses**
- Many candidates demonstrated understanding of the required concepts in this question. To improve, candidates must ensure that all steps are completed accurately and that their final answers are fully supported by prior steps.
- Candidates generally incorrectly sketched a full circle centred on the origin with a radius 9.

Limited responses
- Some candidates shaded the part of the circle in the quadrant $\pi/2 \leq \arg z \leq \pi$, not realising that the locus of the points was the arc of the circle in the correct quadrant. Perhaps not understanding $|z| = 9$.

(b) **Comprehensive responses**
- Some candidates demonstrated partial understanding by sketching a circle centred on (3, 0) with radius 9.

Limited responses
- Others translated the circle from part **(a)** in the wrong direction (either towards the left or upwards).
- Very few candidates recognised that the correct locus was the reflected arc shown in part **(a)**.

Question 3

Comprehensive responses
- The most straightforward method was to substitute for sin2*x* and then use integration by recognition or by substitution.
- Candidates who managed to carry out the integration did know to give the answer in the required form.

Limited responses
- Many candidates needed to demonstrate a clearer understanding of the concepts assessed in this question.
- Some candidates used integration by parts, of these very few managed to integrate completely, with most only applying the formula once.
- Many candidates did substitute for sin2*x* and then did not know how to proceed.
- Some candidates needed to demonstrate a clear understanding of integration of trig functions.

Question 4

(a) **Comprehensive responses**
- If candidates were to re-write the expression as $(1 + (-ax))^{2/5}$ first they would know how to use $-ax$ accurately.
- Many candidates presented a completely correct solution.

Limited responses
- Some candidates needed to demonstrate a clearer understanding of the concepts assessed in this question since they used ax rather than $-ax$ in their expansion.

(b) **Comprehensive responses**
- Most knew to find the next term in the expansion in part **(a)** and then to combine with $2x+1$ to find the required answer.

Limited responses
- A significant number of candidates simply used their answer to part **(a).** Candidates needed to recognize that a different approach or calculation was required for this part of the question.

(c) **Comprehensive responses**
- Those candidates who attempted part **(a)** generally knew that the answer to this part was related and many gave the answer correctly as $|x| < 2/5$.

Limited responses
- Some omitted the modulus symbol.
- A significant number of candidates did not attempt this part.

Question 5

(a) **Comprehensive responses**
- Candidates approached this part by multiplying the numerator and denominator by $\lambda - 2i$ and, having done this then used $i^2 = -1$ to demonstrate understanding of part of the required process.
- Candidates who knew the significance of $\arg z = \pi/4$ were able to derive the correct quadratic equation, solve it and then work out which solution was valid

Limited responses
- Some candidates gave both solutions to the quadratic equation.
- Others incorrectly equated their z to $\pi/4$.

(b) **Comprehensive responses**
- Many candidates who were not completely correct in part **(a)**, showed some correct steps in their response here by using their 'incorrect' value for λ and by showing their method for calculating $|z|$.
- Most candidates substituted their λ into their $(5\lambda + (\lambda^2 - 6)i))/(\lambda^2 + 4)$ and used this to find $|z|$.
- A few candidates used $z=|3 + \lambda i|/|\lambda + 2i|$.

Limited resources

- Some candidates either omitted the square root sign, or to square and add each term inside the square root sign.
- Other candidates gave moduli for both their values of λ.

Question 6

(a) Comprehensive responses

- The candidates who expanded quadratic brackets and compared coefficients tended to be more successful than the ones who attempted algebraic long division.
- Those who attempted long division obtained a quotient of $2x^2 + kx$, although then often did not fully complete the process.
- Another method occasionally seen was to find the complex roots of the polynomial from the given quadratic factor. Candidates then correctly substituted one of these roots back into the quartic although often fully relied on a calculator to expand brackets and did not show sufficient working.

Limited responses

- Other candidates, having completed the division, did not then know what to do with the remainder of $(b - 2)x + a - 7$.
- Again, often having completed the substitution mentioned above, candidates did not then know they needed to equate real and imaginary parts of the resulting equation to find the values of a and b.

(b) Comprehensive responses

- Candidates who had not correctly found the values for a and b from **6(a)** could still demonstrate understanding of the required process in this part.
- Various methods were seen here, the most common being to substitute x = –3 into the original quartic, or into the quotient found in **6(a)**.

Limited responses

- Some candidates incorrectly substituted $x = 3$, whilst others just quoted the summation of all the terms at $x = -3$ to be zero without undertaking the required arithmetic.

Question 7

(a) Comprehensive responses

- Several candidates correctly sketched a graph of y = cosec(x/2) and a graph of y = ln x, and showed there was just one intersection in the given interval.

Limited responses

- Too many candidates produced graphs that failed to resemble either of the required graphs in any way whatsoever.
- Candidates were expected to show that the log graph crossed the x-axis at 1 and the cosec graph had a minimum at $(\pi, 1)$.
- Often graphs were missing one or both these important indicators, and even those solutions which contained fully correct graphs often did not identify the single root in the given interval.

(b) Comprehensive responses

- Most candidates were able to correctly evaluate a relevant expression at both x = 2.6 and x = 2.9 and go on to state there was a sign change within this interval.

Limited responses

- Some candidates thought cosec was the reciprocal of cosine rather than the reciprocal of sine, and some worked in degrees rather than in radians.

Question 8

Comprehensive responses
- Almost all candidates could successfully separate the variables correctly.
- Most candidates could then identify a need for integration by parts for that side of the integral, although a few solutions omitted the minus sign or the 3.
- Those candidates who realised a need to divide through by y + 5 were often successful splitting the fraction and going on to correctly integrate the two resulting terms.
- Those candidates who had an equation resulting in terms of the correct form, usually did not make any errors when going to find the constant of integration.
- Similarly, very few errors were then seen when substituting the constant of integration back into the resulting equation, which was accepted in any correct equivalent form.

Limited resources
- Occasionally there were incorrect coefficients following the attempt at integration by parts.
- Candidates were less successful when trying to integrate the other side of the integral.

Question 9

(a) **Comprehensive responses**
- Many candidates produced a correct solution.
- This is a 'show that question' hence full and detailed working is required.
- Candidates need to show a little more exam understanding and realise that the question is arranged to give a direct and suitable answer to be carried forward to the next part.

Limited responses
- Common to see little or no working for changing the limits, as well as incorrect expressions for $\mathrm{d}x/\mathrm{d}u$ or $\mathrm{d}u/\mathrm{d}x$. Candidates showed some understanding of this part.

(b) **Comprehensive responses**
- Some candidates introduced partial fractions into the original integral rather than following the guidance in part **(a)**. This approach was a valid method and led to correct solutions in some cases.
- Some candidates managed to integrate $\tan x.\sec^2 x$ using a substitution. Others recognised it to be the derivative of $(1/2)\tan^2 x$ or $(1/2)\sec^2 x$.

Limited responses
- Candidates needed to develop their approach further to make progress with this part.
- Some candidates replaced $\tan^2 x$ with $\sec^2 x - 1$, although often with a sign error, and managed to integrate the $\tan x$ part and not the $\tan x.\sec^2 x$ part.

Question 10

(a) **Comprehensive responses**
- Many candidates had partial success and did not know how to remove k.

Limited responses
- For others the k just disappeared from their working.
- Those candidates who realised that k could be removed by isolating it before undertaking the differentiation, often had inaccuracies in their differentiation.

(b) **Comprehensive responses**
- Most candidates obtained the y and $\mathrm{d}y/\mathrm{d}x$ values.
- Some used the formula $\tan(A - B)$.

Limited responses
- Only the most able candidates could construct a diagram from which the answer may be simply written down.
- Those quoting the formula often quoted it incorrectly.

Question 11

(a) **Comprehensive responses**

- This question was dealt with well by most candidates despite running out of time.

Limited responses

- Common errors were arithmetic ones in find vector ***AP*** and using vector ***OA*** instead of vector ***OB*** in their scalar product.

(b) **Comprehensive responses**

- If time allowed an attempt at this question, it usually resulted in a successful outcome.

Limited responses

- Most candidates knew what was required. Some made errors, for example sign error in finding vector ***AB*** and muddling of the modulus of vector ***OC***.

MATHEMATICS

Paper 9709/42
Mechanics

Key messages

What candidates did well
- Many candidates demonstrated excellent understanding and application of the syllabus content.
- Many candidates worked to a sufficient degree of accuracy to give non-exact answers to three significant figures.
- Many candidates drew a fully labelled force diagram when this was helpful in answering a question.

What candidates needed to improve
- Candidates need to follow the rubric of the paper in only approximating non-exact answers to three significant figures.
- When more than one attempt is made to a question, candidates should cross out the work that is not to be assessed.
- Full working should be shown in questions where an answer is given.

Comments on specific questions

Question 1

Comprehensive responses
- Most candidates applied the correct formula to find the frictional force.
- Many candidates used the correct formula relating power, force and speed.

Limited responses
- Some candidates included the weight of the block when calculating the tension in the rope.

Question 2

(a) Comprehensive responses
- Most candidates found the correct speed of the cyclist after 10 seconds.
- Many candidates used the correct constant acceleration formula to find the value of v.

Limited responses
- A few candidates used an initial speed of 0 m s^{-1} when calculating the value of v.

(b)(i) Comprehensive responses
- Many candidates found one of the missing distances using either constant acceleration or the area under a velocity-time graph.
- Most candidates added the distances from the four phases of the motion.

Limited responses
- Some candidates considered only 3 phases of the motion in finding the total distance.

(ii) Comprehensive responses
- Many candidates found one of the missing times using either constant acceleration or the gradient of a velocity-time graph.
- Most candidates added the times from the four phases of the motion.
- Most candidates used a correct formula to find the average speed.

Limited responses

- Some candidates considered only 3 phases of the motion in finding the total time.

Question 3

Comprehensive responses

- Many candidates resolve in two directions with the correct number of terms.
- Most candidates solved their equations simultaneously to find the value of F and the value of θ.

Limited responses

- Some candidates omitted a force in one of the resolved equations.
- Some candidates gave the wrong components of a forces.
- Some candidates used an incorrect trigonometrical relationship $\tan\theta = \frac{\cos\theta}{\sin\theta}$ in solving for θ.

Question 4

Comprehensive responses

- Most candidates found a correct change in potential energy.
- Many candidates applied the correct formula to find kinetic energy.
- Most candidates formed a work-energy equation with the correct number of terms.

Limited responses

- Some candidates had incorrect signs with some terms in the work energy equation.
- A few candidates include the potential energy twice by a misconception that the work done by the weight is different from the change in potential energy.
- Some candidates gave a final answer of the braking force, when the request was for the work done by the braking force.
- Some candidates using a Newton's second Law and constant acceleration rather than the instructed energy method to solve the request.

Question 5

(a) **Comprehensive responses**

- Many candidates drew a correctly labelled full force diagram.
- Most candidates found the normal reaction to at least one of the particles from the plane.
- Most candidates formed a Newton's second Law equation with the correct number of terms.
- Many candidates had 2 equations in acceleration and tension that were correctly solved simultaneously.

Limited responses

- Some candidates omitted the friction when applying Newton's second Law.
- Some candidates included the 12 N force in the Newton's second Law equation for particle P.
- A few candidates had an inconsistent direction of the tension or acceleration when forming two equations.

(b) **Comprehensive responses**

- Some candidates found a deceleration now that there is no tension and used this in the correct constant acceleration formula to get $0 = 2 - (8.464\ldots)t$.

Limited responses

- Some candidates used the acceleration found in **(a)** here.
- Some candidates used a positive acceleration to form $0 = 2 + (8.464\ldots)t$ which leads to a negative time, and often the negative sign was omitted without justification.

Question 6

(a) (i) **Comprehensive responses**

- Many candidates formed a correct equation using conservation of linear momentum.

Limited responses

- Some candidates did not show enough working to show the given answer $v = 6$.
- A few candidates used a value for m as 1.

(ii) Comprehensive responses

- Many candidates found a correct change in kinetic energy due to the collision.

Limited responses

- Some candidates equated a change in kinetic energy to the given 45 J using an incorrect sign which gave $m = -2$.

(b) Comprehensive responses

- Many candidates found that particle Q takes 1.5 seconds to reach the wall after the collision with particle P.
- Many candidates found the correct distance of 3 metres that particle P travels from its collision with Q, until Q hits the wall.
- A few candidates used constant speeds correctly to get the equation $2t + 0.5t = 6$, solved for t and then found the distance from the wall at which the second collision occurs.

Limited responses

- Some candidates used constant acceleration formulae with a non-zero acceleration.

Question 7

(a) Comprehensive responses

- Most candidates differentiated the given function correctly.
- Most candidates substituted $t = 4$ in their function in t and evaluated correctly.

Limited responses

- A few candidates integrated the given function.
- Some candidates gave an answer of the magnitude of the acceleration.

(b) Comprehensive responses

- Most candidates used $v = 0$ and attempted to solve the resulting equation using a suitable method.

Limited responses

- Some candidates used a substitution of $u = t^{\frac{1}{2}}$, solved for u correctly and then evaluated t as $\sqrt{4}$ and $\sqrt{3}$.

(c) Comprehensive responses

- Most candidates integrated the given function correctly.
- Few candidates used the values of t found in **(b)** to evaluate the integral for 3 distinct time periods.

Limited responses

- Many candidates found the displacement by evaluating the integral from $t = 0$ to $t = 25$.

MATHEMATICS

Paper 9709/52
Probability & Statistics 1

Key messages

What candidates did well
- Many demonstrated good understanding of content and presented their solutions well with good supporting work.
- Candidates applied standard processes in routine contexts well.

What candidates needed to improve
- Candidates need to work to a sufficient degree of accuracy to be able to provide non-exact answers to 3 significant figures.
- Candidates need to manage time effectively so that all questions can be attempted.
- Interpretation of success criteria, which is essential in this component.
- Fully correct numerical expressions were sometimes inaccurately evaluated, or left not evaluated.

Comments on specific questions

Question 1

Comprehensive responses
- Most candidates used the combinations approach for probabilities.
- Many candidates identified clearly the possible scenarios.

Limited responses
- Many candidates omitted at least one possible scenario.
- Some candidates assumed selection with replacement.
- A few candidates using the combination approach calculated the number of appropriate outcomes correctly and did not calculate the total number of outcomes for the denominator.

Question 2

(a) **Comprehensive responses**
- The most common approach was to calculate the total number of ways with Es together and subtract the number of ways with Es together and Ts together.
- Many candidates included a simple visual representation of the given conditions.
- Most candidates communicated clearly the scenarios being calculated.

Limited responses
- The most common approach was to attempt to calculate the number of ways with Ts not together + number of ways with 2 Ts together, and only included one term.
- Many candidates did not divide by 3! to remove the effect of the 3Ts present in the arrangements.
- Some candidates omitted the Es from arrangements and used $\frac{7!}{3!}$ and $5!$.

(b) **Comprehensive responses**
- Most candidates clearly identified the possible scenarios, listing in a logical order.
- Many candidates included simple diagrams to assist with finding the total number of outcomes for each scenario.
- The most common approach added the outcomes for 4, 5, 6 and 7 letters between the Es.

Limited responses

- Some candidates only found the outcome for 4 letters between the Es.
- Some candidates did not consider the number of possible positions for the Es given the number of letters between the 2 Es.
- Many candidates did not divide by 3! to remove the effect of the 3Ts present in the arrangements.
- A few candidates attempted to use a single diagram to represent all the different scenarios.

Question 3

(a) **Comprehensive responses**

- Nearly all candidates listed the cumulative frequencies clearly, often by the data table.
- Most candidates used sensible scales on the axes, with a vertical cumulative frequency scale of 1 cm = 10 students on the vertical axis and 2 cm = 10 minutes on the horizontal axis being most common.
- Most candidates plotted the points using + or × accurately, often with the axis scaled so that the upper boundary was on a main grid line.
- All candidates used a smooth curve from (4.5, 0) to connect the points.
- Most candidates labelled the axes clearly, including minutes on the time axis.

Limited responses

- Many candidates did not state the cumulative frequencies, plotting directly on the graph. The scale needed to be sufficient to allow the values to be confirmed.
- Some candidates used scales which made plotting points accurately difficult, for example 2 cm = 15 students for the cumulative frequency. A very small number used 16 cm = 150 students.
- Many candidates plotted points at the upper class value given in the data table, for example (9, 4).
- Most candidates started the curve at their first plotted upper boundary point rather than initial lower boundary point.
- Some candidates omitted ‘minutes’ on the time axis.

(b) **Comprehensive responses**

- Many candidates calculated the term value of the 60th percentile.
- All candidates indicated clearly on the grid the use of their graph.
- Most candidates interpreted the scales accurately.

Limited responses

- Many candidates did not indicate how the graph was being used.
- Some candidates read the cumulative frequency scale at 60.
- Some candidates did not read the value accurately from their graph.

(c) **Comprehensive responses**

- Most candidates listed clearly the midpoints, often by the data table in **3(a)**.
- Most candidates presented an unsimplified calculation for the approximate mean, with the denominator stated as 150.
- Almost candidates all stated a decimal or mixed fraction answer, which were appropriate for the context.
- Some candidates used the ‘table’ approach with clear labelling to identify the operations being performed.

Limited responses

- Some candidates found the midpoints of the first and last class inaccurately, with 7.25 and 64.5 being the most common errors.
- Some candidates incorrectly used the upper boundary or upper class value in their calculation.
- Some candidates incorrectly used the cumulative frequency in their calculation.
- A small number of candidates used the frequencies to recalculate the total number of students and did not obtain 150.

Question 4

(a) **Comprehensive responses**

- Most candidates stated the calculation before evaluating.
- Many candidates obtained the accurate fraction; some then converted to the less accurate rounded decimal.

Limited responses

- Most candidates did not state a calculation, simply giving 0.08, 1 significant figure answer, or a more accurate and truncated decimal including 0.079, 0.0799.

(b) **Comprehensive responses**

- Most candidates stated a clear, unsimplified calculation.
- Many calculated the probability that a yellow disc was obtained once in the first 4 selections and then multiplied by the probability of selecting a yellow disc on the 5th selection.
- Many candidates stated the accurate fraction before converting to the less accurate decimal.

Limited responses

- Many candidates assumed that the yellow discs were obtained on the 4th and 5th selections.
- Some candidates assumed that the discs were not replaced.
- Some candidates tried to use a geometric approximation approach.

(c) **Comprehensive responses**

- Some candidates identified the possible outcomes and calculated the probabilities before creating the probability distribution table.
- Many candidates used a common denominator for the probabilities, as this can be helpful when interpreting the information.
- Most candidates included appropriate labelling for the probability distribution table.
- Some candidates used a ruler to create the probability distribution table, which often produced a clearer solution.

Limited responses

- Many candidates assumed that the discs were replaced after selection.
- Many candidates did not confirm that the probability distribution table summed to 1.
- Some candidates misinterpreted the question and formed a probability distribution table with the variable being the number of red discs drawn before obtaining a green disc.
- Some candidates stated the probabilities as decimals, not always rounded accurately to 3 or more significant figures.

(d) **Comprehensive responses**

- Many candidates stated clear, unsimplified expressions for both the mean and variance calculations.
- Some candidates used a ‘table’ approach with clear labelling to identify the processes being used.
- All candidates used the probabilities obtained in **4(c)**.

Limited responses

- Many candidates stated the mean without showing their working. It is important to include supporting calculations to demonstrate understanding and ensure the method is clear.
- Many candidates did not use the variance formula accurately. The common errors were squaring the probabilities rather than the outcome or not squaring the mean.
- Most candidates did not show unsimplified calculations, which when the probability distribution table was incorrect in **4(c)**. It is important to include supporting calculations to demonstrate understanding and ensure the method is clear.

Question 5

(a) **Comprehensive responses**

- Most candidates completed the tree diagram with correct labelling and probabilities.
- Some candidates used a ruler to draw the lines which provided a clear diagram to support other questions.

Limited responses

- Some candidates only extended the tree diagram from one of the Theory Pass branches.
- Some candidates included branches after the driving qualification had been failed.
- A few candidates did not label the branches fully, with the omission of Pass and Fail the most common error.
- Some candidates only allowed one attempt at the skills test.

(b) **Comprehensive responses**

- Almost all candidates stated the required calculation in the appropriate order to show clearly the required statement.

Limited responses

- A few candidates did not identify the probabilities used when not in the order that could be obtained from the tree diagram.
- A few candidates added rather than multiplied the fractions, and stated that 0.112 was the answer. It is important to use the correct method and show all working, as this demonstrates understanding.

(c) **Comprehensive responses**

- Most candidates provided clear calculations in a systematic order.
- Some candidates used the tree diagram in **5(a)** to identify the possible scenarios required.
- Many candidates calculated all the probabilities for obtaining the driving qualification.
- A few candidates used the more efficient 1 – P (does not qualify) with clear supporting calculations.

Limited responses

- Some candidates did not evaluate accurately their correct unsimplified calculation.
- Some candidates omitted one or more possible scenario for obtaining the driving qualification.

(d) **Comprehensive responses**

- Many candidates used values that had been calculated in **5(c)** appropriately to at least 4 significant figures.
- Most candidates stated a clear unsimplified conditional probability formula before evaluation.
- Some candidates calculated the values required for the numerator and denominator separately and then substituted into the conditional probability formula.

Limited responses

- Many candidates did not use a denominator of at least 4 significant figures in the conditional probability formula, substituting the 3 significant figure answer from **5(c)** which is not sufficient to justify an answer to 3 significant figures.
- Some candidates only calculated P (passes theory test on first attempt and obtained the advance driving qualification) and did not attempt to calculate the conditional probability.

Question 6

(a) **Comprehensive responses**

- Most candidates stated the normal standardisation formula with the correct values substituted.
- Most candidates showed the numerical calculation required to obtain the correct probability area.
- Most candidates stated a 4 significant probability for the success criteria.
- Many candidates showed the calculation required for the expected number and interpreted the decimal answer as a single integer value.

Limited responses

- Some candidates partially evaluated or omitted the normal standardisation formula.
- A few candidates did not provide a numerical calculation for the probability area.
- Many candidates did not attempt to find the expected number of bags with a mass less than 2.32 kg and simply stated the probability.
- Many candidates indicated that the answer was obtained by rounding or truncating their decimal value rather than interpreting within context.

(b) **Comprehensive responses**

- Most candidates formed 2 appropriate normal standardisation equations using μ and σ and z-values.
- Many candidates presented a clear algebraic solution of the simultaneous equations formed.
- Most candidates stated μ and σ to at least 3 significant figures.
- Some candidates included a simple normal distribution diagram to identify the probability regions given.

Limited responses

- Some candidates equated the normal standardisation formula to one of the probabilities given in the equation.
- Many candidates used the normal tables inappropriately to obtain an incorrect z-value, for example 0.80 to obtain 0.7881, which is a probability.
- A few candidates misinterpreted the probability area for 40% and equated to 0.253 and not –0.253.
- Some candidates found only one of μ and σ.

(c) **Comprehensive responses**

- Almost all candidates clearly stated the unsimplified binomial approximation expression for the required success criteria.
- Many candidates included the values for each term, which is not anticipated at this level.
- Most candidates stated the probability to at least 3 significant figures.

Limited responses

- Some candidates misinterpreted the success criteria and found P (7, 8).
- Some candidates omitted the binomial coefficient in each term.
- A few candidates attempted to use the normal approximation for a binomial distribution, although the conditions required were not met.
- A few candidates attempted to use the geometric approximation, which was inappropriate for the context.

MATHEMATICS

Paper 9709/62
Probability & Statistics 2

Key messages

What candidates did well
- Many candidates showed good mathematical knowledge across a wide range of topics.
- Mathematical notation was generally correctly used.
- Solutions were generally well presented.

What candidates needed to improve
- Candidates should read questions carefully and extract the relevant information and then reread the question when they have completed their solution to ensure that all the requirements have been fulfilled.
- Candidates should show sufficient method to justify answers, particularly on questions that require a given answer to be shown.
- Candidates need to keep to at least 4sf accuracy during working in order to ensure a final answer is accurate to 3sf.
- Errors should be crossed out rather than being written over.

Comments on specific questions

Question 1

(a) **Comprehensive responses**
- Many candidates successfully calculated unbiased estimates of the population mean and variance.

Limited responses
- Some candidates confused the two formulae for the unbiased estimate of the variance.
- A few candidates found the biased variance rather than the unbiased variance.

(b) **Comprehensive responses**
- Many candidates used a correct formula for the confidence interval using a correct value for z.
- The question requested a necessary assumption to be made; most candidates gave either an incorrect response or no response at all.

Limited responses
- Some candidates used an incorrect value for z.
- Some used the estimate for the variance calculated in part a rather than the true value now given.
- Some candidates did not give an answer to one decimal place as requested in the question.

Question 2

(a) **Comprehensive responses**
- Most candidates were able to give at least one correct reason.

Limited responses
- Many candidates did not give two different reasons.
- Some gave limited explanations, for example just stating that the choice was not representative without explaining why.

(b) Comprehensive responses

- Some candidates realised that the standard deviation should be unchanged.

Limited responses

- Many candidates gave incorrect assumptions.
- Some stated the hypotheses for the test or considered conditions for a random sample again.

(c) Comprehensive responses

- Many candidates successfully carried out the test.
- Hypotheses were generally correct.
- Many candidates clearly stated a comparison of z values (or areas) and a conclusion.

Limited responses

- Not all candidates gave their conclusions in context with the required amount of uncertainty in the language used.
- Some candidates gave invalid comparisons for example comparing a z value with an area.
- On occasions, a one-tail test was followed.

Question 3

(a) (i) Comprehensive responses

- The majority of candidates successfully found the probability of selling a total of 5 cell phones.

Limited responses

- A minority incorrectly calculated the probability of less than 5 cell phones.
- On occasions candidates used an incorrect value for λ.

(ii) Comprehensive responses

- Many candidates attempted to find P (5, 0) + P (0, 5).

Limited responses

- Some candidates only found P (5, 0).
- Some candidates only found P (5) +P (5).
- A few candidates did not keep to 3sf accuracy in their final answer (possibly due to confusion between 3sf and 3dp).

(b) (i) Comprehensive responses

- N (36.6, 36.6) was used by most candidates.
- Standardising was generally well attempted.

Limited responses

- Errors when standardising included an incorrect or no continuity correction.
- Some candidates found the wrong probability area.

(ii) Comprehensive responses

- Many candidates stated λ was greater than 15; this needed to be put in context by saying that this was true here because λ was 36.6. Not all candidates did this.

Limited responses

- Common incorrect answers were $\lambda > 5$ and $n \geq 30$.

(c) Comprehensive responses

- Many candidates gave a correct initial expression for P (X = 0) = 2P (Y = 1)

Limited responses

- Some candidates were unable to apply the laws of logs correctly to find the required expression.

Question 4

Comprehensive responses
- Candidates attempted to find E(profit) and Var(profit).
- Standardising with their values was generally done well.

Limited responses
- When calculating var(profit) (= $0.15^2 \times 50 \times 0.32^2$) various errors were made by candidates not correctly appreciating which terms were squared and which were not.
- Some candidates found the wrong probability area.

Question 5

(a) **Comprehensive responses**
- Many candidates used a correct method to show that var(X) was 35/12
- Sufficient working was shown by most candidates.

Limited responses
- Some candidates did not know how to find var(X) and used an incorrect formula.
- A few candidates used non-exact decimal values in their solutions and therefore did not show 35/12 exactly.

(b) **Comprehensive responses**
- Most candidates used a normal distribution and attempted to standardise.

Limited responses
- Some candidates omitted $\sqrt{50}$ in their standardising equation.
- Some candidates confused variance and standard deviation.
- Some candidates found the wrong probability area.
- A small number of candidates did not know how to proceed to find a Type II error here.

(c) **Comprehensive responses**
- Many candidates used the context of the question in their answer.

Limited responses
- Some candidates used an incorrect definition of a Type II error.
- Some candidates did not state clearly that the dice was not actually fair if a Type II error had been made.
- A few candidates did not use the context of the question and just stated the definition of a Type II error without applying it to the question.

(d) **Comprehensive responses**
- Most candidates used the information given along with the definition of a Type II error.

Limited responses
- Some candidates applied the definition and correctly stated that the null hypothesis would be rejected.

Question 6

(a) **Comprehensive responses**
- Most candidates correctly and convincingly showed that k was equal to $6/a^3$.

Limited responses
- There were occasions when candidates did not give a fully accurate solution, for example omitting brackets when substituting limits.

(b) **Comprehensive responses**
- Many candidates were able to state the correct value of E(X).

Limited responses

- Some candidates did not use the symmetry of the function to find E(X).
- Some candidates just stated how to find E(X) by integration.

(c) (i) Comprehensive responses

- Many candidates attempted to integrate f(x) with correct limits.

Limited responses

- Numerical errors were commonly made when limits were substituted.
- There were occasional errors made when integrating.
- Some candidates set their integral equal to 0.25 or sometimes 1.

(ii) Comprehensive responses

- Many candidates used the symmetry of the function and their previous answer to find the required probability.

Limited responses

- Some candidates did not follow the instruction 'use your answer to **6(c)(i)**' as given in the question and found the probability by integrating between $\frac{1}{4}$ a and $\frac{3}{4}$ a.
- Some candidates integrated between 0 and $\frac{3}{4}$ a then used their answer to the previous part to find the required probability.
- A few candidates did not know how to approach this question.