Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2026**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2026 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

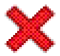
Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
^	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
FT	Follow through
〰	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
J	Judgement
JU	Judgement
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
MR	Misread
O	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
PE	Judgment made by the PE
Pre	Premature approximation
SC	Special case
SEEN	Indicates that work/page has been seen

Annotation	**Meaning**
SF	Error in number of significant figures
✓	Correct
TE	Transcription error
XP	Correct answer from incorrect working

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

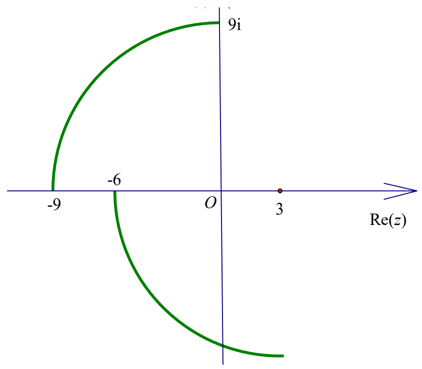
FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Form a quadratic equation in 2^x	**M1***	Need not have all terms on one side. Allow 2^{2x} but not $2^x \times 2^x$.
	Obtain $4 \times 2^2 \times \left(2^x\right)^2 - 3 \times 2^x - 5 \times 4 = 0$ Needs $(2^x)^2$ but may be implied if progressing to solve the quadratic	**A1**	Or three-term equivalent, e.g. $\left(2^{x+4}\right)^2 - 3 \times 2^{x+4} - 320 = 0$. Terms on LHS can be in any order, e.g. $16 \times \left(2^x\right)^2 - 3 \times 2^x - 20 = 0$.
	Solve for x (do not need to solve quadratic correctly for this mark) Allow, e.g., $x\ln 2 = \ln\ldots$ to score M1 at this stage	**DM1**	Requires correct method for solving $2^x = k$ $\left(2^x = 1.2157\ldots\right)$. Need to see some working, such as $x\ln 2 = \ln 1.2157$ or $\log_2(1.2157)$.
	Obtain $x = 0.282$ only final answer	**A1**	No working at all scores **M0*A0M0A0**. From a quadratic equation directly to correct answer scores **M1*A1M0A0**. From solution to quadratic to correct answer **M1*A1M0A0,** then **SCB1**.
		4	

Question	Answer	Marks	Guidance
2(a)	Show an arc (may be complete circle) centre the origin and radius 9. Some indication of the correct radius required.	**B1**	
	Show an arc centre the origin for $\frac{1}{2}\pi \leq \arg z \leq \pi$	**B1**	
		2	
2(b)	Show a quarter circle or arc (may be complete circle) centre 3 on real axis and radius 9 FT *their* arc from **2(a)**	**B1FT**	
	Show the reflection of *their* **2(a)** in the real axis, provided arc (**not** a complete circle) centre the origin in **2(a)**	**B1FT**	Shading not allowed for full marks in **2(a)** or **2(b)**.
		2	

Question	Answer	Marks	Guidance
3	Use correct double angle formula	M1*	$\int 3\sin x \sin 2x \,dx = \int 6\sin^2 x \cos x \,dx$
	Integrate to obtain $k\sin^3 x\,(+C)$ [$k = 2$]	DM1	Constant of integration not required.
	$k\left(\frac{\sqrt{3}}{2}\right)^3 - k\left(\frac{1}{\sqrt{2}}\right)^3$, e.g. $\frac{3}{4}\sqrt{3} - \frac{1}{\sqrt{2}}$	DM1	Use exact values for correct limits correctly in expression of the correct form. May be implied by correct answer after $2\sin^3 x$
	Obtain $\frac{3}{4}\sqrt{3} - \frac{1}{2}\sqrt{2}$	A1	OE ISW Must be of the form $p\sqrt{3} + q\sqrt{2}$.
	Alternative Method for Question 3		
	$I = \int 3\sin x \sin 2x \,dx$ $= -3\cos x \sin 2x + \int 6\cos x \cos 2x \,dx$ or $= -\frac{3}{2}\sin x \cos 2x + \frac{3}{2}\int \cos x \cos 2x \,dx$	M1*	Apply integration by parts once. Condone errors in the coefficients.
	$= -3\cos x \sin 2x + 6\sin x \cos 2x + \int 12\sin x \sin 2x \,dx$ $= -3\cos x \sin 2x + 6\sin x \cos 2x + 4I$ or $= -\frac{3}{2}\sin x \cos 2x + \frac{3}{4}\cos x \sin 2x + \frac{3}{4}\int \sin x \sin 2x \,dx$ $= -\frac{3}{2}\sin x \cos 2x + \frac{3}{4}\cos x \sin 2x + \frac{1}{4}I$	DM1	Apply integration by parts a second time in the "same direction". Condone errors in the coefficients. Must get to $I = \ldots + aI$.
	$\mathrm{I} = \cos x \sin 2x - 2\sin x \cos 2x$ $= \frac{1}{2} \times \frac{\sqrt{3}}{2} + 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{\sqrt{2}}{2} \times 1$, e.g. $\frac{3}{4}\sqrt{3} - \frac{1}{\sqrt{2}}$.	DM1	Simplify and use exact values for correct limits correctly in expression of the correct form. May be implied by correct answer after correct integral seen.
	Obtain $\frac{3}{4}\sqrt{3} - \frac{1}{2}\sqrt{2}$	A1	OE ISW Must be of the form $p\sqrt{3} + q\sqrt{2}$.
		4	

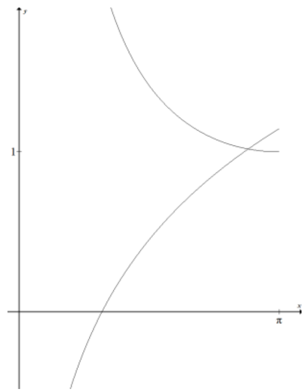
Question	Answer	Marks	Guidance
4(a)	Coefficient of $x^3 = \dfrac{\frac{2}{5}\times\frac{-3}{5}\times\frac{-8}{5}}{3!}(-a)^3$	**B1**	SOI Not, e.g., $\left(\dfrac{2}{5}-1\right)$ for $-\dfrac{3}{5}$.
	Obtain $a=-\dfrac{5}{2}$ only	**B1**	$\left(a^3=-\frac{125}{8}\right)$
		2	
4(b)	Coefficient of $x^4 = \dfrac{\frac{2}{5}\times\frac{-3}{5}\times\frac{-8}{5}\times\frac{-13}{5}}{4!}(-a)^4$ or $\dfrac{\frac{2}{5}\times\frac{-3}{5}\times\frac{-8}{5}\times\frac{-13}{5}\times\left(\frac{5}{2}\right)^4}{4!}=-\dfrac{26}{16}=-\dfrac{13}{8}$	**B1FT**	SOI Not, e.g., $\left(\dfrac{2}{5}-1\right)$ for $-\dfrac{3}{5}$. Follow through $-\dfrac{26}{625}$ (–*their a*)^4.
	Correct method for coefficient of x^4 in the product	**M1**	$2\times1-\frac{26}{625}a^4$ May see $(1)\left(\dfrac{\frac{-13}{5}}{4}\right)\left(\dfrac{5}{2}\right)x^4$, following that x^3 has a coefficient of 1 $\left[1=\dfrac{\frac{2}{5}\times\frac{-3}{5}\times\frac{-8}{5}}{3!}(-a)^3\right]$.
	Obtain $\frac{3}{8}\left(x^4\right)$	**A1**	Accept even if $a=+\dfrac{5}{2}$ in **4(a)**.
		3	
4(c)	$\lvert x\rvert<\left\lvert\dfrac{1}{\textit{their } a}\right\rvert$ must be numerical, strict FT for *their a*	**B1FT**	OE Follow through *their a*. $\lvert x\rvert<0.4$. Allow, e.g. $\lvert x\rvert<\dfrac{1}{2.5}$.
		1	

Question	Answer	Marks	Guidance
5(a)	Multiply numerator and denominator by $\lambda - 2\mathrm{i}$	***M1**	Must attempt both.
	Use $\mathrm{i}^2 = -1$ $\left(z = \dfrac{5\lambda + \mathrm{i}\left(\lambda^2 - 6\right)}{\lambda^2 + 4}\right)$	**DM1**	At least once.
	Use, e.g., $\arg z = \frac{1}{4}\pi$ and $y/x = 1$, or Re = Im to obtain three-term quadratic in λ only	**M1**	E.g. $\left(5\lambda = \lambda^2 - 6\right)$.
	Obtain $\lambda = 6$ only	**A1**	CWO After ***M1 DM1 M0**, allow **SCB1** for correct answer if three-term quadratic not seen but a correct equation in λ seen.
	Alternative Method for Question 5(a)		
	Equate z to $k\left(1 + \mathrm{i}\right)$	***M1**	$\dfrac{3 + \lambda\mathrm{i}}{\lambda + 2\mathrm{i}} = k\left(1 + \mathrm{i}\right)$ Must include k.
	Compare real and imaginary parts	**DM1**	E.g. $3 = k\left(\lambda - 2\right)$, $\lambda = k\left(\lambda + 2\right)$.
	Form a three-term quadratic in λ only	**M1**	E.g. $\lambda^2 - 5\lambda = 6$.
	Obtain $\lambda = 6$ only	**A1**	CWO
		4	

Question	Answer	Marks	Guidance
5(b)	Use correct method for modulus e.g. $\left\|\dfrac{30+30\text{i}}{40}\right\| = \sqrt{\left(\dfrac{3}{4}\right)^2 + \left(\dfrac{3}{4}\right)^2}$ with *their* λ	**M1**	Allow, e.g., $\dfrac{\|3+\lambda i\|}{\|\lambda+2i\|} = \dfrac{\sqrt{9+\lambda^2}}{\sqrt{\lambda^2+4}}$ with λ not substituted.
	Obtain $\dfrac{3\sqrt{2}}{4}$	**A1**	ISW Or simplified exact equivalent, e.g. $\dfrac{3}{2\sqrt{2}}$. No marks available if no method seen.
		2	

Question	Answer	Marks	Guidance
6(a)	Complete division by x^2+x+1 as far as quotient $=2x^2+kx+...$ $\begin{array}{r\|rrrrr} & & & 2x^2 & +(a-2)x & -3 \\ \hline x^2+x+1 & 2x^4 & ax^3 & 4x^2 & bx & -3 \\ & 2x^4 & 2x^3 & 2x^2 & & \\ & & (a-2)x^3 & 2x^2 & & \\ & & (a-2)x^3 & (a-2)x^2 & (a-2)x & \\ & & & (4-a)x^2 & (b-a+2)x & -3 \\ & & & -3x^2 & -3x & -3 \end{array}$	**M1**	Note: The –3 is obtained by comparing the constant terms. Alternative by completing the division gives the quotient as $2x^2+(a-2)x+(4-a)$.
	Equate (part of the) remainder to 0 and solve for a or b	**M1**	
	Obtain $a=7$	**A1**	E.g. from $4-a=-3$.
	Obtain $b=2$	**A1**	E.g. from $b-a+2=4-a$.
	Alternative Method for Question 6(a)		
	State $\mathrm{p}(x)=(x^2+x+1)(2x^2+px-3)$	**B1**	By inspection.
	Compare coefficients to obtain a or b	**M1**	$2-3+p=4,\ a=2+p,\ b=-3+p$
	Obtain $a=7$	**A1**	
	Obtain $b=2$	**A1**	
		4	

Question	Answer	Marks	Guidance
6(b)	Obtain or substitute $x = -3$ into *their* quotient or into original expression	**M1**	Allow with *a* and *b* non-numerical.
	Must evaluate $(-3)^4$ etc. when substituting, and reach = 0 Conclude that $x+3$ is a factor Statement needed either initially or finally	**A1**	AG CWO Need not evaluate, e.g. 2 × 81.
	Alternative Method for Question 6(b)		
	Factorise correct quotient to obtain $(2x-1)(x+3)$	**M1**	Implied by $(x^2+x+1)(2x-1)(x+3)$.
	Obtain $(x^2+x+1)(2x-1)(x+3)$, or 'therefore, $(x + 3)$ is a factor of $\mathrm{p}(x)$'	**A1**	AG CWO
		2	

Question	Answer	Marks	Guidance
7(a)	Sketch a relevant graph e.g. $y = \operatorname{cosec}\frac{1}{2}x$ cosec graph needs $y = 1$ marked on y-axis and graph appears to reach $(\pi, 1)$. Graph starts reasonably near y-axis. ln graph must start below x-axis, near to negative y-axis, and have positive gradient everywhere, not going horizontal. Either $x = 1$ marked or if scale in π on x-axis must go through what appears to be $x = 1$	**B1**	
	Sketch a second relevant graph and justify the given statement	**B1**	**SC B1:** No values on either axis but both graphs appear in correct positions and intersection point marked.
		2	

Question	Answer	Marks	Guidance
7(b)	Calculate the values of a relevant pair of expression or the value of a relevant expression at $x = 2.6$ and $x = 2.9$. Must be using radians. Values correct to at least 2 s.f. rounded or truncated. Allow use of a smaller interval For first method, must attempt all values and have at least 3 out of 4 values correct when rounded or truncated For second or third method, must attempt both values and have at least 1 out of 2 values correct when rounded or truncated	**M1**	E.g. comparing $\ln x$ and $\operatorname{cosec}\left(\frac{1}{2}x\right)$: at 2.6: 0.96 < 1.03 at 2.9: 1.06 > 1.01. If using $f(x) = \ln x - \operatorname{cosec}\left(\frac{1}{2}x\right)$: $f(2.6) = -0.0823...$ $f(2.9) = 0.05737...$ If using $g(x) = e^{\operatorname{cosec}\left(\frac{1}{2}x\right)} - x$: $g(2.6) = 0.223...$ $g(2.9) = -0.162...$
	Complete the argument correctly with correct calculated values	**A1**	If comparing with zero can either indicate different signs or a negative product. Clear comparison for *their* expression; **SC B1** if only 1sf rounded or truncated for second case only.
		2	

Question	Answer	Marks	Guidance
7(c)	Use the iterative process correctly at least once	**M1**	Must be working in radians.
	Obtain final answer 2.767	**A1**	Must not have iterative term = 2.767; do not allow, e.g., $x_{n+1} = 2.767$.
	Show sufficient iterations to at least 5 decimal places to justify 2.767 to 3 decimal places, or show that there is a sign change in the interval $(2.7665, 2.7675)$. 2.60, 2.82306, 2.75335, 2.77082, 2.76609, 2.76734, 2.76701… 2.65, 2.80383, 2.75780, 2.76959, 2.76641, 2.76726, 2.76703… 2.70, 2.78676, 2.76198, 2.76845, 2.76671, 2.76718… 2.75, 2.77175, 2.76584, 2.76741, 2.76699… 2.80, 2.75872, 2.76934, 2.76648, 2.76724, 2.76704… 2.85, 2.74759, 2.77243, 2.76566, 2.76746, 2.76698… 2.90, 2.73831, 2.77508, 2.76497, 2.76764, 2.76693, 2.76712…	**A1**	Allow truncation. Allow recovery.
		3	

Question	Answer	Marks	Guidance
8	Separate variables correctly: $\int xe^{-3x}\,dx = \int \frac{y}{y+5}\,dy$	**B1**	OE Condone missing dx and dy or missing integral signs, but not both. Accept $\int \frac{x}{e^{3x}}\,dx$ in place of $\int xe^{-3x}\,dx$.
	Use integration by parts to obtain $axe^{-3x} - b\int e^{-3x}\,dx$	**M1**	b may be positive or negative.
	Obtain $-\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}\,dx$	**A1**	
	Complete integration by parts to obtain $-\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x}$	**A1**	Condone absence of constant of integration.
	Correctly split the fraction in y and integrate with one term correct	**M1**	$\int\left(1-\frac{5}{y+5}\right)dy$
	Obtain $y - 5\ln(y+5)$	**A1**	
	Use $x=0$, $y=0$ to evaluate the constant of integration or as limits in definite integrals, in an expression containing pxe^{-3x}, qe^{-3x}, ry and $s\ln(y+5)$	**M1**	
	Obtain $-\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} = y + 5\ln\frac{5}{y+5} - \frac{1}{9}$	**A1**	OE ISW Allow 5ln5 – 5ln(y +5) for $5\ln\frac{5}{y+5}$, and allow 46/9 – 5 for $\frac{1}{9}$.
		8	

Question	Answer	Marks	Guidance
9(a)	State or imply $\dfrac{\mathrm{d}x}{\mathrm{d}u} = \sqrt{3}\sec^2 u$	**B1**	E.g. $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{\sqrt{3}}{3+x^2}$
	$x=1 \Rightarrow \tan u = \frac{1}{\sqrt{3}} \Rightarrow u = \frac{1}{6}\pi$ and $x=3 \Rightarrow \tan u = \sqrt{3} \Rightarrow u = \frac{1}{3}\pi$ or better for **both** limits	**B1**	Need to see evidence to justify the change of limits, e.g. $u = \tan^{-1}\dfrac{x}{\sqrt{3}}$.
	Substitute correctly and fully for *their* $\dfrac{\mathrm{d}x}{\mathrm{d}u}$ to obtain an integral in u Need at least one of the integral sign and du	**M1**	$\displaystyle\int \frac{\left(\sqrt{3}\tan u\right)^3 \sqrt{3}\sec^2 u}{3+3\tan^2 u}\,\mathrm{d}u$
	Obtain $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 3\tan^3 u\,\mathrm{d}u$ from full and correct working	**A1**	**AG** **SC** Allow if only **one** limit shown with enough working (so may score **B1B0M1A1**).
		4	

Question	Answer	Marks	Guidance
9(b)	$\int 3\tan^3 u\mathrm{d}u = \int 3\tan u\left(a\sec^2 u - b\right)\mathrm{d}u$	**B1**	Need $a = 1$ and $b = 1$.
	Obtain $\frac{3}{2}a\tan^2 u\ \ 3b\ln\cos u$	**B1FT**	a and b non-zero.
	Obtain $\frac{3}{2}a\tan^2 u$ and $3b\ln\cos u$	**B1FT**	$\frac{3}{2}\tan^2 u\ +\ 3\ln\cos u$ if correct a and b (non-zero).
	Use correct limits correctly in $c\tan^2 u + d\ln(\cos u)$	**M1**	$\frac{3}{2}\left(3-\frac{1}{3}\right)+3\left(\ln\frac{1}{2}-\ln\frac{\sqrt{3}}{2}\right)$
	Obtain $4-\frac{3}{2}\ln 3$	**A1**	OE of valid form, e.g. $4-\frac{1}{2}\ln 27$. Must be of the form $p + q\ln r$, where p, q and r are rational.
	Alternative Method for Question 9(b)		
	$\int 3\tan^3 u du = \int 3\tan u\left(a\sec^2 u - b\right)du = \int\left(3a\tan u\sec u(\sec u) - 3b\tan u\right)\mathrm{d}u$	**B1**	Need $a = 1$ and $b = 1$.
	Obtain $\frac{3}{2}a\sec^2 u$ or $3b\ln\cos u$	**B1FT**	a and b non-zero.
	Obtain $\frac{3}{2}a\sec^2 u$ and $3b\ln\cos u$	**B1FT**	$\frac{3}{2}\sec^2 u\ +3\ln\cos u$ if correct a and b (non-zero).
	Use correct limits correctly in $c\sec^2 u + d\ln(\cos u)$	**M1**	$\frac{3}{2}\left(4-\frac{4}{3}\right)+3\left(\ln\frac{1}{2}-\ln\frac{\sqrt{3}}{2}\right)$
	Obtain $4-\frac{3}{2}\ln 3$	**A1**	OE of valid form, e.g. $4-\frac{1}{2}\ln 27$. Must be of the form $p + q\ln r$, where p, q and r are rational.

Question	Answer	Marks	Guidance
9(b)	**Alternative Method 2 for Question 9(b)**		
	$\int \frac{x^3}{3+x^2}\,\mathrm{d}x = \int ax - \frac{3bx}{3+x^2}\,\mathrm{d}x$	**B1**	Need $a = 1$ and $b = 1$.
	Obtain $\frac{a}{2}x^2$ or $-\frac{3}{2}b\ln\left(3+x^2\right)$	**B1FT**	a and b non-zero.
	Obtain $\frac{a}{2}x^2$ and $-\frac{3}{2}b\ln\left(3+x^2\right)$	**B1FT**	$\frac{1}{2}x^2 \ -\frac{3}{2}\ln\left(3+x^2\right)$ if correct a and b (non-zero).
	Use correct limits correctly in $cx^2 + d\ln(3+x^2)$	**M1**	$\frac{1}{2}(9-1)-\frac{3}{2}(\ln 12-\ln 4)$
	Obtain $4-\frac{3}{2}\ln 3$	**A1**	OE of valid form, e.g. $4-\frac{1}{2}\ln 27$. Must be of the form $p + q\ln r$, where p, q and r are rational.
		5	

Question	Answer	Marks	Guidance
10(a)	Use correct quotient rule or correct product rule to differentiate $\dfrac{x-2}{x+2}$ or *their* remaining expression	**M1**	
	Obtain $\dfrac{(x+2)-(x-2)}{(x+2)^2}$ or their remaining expression differentiated correctly	**A1**	OE, e.g. $2y\dfrac{dy}{dx}=\dfrac{4k}{(x+2)^2}$ or $\dfrac{k-y^2}{x+2}$.
	State or imply that $\dfrac{d}{dx}y^2=2y\dfrac{dy}{dx}$ Or, if rearrange to make y the subject: correct expression for $\dfrac{dy}{dx}$ following correct chain rule and correct quotient rule	**B1**	
	Eliminate k to obtain $\dfrac{dy}{dx}$ in terms of x and y $\left[k=\dfrac{(x+2)y^2}{x-2}\right]$	**M1**	$\dfrac{dy}{dx}=\dfrac{2y^2(x+2)}{(x-2)(x+2)^2y}$
	Obtain $\dfrac{dy}{dx}=\dfrac{2y}{x^2-4}$ from full and correct working	**A1**	**AG** Working backwards and guessing k scores 0/5. Using square root scores max 4/5, since cannot clarify chosen sign.
		5	

Question	Answer	Marks	Guidance
10(b)	$x = 3,\ y = \pm 1$	**B1**	Allow $y = \pm\sqrt{1}$.
	$\Rightarrow$ gradients of tangents are $\pm\frac{2}{5}$	**B1**	Allow B1 for one y-value and the corresponding gradient.
	Use of $\tan(A - B)$ to find the angle between the two lines	**M1**	OE, e.g. $[\tan\theta =]\ \dfrac{\frac{2}{5} - \left(-\frac{2}{5}\right)}{1 + \frac{2}{5} \times \left(-\frac{2}{5}\right)}$.
	Obtain $(\theta =) \tan^{-1}\dfrac{20}{21}$	**A1**	Accept $\tan^{-1}\left(\pm\dfrac{20}{21}\right)$. Must be of the form $a\tan^{-1}\left(\dfrac{b}{c}\right)$ with a, b and c integers.
	Alternative Method for Question 10(b)		
	$x = 3,\ y = \pm 1$	**B1**	Allow $y = \pm\sqrt{1}$.
	$\Rightarrow$ gradients of tangents are $\pm\frac{2}{5}$	**B1**	Allow B1 for one y-value and the corresponding gradient.
	Use of right-angled triangle to find $\tan\left(\dfrac{\theta}{2}\right)$	**M1**	$\tan\left(\dfrac{\theta}{2}\right) = \dfrac{2}{5}$
	Obtain $(\theta =)\ 2\tan^{-1}\dfrac{2}{5}$	**A1**	Must be of the form $a\tan^{-1}\left(\dfrac{b}{c}\right)$ with a,b and c integers.
		4	

Question	Answer	Marks	Guidance
11(a)	Find $\overrightarrow{AP}$ for a general point P on OB	**B1**	$\overrightarrow{AP} = \begin{pmatrix} -2+4\lambda \\ -2+2\lambda \\ 1+4\lambda \end{pmatrix}$ Allow for use of $\overrightarrow{PA}$ or $\overrightarrow{AP} = \begin{pmatrix} -2+4+4 \\ -2+2+2 \\ 1+4+4 \end{pmatrix}$.
	Equate the scalar product of *their* $\overrightarrow{AP}$ and $\overrightarrow{OB}$ to zero and obtain an equation in λ or α	**M1**	E.g. $4(-2+4\lambda)+2(-2+2\lambda)+4(1+4\lambda)=0$.
	Solve and obtain $\lambda = \frac{2}{9}$ or $\alpha = -\dfrac{7}{9}$	**A1**	
	Use correct method to calculate $\lvert their\ \overrightarrow{AP}\rvert$ where *their* $\overrightarrow{AP}$ is found from use of a scalar product	**M1**	E.g. $\sqrt{\left(\frac{-10}{9}\right)^2+\left(\frac{-14}{9}\right)^2+\left(\frac{17}{9}\right)^2}$.
	Obtain $\dfrac{1}{3}\sqrt{65}$ from full and correct working	**A1**	**AG** Allow, e.g., $\sqrt{\left(\frac{-10}{9}\right)^2+\left(\frac{-14}{9}\right)^2+\left(\frac{17}{9}\right)^2}$ leading to $\dfrac{1}{3}\sqrt{65}$.

Question	Answer	Marks	Guidance
11(a)	**Alternative Method for Question 11(a)**		
	Correct method for the scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OB}$	**M1**	$8+4-4=8$
	Correct use of the scalar product to obtain $\cos\angle AOB$	**M1**	$\cos\angle AOB=\dfrac{8}{3\times 6}$
	Obtain $\cos\angle AOB=\dfrac{4}{9}$	**A1**	
	Use $d=\left\|\overrightarrow{OA}\right\|\sin\angle AOB$ where *their* $\overrightarrow{OA}$ is found from use of a scalar product	**M1**	$3\sqrt{1-\frac{16}{81}}$
	Obtain $\dfrac{1}{3}\sqrt{65}$ from full and correct working	**A1**	**AG**
	Alternative Method 2 for Question 11(a)		
	Let $\overrightarrow{AP}$ be the perpendicular Correct method for the scalar product of $\overrightarrow{BO}$ and $\overrightarrow{BA}$	**M1**	$4\times 2+4\times 5=28$
	Correct use of the scalar product to obtain $\cos\angle OBA$	**M1**	$\cos\angle OBA=\dfrac{28}{\sqrt{29}\times 6}$
	Obtain $\overrightarrow{BP}=\dfrac{28}{6}=\dfrac{14}{3}$	**A1**	
	Obtain $d=\sqrt{(AB)^2-(BP)^2}$, where *their* $\overrightarrow{BP}$ is found from use of a scalar product	**M1**	$\sqrt{29-\left(\frac{14}{3}\right)^2}$ if correct.
	Obtain $\dfrac{1}{3}\sqrt{65}$ from full and correct working	**A1**	**AG**
		5	

Question	Answer	Marks	Guidance
11(b)	$\overrightarrow{AB} = 2\mathbf{i} + 5\mathbf{k}$	**B1**	SOI (4, 2, 4) – (2, 2, − 1).
	Use the correct method to calculate the scalar product of $\overrightarrow{OC} = 3\mathbf{i} + p\mathbf{j} + q\boldsymbol{k}$ and *their* $\overrightarrow{AB} = 2\mathbf{i} + 5\mathbf{k}$	**M1**	Use of $\overrightarrow{OA}$ or $\overrightarrow{OB}$ perpendicular to $\overrightarrow{OC}$ gives "correct" values for p and q but is an incorrect method.
	Equate the scalar product to zero and obtain $q = -\frac{6}{5}$	**A1**	
	Use the correct method to find $\cos AOC$ or $\cos COB$ May have slips in scalar product	**M1**	May be implicit. $\sqrt{9 + p^2 + q^2}$ may not be shown, but if seen, allow one slip.
	Equate the two cosines to obtain an equation in p and q or p and *their* q having seen the scalar product attempted	**M1**	Allow accuracy errors in up to two of the four moduli.
	Obtain $\dfrac{6 + 2p - q}{\sqrt{9}\sqrt{9 + p^2 + q^2}} = \dfrac{12 + 2p + 4q}{\sqrt{36}\sqrt{9 + p^2 + q^2}}$	**A1**	OE $\sqrt{9 + p^2 + q^2}$ may not be shown, but if seen, must be correct.
	Obtain $p = -\frac{18}{5}$	**A1**	CWO
		7	

