

# MATHEMATICS

Paper 9709/11

Paper 11

## General comments

While presentation of work for many candidates was satisfactory there were significant numbers of candidates whose work was untidy and difficult to follow. The rubric on the front of the Question Paper says 'You are reminded of the need for clear presentation in your answers'. It is generally true that marks cannot be awarded if methods used are not clear.

It is worth noting here a very common error: forgetting the  $\pm$  sign. For example, in **Question 8(ii)**,  $p^2 = 4 \Rightarrow p = \pm 2$ . Mistakes of this kind were also seen in **Questions 2(ii), 7, 9(i)**.

## Comments on specific questions

### Question 1

This question was rather poorly attempted. Significant numbers of candidates made no attempt at it and some of the attempts that were made seem to have been the result of guesswork. It was expected that candidates would note that the curve has been translated in the positive  $y$  direction by 1 unit. Hence  $a = 1$ . Also that the difference between maximum and minimum values is 4 units compared with the standard sine function in which the difference is 2 units. Hence  $b = 2$ .

Answers:  $a = 1$ ,  $b = 2$ .

### Question 2

In part (i), candidates experienced difficulty in dealing with the  $4x^2$  term and with the required form of the answer. A significant number of candidates kept 4 outside the bracket. Very few candidates thought of putting the two expressions identically equal to each other and equating coefficients, which is an alternative way of dealing with such questions. There were two ways of approaching part (ii). Candidates could have used their answer to part (i), taking their value of  $b$  to the other side of the equation to obtain  $(2x - 3)^2 > 16$ . Taking the square root of both sides gives  $2x - 3 > 4$  or  $2x - 3 < -4$ , from which the solutions come very easily. Alternatively, candidates could have made the given inequality into a quadratic equation, solved the equation giving two critical values and deciding whether to take the region between the two critical values or the region outside the critical values. Candidates who chose the first method often made a mistake when taking the square root and wrote  $2x - 3 > \pm 4$  giving one correct solution and one incorrect solution. Candidates who employed the other method often obtained the correct critical values but did not always manage to proceed correctly to obtain the correct solution to the inequality.

Answers: (i)  $(2x - 3)^2 - 9$ ; (ii)  $x < -\frac{1}{2}$ ,  $x > 3\frac{1}{2}$ .

### Question 3

Although most candidates showed some understanding of the binomial expansion, relatively few reached the answer without making at least one error. The majority of candidates wrote down all of the terms of the expansion and amongst these terms, the term that could simplify to a term independent of  $x$  could usually be found. This term was  ${}^8C_6(4x^3)^2(2x)^{-6}$ . However, 4 was often not raised to the power of 2, and 2 was often not raised to the power  $-6$ . In fact, 2 was sometimes raised to the power  $+6$ . The outcome was that this was a low-scoring question.

Answer: 7.

#### Question 4

Where candidates were able to differentiate correctly full marks were often scored. Some candidates attempted to treat the function as a fraction and use the formula for differentiating a fraction. While this is a valid method it is much easier to write the function as  $4(3x + 1)^{-2}$  and use the chain rule. It is most important that candidates are confident about using the chain rule and that they can do so accurately.

Answer:  $y = 3x + 4$ .

#### Question 5

In part (i), most candidates were able to write down a correct expression for at least one of  $S_{100}$  or  $S_{200}$ . However, candidates seemed to find difficulty in writing down a correct equation linking the two expressions. Often the 4 was on the wrong side of the equation. Even more difficulty was experienced in simplifying the equation and finally solving it. Many candidates did not spot that they could divide throughout by 100 at an early stage and got embroiled in an equation involving large numbers, increasing the likelihood of arithmetic errors. In part (ii), most candidates recognised the need to substitute their answer for  $d$  into the expression  $a + 99d$ .

Answers: (i)  $d = 2a$ ; (ii)  $199a$ .

#### Question 6

In part (i), most candidates were able to find the area of the sector, but often made errors in finding the area of the triangle. In part (ii), the length of the arc  $DE$  was usually found correctly, but once again it was trigonometry which defeated many who were unable to obtain a correct expression for  $AC$ . Some candidates also employed an incorrect strategy for finding the perimeter of the shaded region, thinking that this was equal to the perimeter of the triangle – the perimeter of the sector. Those candidates who had successfully obtained all the correct terms for the perimeter often did not notice that the resulting expression could be simplified as it included a  $+2$  term and a  $-2$  term.

Answers: (i)  $8 \tan \alpha - 2\alpha$ ; (ii)  $4 \tan \alpha + 2\alpha + \frac{4}{\cos \alpha}$ .

#### Question 7

The majority of candidates were able to obtain two correct equations and to realise that they needed to substitute from the linear equation into the quadratic equation. Faulty algebra was often to blame for failure to reach the correct quadratic equation in a single variable. Very few candidates spotted that they could, for example, substitute for  $2 - b$  instead of the more usual  $b$  and this led very simply to  $5(a - 3)^2 = 125$  and then to  $a - 3 = \pm 5$ . Nevertheless, there was a pleasing number of correct solutions.

Answer:  $a = -2$  or  $8$ ,  $b = 12$  or  $-8$ .

#### Question 8

In part (i), candidates were generally familiar with the conditions needed for vectors to be perpendicular and were able to set up a quadratic equation in  $p^2$ . Many candidates solved the equation for  $p^2$  correctly, discarding the solution  $p^2 = -1$  and obtaining  $p^2 = 4$ . Unfortunately many candidates then gave the single solution  $p = 2$ . In part (ii), most candidates found the vector **BA** (or in a significant number of cases, **AB**) but were then unable to find the unit vector in the direction of **BA**.

Answers: (i)  $p = \pm 2$ ; (ii)  $\frac{1}{13} \begin{pmatrix} 12 \\ 5 \\ 0 \end{pmatrix}$ .

### Question 9

Part (i) was not very well done. Because this is a 'proof', candidates are expected to show clearly the intermediate steps. The standard method involves putting both terms over a common denominator, using the identity  $\cos^2 \theta + \sin^2 \theta = 1$ , factorising the numerator and cancelling  $(1 - \cos \theta)$ , and finally using the identity  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ . Algebraic errors were often responsible for disappointing outcomes for this part. In

part (ii), the majority of candidates reached  $\tan^2 \theta = \frac{1}{4}$  but many then wrote ' $\tan \theta = \frac{1}{2}$ ', omitting the negative root and therefore missing the second solution for  $\theta$ .

Answer: (ii)  $26.6^\circ$ ,  $153.4^\circ$ .

### Question 10

Part (i) was not really understood by many candidates. Many did not appreciate that the combined range of the two functions was required, whilst yet more candidates combined the two domains. In part (ii), it would have been of great assistance to candidates if they had drawn the line  $y = x$  (a line joining the two end-points of the curve and the origin) before reflecting the given graph in this line. Very few candidates drew the line and hence the graph of  $y = f^{-1}(x)$  was usually incorrectly drawn. In part (iii), reasonable attempts were made at the two inverse expressions, but their domains were often missing or incorrect.

Answers: (i)  $-5 \leq f(x) \leq 4$ ; (iii) LINE:  $f^{-1}(x) = \frac{1}{3}(x+2)$  for  $-5 \leq x \leq 1$ ,

CURVE:  $f^{-1}(x) = 5 - \frac{4}{x}$  for  $1 < x \leq 4$ .

### Question 11

Many candidates were unsure how to proceed with part (i). The expected method (but not the only one) is to eliminate  $y$  between the two equations to obtain a quadratic equation in  $x$  and then to apply the condition for equal roots ( $b^2 = 4ac$ ). While the marks scored for part (i) were disappointingly low, part (ii) was far more successful. Most candidates found the points of intersection correctly and then found the area under the curve by integration and subtracted from this the area under the straight line.

Answers: (i)  $c = 12$ ; (ii)  $1\frac{1}{3}$ .

### Question 12

In part (i), most candidates attempted to integrate although there were errors seen, particularly with attempts to simplify division by the fractions  $\frac{1}{3}$  and  $\frac{1}{2}$ . Some candidates forgot the constant of integration and therefore made no further progress. Part (ii) was very well done with many candidates scoring full marks. In part (iii), candidates generally understood the need to solve the equation  $dy/dx = 0$ , although there were some dubious methods used in finding a solution. The  $y$  coordinate of the stationary point was often incorrect but attempts to determine its nature usually met with success. Overall, candidates scored well on this question.

Answers: (i)  $y = \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - \frac{2}{3}$ ; (ii)  $\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$ ; (iii)  $(1, -2)$  minimum.

# MATHEMATICS

Paper 9709/12

Paper 12

## Key Messages

Teachers should reinforce with all their candidates the need for full presentation of working, especially in questions where candidates are asked to prove a particular result. If necessary steps of working are omitted then full marks cannot be awarded. This applied particularly to **Question 5** where often very good candidates failed to show sufficient working. Whilst Examiners will generally accept steps that can be done mentally or on a calculator, this does not apply to questions that require a rigorous proof.

The practice of splitting a page into two halves and working separately on each half should be positively discouraged. It makes marking the script extremely difficult and painstaking.

## General Comments

Although there were many very good scripts, standards of arithmetic, algebra and presentation remain variable. The attempt at **Question 3** was very poor and showed a basic lack of understanding of basic trigonometric functions. Similarly the lack of success in **Question 7(ii)** showed a poor understanding of the application of the term “magnitude of a vector”. The standards of arithmetic, algebra and presentation remain variable.

## Comments on Specific Questions

### Question 1

This proved to be a good starting question with the majority of candidates obtaining full marks. There were however several serious misunderstandings about the term “perpendicular bisector”. A significant number of candidates failed to realise that the perpendicular bisector was a line passing through the midpoint of the line joining the two points and at right angles to this line. Other errors were to assume that the gradient was the x-step divided by the y-step or to assume that the midpoint was  $(\frac{1}{2}(x_1 - x_2), \frac{1}{2}(y_1 - y_2))$ .

Answer:  $(3\frac{1}{2}, 0)$

### Question 2

This was generally well answered, though arithmetic errors in dealing with  $(-4)^2$ ,  $(-4)^3$ ,  $(\frac{1}{2})^3$  and  $(\frac{1}{2})^4$  were widespread. It was common to see the minus sign omitted throughout and a final answer of “15 + 160” instead of “15 – 160” was often offered. A significant number of candidates failed to realise that two terms in the expansion of  $\left(\frac{x}{2} - \frac{4}{x}\right)^6$  were needed to find the coefficient of  $x^2$ .

Answer: -145

### Question 3

This was poorly answered by nearly all of the candidates.

- (i) (a) Most candidates realised the need to use  $\sin^2\theta = 1 - k^2$ , but only a few realised that, because  $\theta$  was reflex, the negative square root needed to be taken.
- (b) Most candidates obtained the follow through mark from using “ $\tan\theta = \sin\theta \div \cos\theta$ ”.

- (ii) Only a small number of correct solutions was seen. Candidates failed to realise that because  $\theta$  was reflex, and  $\cos\theta > 0$ , then  $270^\circ < \theta < 360^\circ$  and that therefore  $540^\circ < 2\theta < 720^\circ$  leading to  $\sin 2\theta < 0$ . A few attempts were seen using the double angle formulae for  $\sin 2\theta$ , but again these were usually incomplete through failure to realise that because  $\sin\theta < 0$  and  $\cos\theta > 0$ ,  $\sin 2\theta < 0$ .

Answers: (i) (a)  $-\sqrt{1-k^2}$  (b)  $\frac{-\sqrt{1-k^2}}{k}$  (ii) Proof

#### Question 4

- (i) This was often badly answered, depending at which stage candidates used the substitution  $\theta = p\sin\theta$ . Candidates starting with  $\frac{1}{2}r^2\sin\theta = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$  or  $r^2\sin\theta = \frac{1}{2}r^2\theta$  invariably obtained the correct value of  $p$ . Unfortunately many made the substitution at an early stage and were unable to obtain a value of  $p$  because of errors such as  $\frac{1}{2}r^2\sin(p\sin\theta) = \frac{1}{2}r^2p\sin^2\theta$ . Many candidates opted to use the formula “ $\frac{1}{2} \times \text{base} \times \text{height}$ ” for the area of a triangle and this led to expressions containing sine or cosine of “ $\frac{1}{2}\theta$ ”, from which there was usually no recovery.
- (ii) Most candidates correctly evaluated the arc length  $AXB$ . Although there were many correct solutions for the perimeter, common errors were writing  $AB$  as  $2 \times 8 \times \sin 2.4$ ; forgetting to multiply  $8\sin 1.2$  by 2; and premature approximation leading to 34.2.

Answers: (i)  $p = 2$  (ii) 34.1 cm

#### Question 5

- (i) It was pleasing to see many perfectly correct solutions. Most candidates added the fractions correctly and used “ $\sin^2\theta + \cos^2\theta = 1$ ” to obtain the fraction  $\frac{\sin^2\theta + \sin\theta}{\cos\theta(1 + \sin\theta)}$ . At least a third of all candidates did not manage to factorise this to give  $\frac{\sin\theta}{\cos\theta}$  and then to deduce the answer. Some very good candidates failed to make this last step and it was impossible to give the final 2 marks. Candidates must realise the need to show all working and explanations in this type of question.
- (ii) This was very well answered with only a small number of candidates failing to give both answers in the range or selecting the incorrect quadrants. Several candidates ignored the rubric which requests answers for an angle in degrees correct to one decimal place, not to three significant figures.

Answers: (i) Proof (ii)  $116.6^\circ$ ,  $296.6^\circ$

#### Question 6

- (i) Most candidates recognised that  $8r = 8 + 8d$  and that  $8r^2 = 8 + 20d$ , though common errors were to misinterpret arithmetic for geometric or to assume incorrectly that the  $n^{\text{th}}$  term of a geometric progression is  $ar^n$ . Elimination of  $r$  or  $d$  caused many candidates considerable difficulty and was often accompanied by a mass of algebra. Use of  $\frac{8}{8+8d} = \frac{8+8d}{8+20d}$  as a starting point generally led to correct values for  $d$  and then  $r$ .
- (ii) The follow through mark for the 4<sup>th</sup> term of the geometric progression and the method mark for the 4<sup>th</sup> term of the arithmetic progression were nearly always gained.

Answers: (i)  $r = 1.5$  (ii) 27 and 9.5

### Question 7

- (i) This was generally well answered with most candidates showing confidence in using the scalar product and realising it needed to be zero. Some candidates assumed that  $\overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OB}$  instead of  $\overrightarrow{OA} - \overrightarrow{OB}$ . Use of vector notation was much improved from previous years.
- (ii) This was rarely correct and often not attempted. Only about a third of all candidates realised the need to find the unit vector in the direction of  $\overrightarrow{BA}$  and then multiply this by 12 to find  $\overrightarrow{CD}$ . Many candidates thought that  $\overrightarrow{CD}$  was the position vector of D.

Answers: (i) Proof (ii)  $\begin{pmatrix} 12 \\ 9 \\ -2 \end{pmatrix}$

### Question 8

Most candidates realised the need to integrate twice and the standard of integration was good. Unfortunately, failure to include a constant of integration (either once or twice) meant that most of the marks were unavailable. Some candidates used the same letter for both integrations. About a half of all candidates, whilst attempting to use  $y = -10$  when  $x = 3$ , failed to realise that  $\frac{dy}{dx} = 0$  when  $x = 3$ .

Answer:  $(-2, 10\frac{5}{6})$

### Question 9

This was a well answered question and there were many correct solutions.

- (i) The majority of candidates realised that the function was composite and recognised the need to multiply by the differential of the bracket  $(-1)$  to find the differential and to divide by  $(-1)$  to find the integral. Surprisingly many attempts were seen in which “8” was neither differentiated nor integrated.
- (ii) Again this was usually correct, though with weaker candidates it was common to see the gradient left as an algebraic expression or to offer the equation of the normal instead of the tangent.
- (iii) The majority of candidates knew what was expected, though a few used the formula for volume instead of area. In evaluating the area under the curve, a common error was to assume that because the lower limit was 0, then the value of the integral was automatically 0. Other common misunderstandings were to ignore the area under the line or to use the area of a trapezium with 6 and 7 as the vertical heights instead of  $5\frac{1}{2}$  and 7. Those attempting to find the area between the curve and the  $y$ -axis were rarely successful, either because they were using incorrect limits or unable to cope with the integration.

Answers: (i)  $\frac{1}{2\sqrt{4-x}}$ ,  $8x + \frac{2}{3}(4-x)^{\frac{3}{2}}$  (ii)  $y = \frac{1}{2}x + 5\frac{1}{2}$  (iii)  $\frac{7}{12}$

**Question 10**

- (i) This part was extremely well answered with candidates coping comfortably with the composite function. Occasionally ff was given as  $2(2x - 3)$  instead of  $2(2x - 3) - 3$ .
- (ii) This caused more difficulty with many candidates factorising  $x^2 + 4x$  to obtain  $-4$  fortuitously. The successful attempts (from less than a half of candidates) came from either completing the square or using calculus to find  $x = -2$  and then deducing that  $y \geq -4$ .
- (iii) The solution of  $x^2 + 4x - 12 = 0$  was generally correct, though " $x(x + 4) = 12$  implying  $x = 12$  or  $(x + 4) = 12$ " was a serious error. Solving the inequality proved to be more difficult and it was very common to see such incorrect answers as " $x > 2$ ,  $x > -6$ " or " $-6 < x < 2$ ".
- (iv) As in part (i), most candidates coped well with finding gf, though " $(2x - 3)^2 + 4x$ " was a common incorrect expression. Virtually all candidates realised the need to use the discriminant ( $b^2 - 4ac$ ) but a significant number of solutions were seen in which " $= p$ " was only introduced after the discriminant had been used.
- (v) Only a small proportion of candidates realised that the smallest value of  $k$  was  $-2$ , the value at which the curve had a minimum point.
- (vi) This was very well answered, with most candidates realising the need to use "completing the square" to express  $x$  in terms of  $y$ . Marks were lost through failing to remove the  $\pm$  in the answer or to leave the answer as a function of  $y$  instead of  $x$ .

Answers: (i) 5 (ii) Range  $\geq -4$  (iii)  $x < -6$ ,  $x > 2$  (iv)  $p = -4$  (v)  $-2$  (vi)  $h^{-1}(x) = \sqrt{x + 4} - 2$

# MATHEMATICS

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Paper 9709/13

Paper 13

## General comments

Most candidates were able to demonstrate fully in this paper their understanding and knowledge of the subject material.

Setting out was usually satisfactory although candidates should be advised not to divide their written answer page vertically as this often makes it much more difficult to follow the candidate's work and their arguments. In **Question 11**, setting out sometimes fell below the general standard. Occasionally a candidate would attempt to find, for example, several equations without making it clear which lines were under consideration, or find the intersection of two lines without making it clear which point was being sought.

As reported in individual questions, where the required answer is given in the question, candidates did not always appreciate that intermediate steps need to be shown explicitly, even if some of the steps can be done mentally.

It was also clearly the case that some candidates had not read the questions carefully enough. For example, **Question 11** requires a calculation method. Answers obtained by measurement, drawing or pure guesswork are not acceptable. Similarly, **Question 10** requires all necessary working to be shown.

## Comments on specific questions

### Question 1

This question was well done by the majority of candidates. Almost all candidates identified the correct term of the expansion but a significant number omitted the minus sign when evaluating  $(-2/x)^3$ .

*Answer:*  $-80$ .

### Question 2

This question was very well done with many candidates scoring full marks. In part (i), a few candidates divided the terms the wrong way round and a similar number used 32 as the value for the first term. In part (ii), a few candidates used  $d = +4$  and there were some arithmetic errors in evaluating the value of  $n$ .

*Answers:* (i) 324; (ii) 19.

### Question 3

In part (i), the length of the major arc was required and this caused some confusion, some candidates using an angle of just 2.2 and others using  $\pi - 2.2$ . Some candidates converted radians to degrees for the calculation and this sometimes caused arithmetic errors. A disappointing number forgot to add the two radii. In part (ii), most candidates successfully found the area of the major sector although some arithmetic errors arose particularly from those who found the area of the circle minus the area of the minor sector. Errors were also seen in the calculation for the area of the triangle particularly from the method half base  $\times$  height. The use of  $\frac{1}{2}ab\sin C$  seems to be more secure. However, the most common error was to assume that the ratio required an integer answer.

*Answers:* (i) 36.5; (ii) 5.05.

#### Question 4

The majority of candidates clearly knew how to prove the identity in part (i) but significant numbers lost marks by omitting to show intermediate steps in their proof. When candidates have to show a certain result is true, the answer is given and, in these circumstances especially, Examiners need to satisfy themselves that each candidate really does understand how to obtain the required result. The onus is on the candidate to make sure all the steps in the working are shown. There are several ways in which this identity can be proved but most methods require the use (twice) of the identity:  $\tan = \sin/\cos$  and the use of the identity:  $\sin^2 + \cos^2 = 1$  with, of course, no algebraic errors. Part (ii) was usually completed correctly but significant numbers of candidates lost marks by giving their answers in degrees. When the question requests the solutions to be in a range which involves  $\pi$ , this implies that the solutions must be in radians. Furthermore, radian solutions must be given correct to 3 significant figures, unless otherwise directed. A small minority of candidates lost marks by giving only 2 significant figures.

Answer: (ii) 0.983, 4.12.

#### Question 5

This question was not very well done. There was confusion in the minds of some candidates between  $f'$  and  $f^{-1}$  and these candidates were able to score very few marks for this question. In part (i) the derivative was usually found correctly but most candidates were not able to explain why  $f$  has an inverse, with many employing the stock answer that  $f$  is one-one. In part (ii), finding an expression for the inverse function was well handled although many were not able to find the domain and range successfully. When both domain and range are required, for example, candidates should make sure that they identify which is which and to use the correct combination of letters in the inequalities.

Answers: (i)  $\frac{-30}{(2x+3)^2}$ ; (ii) Domain:  $1 \leq x \leq 5$ , range:  $0 \leq f^{-1}(x) \leq 6$ .

#### Question 6

The point (2, 14) did not lie on the line  $3y + x = 5$  but, for part (i), candidates were only required to use the gradient of the normal ( $-\frac{1}{3}$ ) from the equation of the line. Examiners ensured that no candidate was disadvantaged and part (ii) was unaffected. (This equation has now been amended on the question paper and mark scheme.) Candidates were expected to state that the gradient of the tangent is 3 and equate this with the given gradient function, substituting  $x = 2$  and showing (without missing out steps in the working) that  $a = 8$ . In part (ii), most candidates realised the need to integrate and most were successful in this. A small minority forgot the constant of integration and could get no further, but most candidates were able to use the point (2, 14) and achieve the correct answer.

Answer: (ii)  $y = 6(4x + 8)^{\frac{1}{2}} - 10$ .

#### Question 7

In part (i), most candidates were able to find vectors **AB** and **AC** (or **BA** and **CA**) correctly and this almost always led to full marks for this part. Some had one of the vectors reversed, effectively finding the obtuse angle  $BAC$ . Part (ii), however, was poorly attempted with most candidates not appreciating the fact that the word 'exact' did not allow them simply to evaluate  $\cos^{-1}(\frac{1}{3})$  as a decimal number (correct to 3 significant figures) and then to use this angle to calculate the area of the triangle. The simplest approach, rarely seen, was to use the identity  $\sin^2 A = 1 - \cos^2 A$  to find that  $\sin A = \frac{\sqrt{8}}{3}$  and to find the area of the triangle using the formula  $\frac{1}{2}bc \sin A$ .

Answer: (ii)  $5\sqrt{8}$ .

### Question 8

Both parts of this question were well answered with many candidates achieving full marks. In part (i) the values of  $a$ ,  $b$  and  $c$  were usually correct. For the minimum value quite a number stated the  $x$  value while others gave both  $x$  and  $y$  as a pair of coordinates. In part (ii) most candidates used the right method for finding the discriminant and knew that this needed to be negative. There were, however, some sign errors when squaring the coefficient of  $x$  and this usually led to the critical values from the quadratic in  $k$  being  $+18$ ,  $+2$  instead of  $-18$ ,  $-2$ . Most candidates correctly chose the region inside the critical values.

Answers: (i)  $2(x - 2\frac{1}{2})^2 - 4\frac{1}{2}$ , minimum value  $-4\frac{1}{2}$ ; (ii)  $-18 < k < -2$ .

### Question 9

Part (i) was not done very well. Candidates first had to let the height of the cuboid be  $y$  and then to establish that  $y = \frac{288}{3x^2}$ . Since the answer is given, candidates had to show intermediate steps of working in order to achieve the desired result. In part (ii), most candidates differentiated correctly and were able to obtain  $x = 4$  at the stationary point. A large number of candidates clearly thought that 4 was the stationary value of  $A$ . Most candidates attempted to find the second derivative and almost all of these were successful in showing that the stationary point is a minimum.

Answers: (ii) 288, minimum.

### Question 10

This question was generally well done with many candidates scoring full marks. Most candidates were able to find  $x = 3$  and  $x = 7$ , the points of intersection of the line and the curve, although some candidates found the intersection of the curve with the  $x$ -axis. The shaded area can be found by subtracting from the integral of the curve the integral of the straight line with limits 3 and 7 in both cases. The question required candidates to show all necessary working. In particular, candidates had to show the result of integrating the required terms and of treating the limits in the appropriate way. Candidates who failed to do this unfortunately lost the majority of the marks for this question. The area under the straight line can, of course, be found by finding the area as a trapezium.

Answer: 10%.

### Question 11

Most candidates managed to find the coordinates of  $A$  and  $C$  correctly. After this there was a variety of methods attempted. A very common error was to take the diagonals of the parallelogram to be perpendicular and candidates who made this assumption very often scored only the first 4 marks. The most popular correct method was to find the equation of the line  $BC$  and then the intersection with  $AB$ . The coordinates of  $D$  can then be found, either by using a similar method or by using the fact that  $E$  is the mid-point of  $BD$ . It must be noted, also, that the wording of the question requires a 'calculation' method. Some answers were given with very little evidence of calculations being carried out, resulting in significant loss of marks.

Answers:  $A(1, 3)$ ,  $B(4, 12)$ ,  $C(12, 14)$ ,  $D(9, 5)$ .

# MATHEMATICS

Paper 9709/21

Paper 21

## Key Messages

Candidates are advised to read the questions carefully and make sure they are actually answering the question. Candidates should also ensure that they are working to the required level of accuracy.

## General Comments

There were many scripts of a high standard submitted showing that candidates had a good understanding of the syllabus objectives and were able to apply techniques learned both appropriately and correctly.

## Comments on Specific Questions

### Question 1

Most candidates chose to square both sides of the inequality and solve the resulting quadratic equation, obtaining the correct critical values in most cases. Many candidates, however, were unable to determine the correct range of values required. It should be noted that it is good practice for discontinuous ranges to be given as 2 discrete expressions (see solution below) rather than one expression (e.g.  $-\frac{1}{2} \geq x \geq 3$ ).

*Solution:*  $x \leq -\frac{1}{2}, x \geq 3$

### Question 2

- (i) Most candidates were able to differentiate  $3\sin x$  correctly. Problems arose when differentiating  $\tan 2x$ , with many candidates offering the response  $\sec^2 x$ , rather than the correct response of  $2\sec^2 2x$ . Unfortunately the former gave a fortuitous answer of 5. Use of the double angle formula for  $\tan 2x$  followed by differentiation of a quotient was another approach.
- (ii) Different approaches were used with varying degrees of success. If candidates realised that the derivative of 6 with respect to  $x$  was zero, then use of the product or quotient rule was often successful. For those candidates who used the chain rule, there was often a missing exponential term of  $2e^{2x}$ .

*Solution:* (i) 5 (ii) -3

### Question 3

- (i) Most candidates adopted the process of algebraic long division, usually with great success. Most candidates that used this approach were able to obtain correct quotients and thus show that there was a remainder of 7. Similarly those candidates that chose to use a method involving an identity were in general, equally successful. Problems arose when candidates attempted to use synthetic division or the remainder theorem with substitutions of  $x = \pm 2$ . The synthetic division approach was acceptable, provided candidates realised that they needed to obtain a cubic expression after division by either  $x - 2$  or  $x + 2$ , and then repeat the process with a further division, using the unused factor, to obtain the quotient and remainder. Use of the remainder theorem with  $x = \pm 2$  yielded a remainder of 7 for both divisions, but this is insufficient and does not result in a quotient being obtained.

- (ii) It was realised by most candidates that they were able to make use of the quotient obtained in part (i) to solve the given quartic equation. Some lost a mark by forgetting to give the solutions  $x = \pm 2$  in addition to those obtained by solving the quadratic quotient equation.

*Solution:* (i)  $6x^2 - x - 2$  (ii)  $\pm 2, -\frac{1}{2}, \frac{2}{3}$ .

#### Question 4

- (i) The standard of curve sketching was generally poor. Most candidates have a good idea of what the graphs should look like, but present them badly. A sketch does not need to be done on graph paper. It is sufficient for it to appear within the body of the rest of the question. Axes are meant to be straight lines. It would be useful and good practice to mark in the coordinates of the points where the curves cross the coordinate axes. The question asked the candidates to show that there is exactly one real root. So a conclusion was expected to be seen stating that, as there is only one point of intersection between the two curves, there is only one real solution to the given equation.
- (ii) Most candidates used the correct approach of considering the value of  $3\ln x - 15 + x^3$  or equivalent when  $x = 2$  and when  $x = 2.5$ . It is necessary to give actual numeric values, rather than just state that the result is either positive or negative. Again, many candidates were not giving a conclusion. A statement was expected to be seen to the effect that, as there was a change of sign, the root was between the two given values.
- (iii) As usual, most candidates were able to obtain full marks for this part of the question, showing a good understanding of the process of iteration. It was surprising to note that there are still candidates who do not make full use of their calculators and the 'Answer' function to make quick work of the iteration process. Ensuring that the final answer is given to the required level of accuracy is also of importance.

*Solution:* (iii) 2.319

#### Question 5

- (i) Provided candidates were able to express the left hand side of the expression as a single fractions, most were able to get full marks. There were also many correct solutions starting with the right hand side of the expression with candidates making use of  $1 = \sin^2 \theta + \cos^2 \theta$ .
- (ii) It was realised by most candidates that they needed to use the result from part (i) and most were able to write the given expressions correctly using part (i). Some candidates are still unaware of the implications of the word 'Hence' and the phrase 'exact value'.

*Solution:* (ii) (a)  $2\sqrt{2}$  (b) 3

#### Question 6

- (a) There was a pleasing number of correct solutions for this part of the question. The fact that candidates were told to show that the integral was equal to  $\ln 125$  prompted most to use a form of  $\ln(2x - 7)$ . Limits were usually substituted in correctly and the laws governing the use of logarithms were usually correct with the exception of the all too frequent incorrect statement  $3\ln 125 - 3\ln 5 = 3\frac{\ln 125}{\ln 5}$ .
- (b) Many completely correct solutions were seen showing a good understanding of the trapezium rule. There were occasional errors in the x-coordinates used, leading on to incorrect y-values being used. Candidates should be reminded to make sure they are giving their answers to the correct level of accuracy.

*Solution:* (b) 13.5

**Question 7**

- (i) Most candidates realised that they had to differentiate implicitly with respect to  $x$ . Any errors usually occurred when trying to deal with the product  $3xy$  or  $3$ . Correct numerical gradients were usually obtained, together with a correct equation of the tangent. Many candidates did not give their answer in the required form, however, leaving their equation with fractional coefficients.
- (ii) Very few correct solutions to this part of the question were seen. Most candidates were able to reach the conclusion that  $4x + 3y = 0$ , but were unsure of what to do with this result. Very few candidates realised that they needed to use this result together with the original equation and thus obtain a result of either  $-\frac{1}{8}y^2 = 3$  or  $-\frac{2}{9}x^2 = 3$ , neither of which has any real solutions. Again a conclusion to this effect was expected.

*Solution:* (i)  $5x + 4y - 6 = 0$

# MATHEMATICS

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Paper 9709/22

Paper 22

## Key Messages

Candidates are advised to read the questions carefully and to make sure they are actually answering the question. Candidates should also ensure that they are working to the required level of accuracy.

## General Comments

Most candidates showed a good understanding of the syllabus objectives and were able to make a reasonable attempt at the paper, which was accessible to all. Some high scoring scripts were proof of good teaching and learning techniques.

## Comments on Specific Questions

### Question 1

- (i) Most candidates were able to obtain  $x = \frac{11}{2}$ , usually by squaring both sides of the given equation.
- (ii) Very few candidates made the link with the first part of the question, in spite of the word 'Hence' being used, to state  $3^y = \frac{11}{2}$  and then calculate the value of  $y$ . Very often candidates started the question again, thus repeating the process already completed for part (i) and then making use of the laws of logarithms. It is important that the meaning and implications of the word 'Hence' is understood by all candidates.

*Solution:* (i)  $\frac{11}{2}$ , (ii) 1.55

### Question 2

Good efforts were made by most candidates, using the correct trigonometric identities, to obtain an equation of the form  $c_1 \sin^2 \theta = c_2$ . There were many completely correct solutions produced, showing a good understanding by candidates of the basic solution of trigonometric equations.

*Solution:*  $35.3^\circ, 144.7^\circ$

### Question 3

- (a) Many answers of the form  $k \sin\left(\frac{x}{3} + 2\right)$  were produced. There were occasional slips with the value of  $k$ , but in general this part of the question was done well by most candidates.

- (b) Historically, questions involving the trapezium rule have been less than well done, candidates making errors in the number of ordinates/strips being used and hence the value of  $h$ . It was pleasing to note that this year has marked a great improvement, with many completely correct solutions being produced.

*Solution:* (a)  $12 \sin\left(\frac{x}{3} + 2\right) + c$ , (b) 79.2

#### Question 4

Most candidates were able to deal successfully with the differentiation, with respect to  $t$ , of the parametric equations, and then go on to find successfully the equation of the tangent. It was evident that many candidates did not read the question thoroughly and did not give their correct, but unsimplified, equation in the required form involving integers.

Some candidates erroneously equated  $\frac{dy}{dx}$  to zero and went on to find a value for  $t$ , having misunderstood the demands of the question. It is important that the question is read carefully before 'habit' takes over and errors are made.

*Solution:*  $2x - y + 4 = 0$

#### Question 5

This question proved to be the most problematic question on the paper, with many candidates failing to score any meaningful marks. There were many misuses of the logarithmic values, for example, incorrect statements such as  $\ln 1.87 = \ln K + \ln 1.35p \ln 2$ , which showed a complete misunderstanding of the question and thus made the awarding of marks difficult. This is clearly a topic of the syllabus which candidates need to work on.

*Solution:*  $p = 1.40$ ,  $K = 1.75$

#### Question 6

- (i) Most candidates were able to find the value of  $a$  correctly, usually by using the factor theorem, but on occasion by algebraic long division. Candidates who made use of algebraic long division were much more likely to make an error in the calculation of the value of  $a$ . Use of the factor theorem should be promoted above the use of algebraic long division when possible.
- (ii) Provided a correct value of  $a$  had been obtained, most candidates were able to obtain a correct quotient of  $x^2 - 2x + 6$ , either by algebraic long division or by observation. However, many candidates appeared to think that, as the quotient could not be factorised, there were no solutions. Very few considered the discriminant or made an attempt to use the quadratic formula to show that the equation  $x^2 - 2x + 6 = 0$  had no real roots, as was intended.

*Solution:* (i)  $a = 12$

#### Question 7

- (i) The fact that candidates had a given answer to work towards helped with the correct solution of this part of the question. There were some arithmetic slips which were then corrected, but contrived results were few.
- (ii) Candidates are still not setting their working out in a clear and logical fashion and then drawing conclusions from it. An ideal solution would be to state  $f(x) = x - \frac{1}{3} \ln(61 - 2x^3)$ , or equivalent. Calculations involving  $f(1)$  and  $f(1.5)$  are then made (many candidates attempted to do this part, but often did not make clear their intentions or explain where their resulting figures come from). It

is important that a conclusion is given, making reference to the change of sign, rather than just leaving the calculations as a result.

- (iii) The iterative process is usually done well and this year was no exception. Candidates still lost marks through not giving their working or their final answer to the required level of accuracy. It is important that the question is read carefully and the levels of accuracy stated are adhered to.

*Solution:* (iii) 1.343

#### Question 8

- (i) Many candidates misread the question and tried to rearrange the given equation into the form stated without any attempt at differentiation first and then rearrangement. Careful reading of any question prior to starting work on it is of paramount importance. For the candidates who did realise that differentiation needed to be done first, most were able to do so correctly using the product rule, but then had problems in using trigonometric identities and attempting to rearrange their often correct result into the required form.
- (ii) This part of the question was not often attempted, probably because the first part of the question had been so problematic. For those candidates who did attempt this part, most failed to spot that the equation can be written as a quartic equation in  $\cos x$  and then as a quadratic equation in  $\cos^2 x$ . For those that did realise this, factorisation was often attempted. Candidates should realise that not all quadratic expressions factorise and that it is sometimes necessary to use the quadratic formula or to complete the square.

*Solution:* (ii) 0.45

# MATHEMATICS

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Paper 9709/23

Paper 23

## Key Messages

Candidates are advised to read the questions carefully and to make sure they are actually answering the question. Candidates should also ensure that they are working to the required level of accuracy.

## General Comments

Most candidates showed a good understanding of the syllabus objectives and were able to make a reasonable attempt at the paper, which was accessible to all. Some high scoring scripts were proof of good teaching and learning techniques.

## Comments on Specific Questions

### Question 1

- (i) Most candidates were able to obtain  $x = \frac{11}{2}$ , usually by squaring both sides of the given equation.
- (ii) Very few candidates made the link with the first part of the question, in spite of the word 'Hence' being used, to state  $3^y = \frac{11}{2}$  and then calculate the value of  $y$ . Very often candidates started the question again, thus repeating the process already completed for part (i) and then making use of the laws of logarithms. It is important that the meaning and implications of the word 'Hence' is understood by all candidates.

*Solution:* (i)  $\frac{11}{2}$ , (ii) 1.55

### Question 2

Good efforts were made by most candidates, using the correct trigonometric identities, to obtain an equation of the form  $c_1 \sin^2 \theta = c_2$ . There were many completely correct solutions produced, showing a good understanding by candidates of the basic solution of trigonometric equations.

*Solution:*  $35.3^\circ, 144.7^\circ$

### Question 3

- (a) Many answers of the form  $k \sin\left(\frac{x}{3} + 2\right)$  were produced. There were occasional slips with the value of  $k$ , but in general this part of the question was done well by most candidates.

- (b) Historically, questions involving the trapezium rule have been less than well done, candidates making errors in the number of ordinates/strips being used and hence the value of  $h$ . It was pleasing to note that this year has marked a great improvement, with many completely correct solutions being produced.

*Solution:* (a)  $12 \sin\left(\frac{x}{3} + 2\right) + c$ , (b) 79.2

#### Question 4

Most candidates were able to deal successfully with the differentiation, with respect to  $t$ , of the parametric equations, and then go on to find successfully the equation of the tangent. It was evident that many candidates did not read the question thoroughly and did not give their correct, but unsimplified, equation in the required form involving integers.

Some candidates erroneously equated  $\frac{dy}{dx}$  to zero and went on to find a value for  $t$ , having misunderstood the demands of the question. It is important that the question is read carefully before 'habit' takes over and errors are made.

*Solution:*  $2x - y + 4 = 0$

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*Solution:*  $p = 1.40$ ,  $K = 1.75$

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*Solution:* (i)  $a = 12$

#### Question 7

- (i) The fact that candidates had a given answer to work towards helped with the correct solution of this part of the question. There were some arithmetic slips which were then corrected, but contrived results were few.
- (ii) Candidates are still not setting their working out in a clear and logical fashion and then drawing conclusions from it. An ideal solution would be to state  $f(x) = x - \frac{1}{3} \ln(61 - 2x^3)$ , or equivalent. Calculations involving  $f(1)$  and  $f(1.5)$  are then made (many candidates attempted to do this part, but often did not make clear their intentions or explain where their resulting figures come from). It

is important that a conclusion is given, making reference to the change of sign, rather than just leaving the calculations as a result.

- (iii) The iterative process is usually done well and this year was no exception. Candidates still lost marks through not giving their working or their final answer to the required level of accuracy. It is important that the question is read carefully and the levels of accuracy stated are adhered to.

*Solution:* (iii) 1.343

#### Question 8

- (i) Many candidates misread the question and tried to rearrange the given equation into the form stated without any attempt at differentiation first and then rearrangement. Careful reading of any question prior to starting work on it is of paramount importance. For the candidates who did realise that differentiation needed to be done first, most were able to do so correctly using the product rule, but then had problems in using trigonometric identities and attempting to rearrange their often correct result into the required form.
- (ii) This part of the question was not often attempted, probably because the first part of the question had been so problematic. For those candidates who did attempt this part, most failed to spot that the equation can be written as a quartic equation in  $\cos x$  and then as a quadratic equation in  $\cos^2 x$ . For those that did realise this, factorisation was often attempted. Candidates should realise that not all quadratic expressions factorise and that it is sometimes necessary to use the quadratic formula or to complete the square.

*Solution:* (ii) 0.45

# MATHEMATICS

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Paper 9709/31

Paper 31

## General comments

The standard of work on this paper varied considerably. No question or part of a question seemed to be too difficult for the more able candidates, and most questions discriminated well within this group. The questions that candidates found relatively easy were **Question 1**, **Question 2**, **Question 3**, **Question 6** and **Question 9**. Those that they found difficult were **Question 4**, **Question 5**, **Question 7** and **Question 10**.

In general the presentation of the work fell below that expected from candidates attempting this paper. A significant number of candidates appeared to have studied only part of the syllabus, hence excellent attempts at **Question 9(ii)** and **Question 10**, both difficult questions, were accompanied by sparse attempts at the other questions.

When attempting a question, candidates need to be aware that it is essential that sufficient working is shown to indicate how they arrive at their answer, whether they are working towards a given answer or an answer that is not given. Candidates who use their calculators to produce an exact answer with no supporting working receive no credit.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

## Comments on Specific Questions

### **Question 1**

- (i) Many candidates seemed unable to express both  $\sin 2\alpha$  and  $\sec \alpha$  in terms of  $\sin \alpha$  and  $\cos \alpha$ .
- (ii) Here, many candidates did not use the double angle formula, and of those that did, many subsequently made errors. Often those who successfully solved the correct quadratic equation did not reject one of their answers.

Answer: (i)  $2\sin \alpha$  (ii)  $\frac{1}{3}$

### **Question 2**

Correct differentiation of  $u$  was usually followed by the omission of the coefficient 3 when candidates attempted their substitution. However, despite this many candidates were able to proceed and acquire the remaining method marks.

Answer:  $\frac{14}{9}$

### Question 3

This question was one that many candidates could make a reasonable attempt at. However, the coefficient 2 was often omitted from their  $\frac{dx}{dt}$  and the value of  $t$  where the curve crossed the  $y$ -axis was wrongly assumed to be  $t = 0$ .

Answer:  $\frac{5}{2}$

### Question 4

Few candidates were successful at this question since the separation of variables was nearly always not undertaken, and even when it was the integration of  $\frac{6e^{3x}}{2 + e^{3x}}$  proved too difficult.

Answer:  $y = 4(2 + e^{3x})^2$

### Question 5

This question was one that few could make any progress with, other than to obtain  $z$  in the form  $x + iy$ . Attempts at part (ii) were very rare.

Answers: (i)  $9e^{\frac{1}{3}\pi i}$ , (ii)  $3e^{\frac{1}{6}\pi i}$ ,  $3e^{-\frac{5}{6}\pi i}$

### Question 6

- (i) Few candidates were able to use correctly both the product and power laws of logarithms. It was common to see candidates ignore the given expression and to just solve this equation instead of showing that the equation follows from the given expression.
- (ii) Some sound work was seen in this section and many candidates scored full marks here.
- (iii) Most candidates did not realise that only  $x = 2$  was a possible solution, with  $x = \frac{1}{4}$  being rejected.

Answer: (ii)  $(x - 2)(4x + 1)^2$  (iii)  $x = 2$

### Question 7

- (i) Many candidates made little progress was made with this question other than to work with the LHS of the plane equation.
- (ii) Despite the various possible methods such as scalar product of unit, normal with point in plane, finding the point of intersection of the perpendicular from the origin to the plane or using formula for the perpendicular distance from a point to a plane, little progress was made by many candidates with this question.
- (iii) Only a minority of candidates recognised that the required planes were of the same LHS as that in (i) and all that was required was the use of the formula for the perpendicular distance to a plane, or something equivalent. Usually candidates produced a correct solution. However several candidates produced an expansion for  $(1 - x)^{1/2}$  instead of  $(1 - x)^{-1/2}$ .

Answers: (i)  $2x - 3y + 6z = 23$  (ii)  $\frac{23}{7}$  (iii)  $2x - 3y + 6z = 121$ ,  $2x - 3y + 6z = -75$

### Question 8

- (i) The sketching of  $y = \operatorname{cosec} x$  proved difficult, although better success was forthcoming for  $y = x(\pi - x)$ . Often candidates who obtained both graphs correct then did not clearly show that there were exactly two real solutions by indicating the roots to be where the graphs crossed.
- (ii) Candidates usually managed to introduce  $\sin x$  into the expression, but then did not realise that they needed to split  $x(\pi - x)$  into  $x\pi$  and  $x^2$  in order to progress correctly to the required form.
- (iii) This numerical work was nearly always correct.
- (iv) Again, when attempted this value was usually correct.

Answers: (iii) 0.66 (iv) 2.48.

### Question 9

- (i) Usually one of the correct forms for the partial fractions was chosen. Despite several candidates making some arithmetical errors in establishing the coefficients of the terms in the partial fractions this section contained some of the best work from the candidates.
- (ii) Whilst most candidates knew how to perform a binomial expansion once they had an expression of the form  $(1 - \frac{x}{3})^{-1}$ ,  $(1 + 2x)^{-1}$  or  $(1 + 2x)^{-2}$ , many did not reach these forms as errors were introduced as the constants 3 and 2 were nearly always incorrectly brought into the numerator.

Answers: (i)  $\frac{1}{3-x} + \frac{3}{2(1+2x)} - \frac{1}{2(1+2x)^2}$  or  $\frac{1}{3-x} + \frac{3x+1}{(1+2x)^2}$  (ii)  $\frac{4}{3} - \frac{8}{9}x + \frac{1}{27}x^2$

### Question 10

- (i) Frequent sign errors were seen in the use of the product rule when differentiating  $\sin 4x$ . However, despite the question being a little unusual, many candidates obtained the correct solutions, other than them being in degrees as opposed to radians.
- (ii) Many students knew how to tackle this question but muddled either their initial value or the step size between the roots.

Answers: (i) 0.362, 1.147 (ii) 33

# MATHEMATICS

Paper 9709/32

Paper 32

## General comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks from zero to full marks. The questions or parts of questions that were generally done well were **Question 3** (trigonometric equation), **Question 4** (parametric differentiation) and **Question 8** (calculus). Those that were done least well were **Question 2** (logarithmic and exponential functions), **Question 5** (polynomial algebra), **Question 6 (i)** (geometry), and **Question 7** (complex numbers).

In general the presentation of work was good. Some candidates appeared to be pressed for time at the end of the paper, probably because they had spent too long on their attempts at **Questions 5** and **7**. Candidates need to ration the time spent on questions that cause them difficulty.

The detailed comments that follow draw attention to common errors though there was a fair number of scripts showing a complete understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

## Comments on Specific questions

### Question 1

This was only fairly well answered. Candidates need to be secure in working on literal equations and inequalities. The presence of literal constants seemed to unsettle many candidates. Rather than obtain the critical values by solving the simple linear equations  $(x + 2a) = \pm 3(x - a)$ , the majority squared the given inequality and then tried to solve a non-modular quadratic equation or inequality. Apart from the error of failing to square the factor of 3, mistakes in handling the literal coefficients and constants were frequent and some gave a value to the variable  $a$ .

Answer:  $\frac{1}{4}a < x < \frac{5}{2}a$

### Question 2

This was well answered by those with a sound understanding of logarithms and indices, but there were many poor attempts. Common errors included taking  $\ln(5 - e^{-2x})$  to be  $\ln 5 - \ln e^{-2x}$  or taking  $e^{\frac{1}{2}} + e^{-2x}$  to be  $e^{-x}$  or, alternatively, to be  $e^{\frac{1}{2} - 2x}$ .

Answer:  $-0.605$

### Question 3

The expansion of  $\cos(x + 30^\circ)$  was usually stated correctly and many candidates went on to find a correct value for  $\tan x$ . Candidates need to be reminded to check this type of question carefully to ensure that their solutions lie within the required interval. Many gave one correct solution, but far fewer gave the correct pair of solutions. A common error was the omission of  $-66.2^\circ$  and another was the inclusion of  $66.2^\circ$ .

*Answer:*  $-66.2^\circ, 113.8^\circ$

### Question 4

This was generally well answered. In part (i) most candidates had the correct approach and differentiated at least one of  $x$  and  $y$  correctly with respect to  $t$ . Common errors included taking the derivative of  $x$  to be  $-\sec^2 t$ , and the derivative of  $y$  to be  $1/\cos t$ . It was quite common for candidates with incorrect derivatives to manipulate their quotient and claim to have reached the given answer. In part (ii) some found a value for  $\tan^{-1}\left(\frac{1}{2}\right)$  in degrees instead of radians. Some did not go on to calculate the value of  $x$ , or simply presented their value of  $t$  as the value of  $x$ .

*Answer:* (ii)  $-0.0364$

### Question 5

The response to part (i) was disappointing. Only a minority obtained a correct derivative of  $f(x)$ . Expressions such as  $2(x - 2)g(x)$  or  $2(x - 2)g'(x)$  were much more common. Of those with a correct derivative nearly all went on to factorise it or to show that  $f'(2) = 0$ .

In part (ii), many started by substituting  $x = 2$  in the given polynomial and obtaining a correct equation in the unknowns. Those who applied the result of part (i), as suggested in the wording of the question, easily obtained a second equation and completed the problem quickly. Failure to differentiate the constant term of the polynomial was the commonest error here. However a large number of time consuming attempts were seen which involved identifying the polynomial with  $(x - 2)^2 (Ax^3 + Bx^2 + Cx + D)$ , or dividing it by  $(x^2 - 4x + 4)$ . Only a few of these attempts sustained accuracy for long enough to make useful progress.

*Answer:* (ii)  $a = -4, b = 3$

### Question 6

Part (i) was poorly answered. It hinged on initially finding that  $AB = 2r\cos x$ , or equivalent. But many candidates took  $AB$  to be equal to  $r$  or  $r\cos x$ . As a result there were not many valid attempts at establishing the given expression. In part (ii) many candidates found the value of  $\cos^{-1}\left(\frac{\pi}{4 + 4x}\right)$  for  $x = 1$  and  $x = 1.5$ ,

but failed to compare their answers with the corresponding values of  $x$ . Some concluded that their two positive answers indicated that there was no root in the given interval and some just changed the sign of one of their values in order to exhibit a sign change. Such candidates need to be reminded that the 'sign change rule' only applies when attempting to approximate solutions of equations of the form  $f(x) = 0$ . Many carried out the iterations well in part (iii) and observed the requirement to give iterates to 4 decimal places, but a surprisingly large number rounded their final answer to 1.20 rather than 1.21. Another common error was to perform the calculations in degree mode rather than in radian mode.

*Answer:* (iii) 1.21

### Question 7

- (a) Although most candidates made a good attempt at substituting  $-1 + (\sqrt{5})i$  in the given equation, many of them failed to maintain perfect accuracy in the subsequent expansion and collection of terms. Most candidates knew that the conjugate was also a root of the equation and stated it correctly. But some omitted to state a second complex root.
- (b) There was a number of different lines of attack here but in general candidates found this a difficult problem and only a few were completely successful. The most popular approach was to substitute  $w = \cos 2\theta + i \sin 2\theta$  and use double angle formulae. It was quite common to see accurate work along these lines reach  $\frac{-\sin \theta + i \cos \theta}{\cos \theta + i \sin \theta}$  and get no further. A less frequently seen approach was to multiply the numerator and denominator of the given expression by  $w^* + 1$ . Substitution and simplification gives  $\frac{2i \sin 2\theta}{2 + 2 \cos 2\theta}$  and the required result follows.

Answer: (a)  $a = -12$ ;  $z = -1 - (\sqrt{5})i$

### Question 8

Part (i) was answered well in general. The majority of candidates obtained the first derivative correctly. The most common source of error seemed to stem from misapplying or not applying the chain rule with, for example, the derivative of  $\cos \frac{1}{2}x$  being taken to be  $-\frac{1}{2} \sin x$  or  $-\sin \frac{1}{2}x$  respectively. The attempts at the second derivative were less successful. Some candidates did not seem to realise that the product rule was needed again, and some solutions contained only two terms. Part (ii) was also generally done well. Many of the attempts at integration by parts had the right structure, reaching an indefinite integral of the form  $a \sin \frac{1}{2}x + b \cos \frac{1}{2}x$ . The main errors seemed to be the inaccurate integration of the two trig functions of  $\frac{1}{2}x$ . At the end some candidates failed to give an exact answer.

Answer: (ii)  $2\pi - 4$

### Question 9

In part (i) many candidates merely verified that the given values fitted the given equation, rather than set up an equation of the form  $\frac{dN}{dt} = kN(1 - 0.01N)$  and then show that the constant of proportionality  $k$  was equal to 0.02. Many candidates failed to give sufficient numerical working to justify the given value of  $k$ .

In part (ii) the separation of variables was usually correct and the majority of candidates realised that some form of partial fractions was required in order to attempt the integration of the term in  $N$ . Among those who integrated correctly, there were some who omitted a constant of integration, some who found an expression for  $N$  in terms of  $t$  and others who never obtained an explicit expression for  $t$  in terms of  $N$ . Nevertheless much good work was seen in this part and some candidates were able to complete part (iii) correctly.

Answer: (ii)  $t = 50 \ln(4N/(100 - N))$ ; (iii)  $t = 49.0$

### Question 10

- (i) Most candidates based their attempt on the scalar products  $\overrightarrow{AB} \cdot \overrightarrow{AC}$  or  $\overrightarrow{BA} \cdot \overrightarrow{CA}$  and there were many correct solutions. The commonest errors were arithmetical slips in calculating the components of the vectors, and the use of products such as  $\overrightarrow{AB} \cdot \overrightarrow{CA}$ , leading to  $-\frac{20}{21}$ . A small number of candidates avoided the use of the scalar product by finding the lengths of the sides of triangle  $ABC$  and applying the cosine rule.
- (ii) Not many candidates derived and used an exact value of the sine of angle  $BAC$  in an attempt to find the exact value of the area of the triangle  $ABC$ . Most used decimal approximations to the

angle, applied the formula  $\frac{1}{2} bcsinA$  and often reached the answer 3.20. The omission of the factor of  $\frac{1}{2}$  in the formula was a fairly common error. Unsound attempts at finding the length of an altitude were also seen.

- (iii) Most candidates understood how to use the normal vector to form the equation of the plane. However a substantial number lacked a sound method for finding a normal vector. This was often because they were unable to state a correct direction vector for the y-axis.

Answer:  $\frac{20}{21}$ ; (ii)  $\frac{\sqrt{41}}{2}$ ; (iii)  $4x + z = 9$

# MATHEMATICS

Paper 9709/33

Paper 33

## General comments

Whilst good attempts were seen for all questions in this paper, **Question 5**, the second part of **Question 6** and **Question 9** proved difficult for many candidates.

In general, the presentation of the work was good and most candidates attempted all questions. As mentioned in reports on previous papers, when attempting a question candidates need to be aware that it is essential that sufficient working is shown to indicate how they arrive at their answer, whether they are working towards a given answer, for example as in **Question 3(i)** and **Question 8(ii)**, or answers that are not given, as **Question 3(ii)** and **Question 9(i)**. Candidates are expected to use only an electronic calculator and if for whatever reason they have a calculator that will produce the solution immediately, such as evaluate

$\frac{1}{2}(e^2 - 1)$  in **Question 9(i)** and the answer of 0.896 for **Question 9 (ii)** with no working, then no marks will be awarded for those sections. In addition, candidates should realise that where they have made an error, as often happened in **Question 9(ii)**, then it is even more necessary to show the details of the solution of their quadratic equation and not just 2 incorrect answers from their calculator. The latter will result in the method mark for the solution to this equation being withheld.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

## Comments on Specific Questions

### Question 1

Most candidates solved the equation successfully, although the use of  $e^2$  was regularly seen. A very common error was an answer of  $x = 11$ , having reached  $99x = 9$ .

Answer:  $x = \frac{1}{11}$

### Question 2

Again many correct answers were seen, however a few candidates made an error in simplifying one of the coefficients. Multiplying the final answer by a factor of 3 to clear the denominator was penalised by the loss of one accuracy mark.

Answer:  $1 - x + 2x^2 - \frac{14}{3}x^3$

### Question 3

Most candidates were successful in obtaining the given answer, although, as mentioned earlier, several candidates did not show sufficient working on their way to the given answer and as a result lost at least one accuracy mark. Fortunately, most candidates obtained the correct quadratic equation that had to be solved, although a few candidates had an additional constant term of  $\sqrt{3}$ . Any error in the solution of the quadratic equation obtained directly from the calculator, as opposed to using the formula approach, meant that no method mark could be awarded.

Answer:  $21.6^\circ$  and  $128.4^\circ$

### Question 4

- (i) Many candidates did not score full marks here for a variety of reasons. Some only looked at the value of the RHS of the equation. Others looked at the values of both the RHS and the LHS but did not explain exactly what was meant by  $LHS > RHS$  and  $LHS < RHS$  at  $x = 1$  and  $x = 2$ . The correct approach is to establish the equation in the form  $f(x) = 0$ , and then to look at the change in sign between the numerical values of  $f(1)$  and  $f(2)$ .
- (ii) Most candidates correctly established that the iterative formula could be converted into the original equation, or vice versa. The use of a common  $x$  was not penalised, although the value of  $x$  should have been replaced by  $\alpha$  throughout the iterative formula and the equation. A few candidates thought that convergence of the sequence meant that the value of  $\alpha$  was required, hence doing what was required in **part (iii)** in **part (ii)**.
- (iii) Most candidates obtained the correct answer, working to the correct number of decimal places, as well as showing convergence to the required accuracy.

Answer: (iii) 1.14

### Question 5

Many candidates found this question difficult. Most could separate correctly, but then experienced trouble since they were unable to convert  $1/(2\cos^2 \theta)$  into  $(1/2)\sec^2 \theta$ . A few candidates managed to obtain part of the expression with  $2\sec^2 \theta$ , however most tried to integrate  $1/(2\cos^2 \theta)$  directly and a wide array of incorrect integrals, many involving logarithms of trigonometric functions, were seen.

Answer:  $x = \frac{1}{8}(\tan \theta + 1)^2 - \frac{1}{2}$

### Question 6

Candidates made things difficult for themselves by multiplying the brackets out prior to attempting their differentiation, hence usually introducing errors. The easiest approach was with implicit differentiation and then with  $dy/dx = 0$  introduced. Most candidates opted to expand, then differentiate, followed by establishing an expression for  $dy/dx$ , before introducing  $dy/dx = 0$ . Usually by this stage,  $dy/dx$  was incorrect. Candidates were then often at a loss as to whether the numerator or the denominator or both should be zero, and then what to do with this/these equations. Often candidates had already established an incorrect value of  $x$  or  $y$  and so the process of substituting the equation they had established back into their original equation was irrelevant.

Answer:  $(\frac{1}{2}\sqrt{3}, \frac{1}{2})$

### Question 7

- (a) Some excellent solutions were seen to this part; errors were rare.
- (b)(i) The centre and circle were usually correct, but the perpendicular bisector was too often either not perpendicular or passed through the centre of the circle. Most candidates opted for the wrong segment of the circle.
- (ii) Few candidates explained which angle they were trying to find, and even those who realised that it had something to do with the tangent to the circle and hence showed a tangent to their circle, then found the angle between this tangent and a line to the centre of the circle. Too many candidates who knew which angle was required believed that using the coordinates of a point on the tangent would produce an accurate value, rather than using simple trigonometry involving the coordinates of the centre of the circle.

Answers: (a)(i)  $-1 - i$  (b)(ii) 0.927 radians or  $53.1^\circ$

### Question 8

- (i) Errors were rare in this part.
- (ii) Integration of the term involving  $A$  often involved incorrect sign in the coefficient, whilst the term involving  $B$  regularly was an incorrect integral, often considering only the part  $\frac{1}{2+x^2}$  of this term so producing an incorrect expression such as  $x \ln(2+x^2)$ . With the answer given it was essential to show ALL the substitution of the limits and the logarithmic work in order to gain the method and the accuracy marks.

Answer: (i)  $A = 3, B = 3, C = 0$

### Question 9

- (i) Many candidates decided to take  $y = e^{2\sin x}$  instead of the given expression for  $y$ . This error was very common, but it was difficult to understand, or to see why it was so prevalent. Despite it saying "find an exact value" many candidates opted for a calculator approach and a decimal answer, so resulting in the loss of at least a couple of marks.
- (ii) Following an incorrect differentiation it was difficult for candidates to obtain a quadratic equation in  $\sin x$  that they could solve. But again many who had errors lost the method mark for the solution of their similar equation by showing no method.

Answers: (i)  $\frac{1}{2}(e^2 - 1)$  (ii) 0.896

### Question 10

- (i) Some good work was seen from most candidates, but, having established a correct equation for  $\lambda$ , too many candidates then made a basic arithmetical error.
- (ii) Most candidates knew what to do, and to use the normal to the plane. However, too often the other vector was not the direction vector of  $l$ . When candidates were successful in choosing the correct vector from  $l$  they too often did not complete the solution, leaving their answer as  $66.8^\circ$ .
- (iii) Many correct answers were seen, but often candidates did not know how to establish the normal to the second plane, or which vectors to use in their method. For the candidates that had a sound method, usually the cross-product approach, sign errors in establishing the normal vector were seen often.

Answers: (i)  $-\frac{1}{2}\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$  (ii)  $23.2^\circ$  (iii)  $4x + 19y + 13z = 29$

# MATHEMATICS

Paper 9709/41

Paper 41

## General Comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen. The presentation of the work was good in most cases.

Some candidates lost marks due to not giving answers to 3 significant figures as requested and also due to prematurely approximating within their calculations leading to the final answer, particularly in **Question 4**. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

One of the rubrics on this paper is to take  $g = 10$  and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct answer unless this value is used.

## Comments on Specific Questions

### Question 1

Most candidates performed very well on this question. They applied the relationship  $P = Fv$  and used the fact that the driving force was equal to the resistive force in this case. A large number of well presented correct answers was seen.

Answer:  $V = 47.5$

### Question 2

- (i) Many candidates solved this problem correctly by resolving forces perpendicular to the plane and equating this to the given normal reaction. An error that was occasionally seen was to use  $\sin \alpha$  instead of  $\cos \alpha$  when resolving forces.

Answer:  $\alpha = 16.3$

- (ii) Most candidates realised that this part of the question involved the application of  $F = \mu R$ . The majority of candidates found  $R$ , substituted into the equation  $F = \mu R$  and solved for  $\mu$ . Others realised that the value of  $\mu$  was simply the tangent of the angle found in part (i).

Answer:  $\mu = \frac{7}{24}$  or 0.292

### Question 3

This question was well done with most candidates resolving the given forces horizontally and vertically. There were some errors in signs and also  $\sin 30$  and  $\cos 30$  or  $\sin 60$  and  $\cos 60$  were sometimes mixed up when resolving. Often very little detail of the calculations involving the components of the forces was shown, with candidates simply quoting a value for the net horizontal and vertical forces. This did not matter provided the forces were found correctly, but for those who made errors in their calculations it could lead to a loss of method marks. It is an illustration of why candidates should be advised to show all of their working. The first 4 marks were often earned and although the resultant force was found by most candidates, a large number did not find the direction of the resultant which lost the final two marks.

Answer: Resultant = 1.23 N, Direction =  $152.9^\circ$  anticlockwise from the positive x-axis (or equivalent)

#### Question 4

Most candidates attempted this question by finding the height travelled by the particle until it came to rest and this was generally found correctly. It was then possible without further calculation to write down the total distance travelled although a few candidates not only doubled the height reached but also the given height above ground. Overall this question was well done by most candidates. A variety of methods was used to find the total time. The most obvious was to determine the time taken to reach the highest point and then to determine the time taken to reach the ground and add these times together. A few candidates realised that, with the upward direction taken as positive, the net displacement of the particle was  $-3.15$  and by using the constant acceleration equation  $s = ut + \frac{1}{2}at^2$  the total time taken could be evaluated in one step.

*Answer:* Total distance travelled = 11.25 m. Total time taken = 2.1 s

#### Question 5

- (i) This question was not generally well done by candidates. Many determined the total potential energy at the top of the incline when in fact the question referred to finding kinetic and potential energies at a distance  $x$  along the incline. This meant that very few candidates achieved the correct value of  $k$ . Although it was possible to answer the question using Newton's second law, the question used the word "hence" which was directing candidates towards using the work/energy method.

*Answer:*  $k = 5.5$

- (ii) There were similar problems in this part of the question and a common error was that as  $x$  was given as the distance travelled then the distance of the particle from  $A$  as it moved along  $AB$  was not  $x$  but  $x - 1760$ . It was possible to solve this problem by using Newton's second law or by using energy principles as no method was referred to in this part.

*Answer:* Given

#### Question 6

- (i) Most candidates made good attempts at this part of the question with the majority writing down Newton's second law for each particle and using it to determine the acceleration. Once candidates had found the acceleration, most simply used the constant acceleration equations to determine correctly the distance travelled.

*Answer:* Acceleration =  $5 \text{ ms}^{-2}$ . Distance travelled = 0.9 m

- (ii)  $V$  was found correctly by most candidates but the value of  $T$  was often wrongly given as 0.3 as candidates forgot that they had to add on the extra 0.6 seen on the graph in the question paper.

*Answer:*  $V = 3$ ,  $T = 0.9$

- (iii) Only a few candidates managed to score full marks on this part. Often the distance travelled upwards was found correctly but errors were made when considering the downward motion.

*Answer:* Distance travelled upwards = 1.35 m, distance travelled downwards = 2.45 m,  $h = 2.45 - 1.35 = 1.1$

#### Question 7

- (i) This part was well done by almost all candidates. Valid confirmation that the distance travelled was 1600 m was completed by most and the speed of the cyclist  $P$  was also found correctly by almost all candidates.

*Answer:* Speed of  $P = 5 \text{ ms}^{-1}$

- (ii) This part was not generally well done. Candidates often continued to assume that cyclist Q was also travelling with constant acceleration although the form of velocity given showed that this was not the case. Only a few candidates correctly integrated the given velocity expression and equated this to the distance travelled. Most were able to show where the maximum speed occurred although the wrong value of  $k$  had already been found.

Answer:  $k = \frac{4}{3}$ , Maximum speed of Q =  $\frac{16}{3} \text{ ms}^{-1} = 5.33 \text{ ms}^{-1}$

- (iii) This part was well done by most candidates with the time taken found correctly by the majority of candidates. Most candidates then used the value of the time taken to travel from B to C along with the given information to attempt to find the acceleration  $a$ , although in many cases the initial velocity had not been found correctly. However, candidates were still able to earn the method marks.

Answer: Time taken to travel from B to C = 280 s,  $a = \frac{11}{420} = 0.0262$

# MATHEMATICS

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Paper 9709/42

Paper 42

## General Comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen. The presentation of the work was good in most cases.

Candidates should be advised not to divide their written answer page vertically as this often makes it much more difficult to follow the candidate's work and their arguments.

Some candidates lost marks due to not giving answers to 3 significant figures as requested and also due to prematurely approximating within their calculations leading to the final answer, particularly in **Question 3**. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

In **Question 3**, triangles were shown in the question such that sines and cosines of angles could be determined to enable candidates to perform exact calculations. However, many candidates often then proceeded to find the relevant angles to 1 decimal place and immediately lost accuracy and in some cases marks.

One of the rubrics on this paper is to take  $g = 10$  and it has been noted that virtually all candidates are now following this instruction. In fact in some cases such as **Question 5 (iii)** it is impossible to achieve the correct answer unless this value is used.

## Comments on Specific Questions

### Question 1

- (i) Most candidates attempted to use the relationship  $P = Fv$  in order to determine the driving force and then proceed to write down Newton's second law for the motion. Some lost marks by misquoting the equation as  $F = Pv$  which lost the majority of the marks. Overall this question was well done by most candidates.

Answer:  $R = 410$

- (ii) This part simply involved seeing that, with  $a = 0$ , the driving force was equal to the resistance force. Multiplying the answer to part (i) by 15 gave the answer and hence the mark.

Answer: Rate of working of car's engine = 6150 W

### Question 2

- (i) There were some good answers to this question but a variety of errors were seen as well as several misunderstandings as to what approach should be taken. The best method was to determine the distance travelled by each particle in time  $T$  and state that the sum of these two expressions was 10. Some used the difference equal to 10, confused by the fact that the particles were travelling in opposite directions. Others equated the distances travelled by each particle, thinking that they collided at the mid-point, but did not introduce the fact that the particles travelled a total of 10 m. It is possible to solve the problem by considering the distance,  $x$ , travelled by one of the particles and  $10 - x$  by the other and a few candidates adopted this approach.

Answer:  $T = 5$

- (ii) This just needed candidates to use the equation  $v = u + at$  with  $u = 0$  for the particle  $P$  and, as it was a follow through mark, most candidates scored here.

Answer: Speed of  $P = 2.5 \text{ ms}^{-1}$

### Question 3

The standard method followed by most candidates here was to resolve forces horizontally and vertically. Some candidates used the given triangles and did not determine the specific angles. However, many found the angles, losing accuracy, and hence often lost the final accuracy mark. A common mistake was to ignore or not realise the existence of the weight of the particle. This error lost candidates half of the available marks. Another error was to assume that the tensions in the two strings were the same which lead to inconsistent equations. In spite of this there were many very good answers to this question.

Answer: Tension in AP is 11.9 N. Tension in BP is 0.5 N

### Question 4

- (i) Most candidates differentiated correctly to find the acceleration, although occasionally the expression for the velocity was integrated and several candidates differentiated correctly but added a spurious '+ c' to their answer. The decrease in acceleration was shown correctly by the majority of candidates.

Answer: Acceleration of  $P = \frac{t^{\frac{1}{3}}}{3}$  for  $8 \leq t \leq 27$  with  $a = \frac{1}{6}$  at  $t = 8$ . Decrease =  $\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

- (ii) The distance travelled was often treated as a single integral from time  $t = 0$  to  $t = 27$  when in fact different conditions held over the first 8 seconds. Some candidates were confused by the two part journey and integrated from  $t = 0$  to  $t = 19$  for the second part. Those candidates who completed the question correctly firstly found the distance travelled over the first 8 seconds to be 8 m and then either used definite integration from  $t = 8$  to  $t = 27$  or used indefinite integration with the constant of integration being found from the distance travelled at  $t = 8$ .

Answer: Distance moved by  $P = 71.3 \text{ m}$

### Question 5

Overall candidates found this question to be the most difficult on the paper. There was often confusion with units and a poor understanding of the principles of work and energy.

- (i) This was straightforward for most but some candidates subtracted the two kinetic energy contributions instead of adding them or did not attempt to combine the two values.

Answer: Kinetic Energy =  $10.5 v^2$

- (ii) (a) Here many only considered the particle of mass 16 kg or did not consider that the 5 kg particle was not moving directly upwards and so needed a component of  $x$  when evaluating its potential energy

Answer: Loss of Gravitational Potential Energy =  $135x$

- (ii) (b) The performance of candidates on this part was much better. Most found the reaction and proceeded to use  $F = \mu R$  correctly. Occasionally dimensional errors were seen with  $R = 5 \cos 30$  used rather than  $R = 50 \cos 30$

Answer: Work done against friction =  $25x$

- (iii) Very few candidates arrived at this part of the question with all three expressions found correctly in their earlier work and so were unable to complete the work energy equation correctly. The method mark was available to candidates who correctly used their own values from parts (i) and (ii) in a work/energy equation.

Answer:  $135x = 10.5v^2 + 25x$  leading to the given expression

### Question 6

This question needed some careful thought but, apart from a couple of misconceptions, was particularly well done by many candidates.

- (i) Most found the speed at which the particle entered the liquid and then used this to find the deceleration. A common error when using Newton's second law to find  $R$  was that the weight of the particle was ignored. Many candidates scored well on this part.

*Answer:* Deceleration =  $67.5 \text{ ms}^{-2}$   $R = 15.5$

- (ii) Again most candidates made reasonable attempts at this part of the question. The acceleration was almost always found correctly but again the weight was frequently ignored when finding the tension in the string using Newton's second law.

*Answer:* Tension = 17.6 N

### Question 7

- (i) This part was well done by some candidates who applied Newton's second law to both particles  $A$  and  $B$ . However many thought that friction was acting directly on these particles which led to incorrect expressions for the tensions.

*Answer:*  $T_A = 2.5 + 0.25a$ ,  $T_B = 7.5 - 0.75a$

- (ii) Candidates used their expressions found in (i) here when applying Newton's second law to particle  $P$ . Some good answers were seen but often either  $T_A$  or  $T_B$  was missing from the calculation or incorrect expressions for the tensions were carried through from part (i).

*Answer:* given

- (iii) Many correctly performed the calculation required to find the height from which the particle fell and then used the correct equation to determine the speed.

*Answer:* Speed =  $1.2 \text{ ms}^{-1}$

- (iv) This part was very poorly done. Candidates not only had to realise that  $T_B$  was now zero but also that  $T_A$  now took on a new value. Many used  $T_A = 3$  from the earlier motion or thought that only friction was acting. Some attempted to use constant acceleration formulae which was not possible in this case due to the fact that the actual forces had to be considered.

*Answer:* Deceleration =  $6 \text{ ms}^{-2}$

# MATHEMATICS

Paper 9709/43

Paper 43

## Key messages

- When solving any problem involving forces e.g. **Question 3**, and in particular when there is an inclined plane e.g. **Question 2**, **Question 5(i)** and **Question 7(i)**, a force diagram may help to prevent the omission of a component of force in the candidate's solution.
- Whilst answers were usually given rounded to 3 significant figures as required, candidates need to maintain sufficient accuracy in their working to achieve a correct 3 significant figure answer.

## General comments

The paper provided the opportunity for candidates to show what they knew with aspects of every question differentiating between stronger and weaker candidates. As usual the full range of marks was seen. Most candidates attempted all questions and many provided clearly written solutions supported by appropriate diagrams and explanation. **Question 2** and **Question 4(i)** were well answered questions. **Question 6** parts (ii) and (iii) and **Question 7(iii)** were found to be more difficult.

## Comments on specific questions

### Question 1

Although many candidates found this a straightforward question and gained full marks, there was some confusion with the language of the question in part (i).

- (i) 'The normal component of the force exerted on B by the ground' was sometimes understood to mean  $X\cos 15^\circ$ , the vertical component of  $X$ .  $X\cos 15^\circ = 70$  and  $X\cos 15^\circ = 70 \rightarrow X = 70/\cos 15^\circ$  were also incorrect answers, suggesting that some candidates did not understand what they had been asked to find.
- (ii) This part of the question was better attempted and even candidates who had scored no marks in part (i) sometimes scored full marks in part (ii). Whilst  $F = \mu R$  was usually applied, common erroneous answers were  $X = 108$  (using  $R = 70$ ) and  $X = 220$  (using  $R = X\cos 15^\circ - 70$ ). Three significant figure accuracy was lost by candidates who used approximations such as  $\sin 15^\circ = 0.97$  in their working.

Answer:  $70 - X\cos 15^\circ$ , 43.4

### Question 2

This question was often well answered with many candidates gaining full marks. Most candidates applied Newton's Second Law up the plane and used Driving Force =  $23000/v$ . Those who had learnt a formula, e.g.  $P/v = (ma + R)$ , omitted the component of weight down the plane and found  $v = 18.8 \text{ ms}^{-1}$ . Others used  $mg\sin\alpha$  instead of  $mgs\sin\alpha$  to obtain  $v = 18.4 \text{ ms}^{-1}$ . Less common errors seen included a sign error or the use of  $\cos\alpha$  instead of  $\sin\alpha$ .  $P/v = mgs\sin\alpha + R$  suggesting equilibrium and  $P = mav$  suggesting acceleration on a frictionless, horizontal surface were also used. A force diagram could help to avoid some of these errors.

Answer:  $15.6 \text{ ms}^{-1}$

### Question 3

Candidates were expected to resolve forces in two perpendicular directions and to solve the resulting equations. Whilst the majority of candidates attempted to do this, a surprising number omitted to consider the weight 1.4 N of the particle  $P$  and a few believed that the tension in the two strings was equal. When drawing a force diagram, candidates should check that they have considered all relevant forces.

The solution of the simultaneous equations was completed in a variety of ways including some neat solutions using exact values for sine and cosine of the appropriate angles. However, many candidates made errors when manipulating their equations, or when using the calculator, or through approximations which led to some inaccuracy.

Whilst Lami's Theorem was not a suitable method of solution for this situation of four forces, a few of the candidates who omitted the weight attempted this method and were able to obtain the same results (1 N and 1.56 N) as those who resolved without 1.4 N.

Answer: 2.5 N     0.26 N

### Question 4

- (i) This was generally well answered using various methods. The briefest solutions used 'Initial PE – KE lost = Final PE' whilst other methods included the calculation of one or more velocities. A few considered the velocity  $8 \text{ ms}^{-1}$  obtained from  $\frac{1}{2}mv^2 = \text{KE loss}$ , found the height of 3.2 m which could be reached with this initial velocity but then did not always realise that the actual height was (5–3.2) m.
- (ii) Candidates were less successful with this part of the question. Whilst many found the time taken to reach the ground correctly as 1 second, errors occurred through not accounting for the energy loss and thus assuming that the velocity on reaching the ground and on leaving the ground were equal ( $10 \text{ ms}^{-1}$ ). Thus  $t_2 = 0.36$  seconds from  $s = \frac{1}{2}(u+v)t$  and  $t_2 = 0.2$  seconds from  $s = ut + \frac{1}{2}at^2$  were common incorrect values given as the time taken for the second stage of motion. Other candidates used  $u = 8$  to obtain  $t_2 = 0.8$  seconds, whilst a few only considered the upward motion and gave the total time as 0.6 seconds.

Answer: 1.6 s

### Question 5

- (i) Successful candidates either attempted a work/energy equation or resolved along the plane to find the resisting force before calculating the work done against the resistance. Some found the resistance (2100 N) but omitted to calculate the work done. Others omitted to consider the effect of the weight of the lorry, missing either the weight component down the plane or the potential energy gain, depending on which method they used.
- (ii) Most candidates, as required, attempted to use an energy method and at least calculated the kinetic energy lost either explicitly or implicitly. Errors included omission of the work done against resistance or inclusion of 'potential energy' even though the surface was horizontal. The few who ignored the requirement to use energy and instead used Newton's Law could gain a correct value for  $F$  but were unable to gain full marks.
- (iii) Candidates who had a correct value for  $F$  in part (ii) often completed two appropriate calculations to show that the power at  $A$  and the power at  $B$  were the same. Others who did not have a correct value for  $F$  could still show a correct method but sometimes did not show enough working, perhaps when they realised that their two values would not be the same.

Answer:  $2.52 \times 10^6 \text{ J}$      3200

### Question 6

- (i) The majority of candidates gained both marks. Occasionally candidates substituted  $t = 59$  and  $t = 61$  into the two equations rather than  $t = 60$  into both. Some found the speed  $1 \text{ ms}^{-1}$  rather than the velocity  $-1 \text{ ms}^{-1}$ . Others calculated only one value for  $v$ , possibly believing that the velocity was the same before and after hitting the wall.
- (ii) Nearly all candidates knew that integration was needed, at least for calculating the distance ( $s_1$ ) before hitting the wall. Many obtained this distance, 54 m, correctly although a few persisted with a limit of 59 instead of 60. The calculation of the distance after hitting the wall ( $s_2$ ) proved more challenging. Those who found  $s_2 = \int v(t) dt$  with limits of 60 and 100 frequently obtained  $s_2 = -20$  and then calculated the total distance to be 34 m ( $54 - 20$ ) rather than 74 m ( $54 + 20$ ). Another common error was to substitute a single value  $t = 40$  ( $100 - 60$ ) into the integrated function to obtain a total distance of 134 m ( $54 + 80$ ). Those who involved a constant of integration were often not aware that they were finding displacement from a particular position rather than distance travelled, e.g.  $s(60) = 54 \rightarrow C = 159 \rightarrow s_2 = 34$  was erroneously interpreted as a total distance of 88 m ( $54 + 34$ ) rather than  $54 + (54 - 34)$ . The use of constant acceleration formulae was seldom seen in this part of the question although it was a suitable method for finding the distance travelled for  $60 < t \leq 100$ .
- (iii) Whilst candidates often found the maximum speed correctly, the graphical part of this question was not well answered. The quadratic part of the graph was frequently shown as linear and the linear part occasionally curved. Many attempts included three or four straight line segments with some showing  $v > 0$  for all values of  $t$ . A vertical line joining  $v_1(60)$  to  $v_2(60)$  was common.

Answer:  $1.2 \text{ ms}^{-1}$      $-1 \text{ ms}^{-1}$     74 m     $1.25 \text{ ms}^{-1}$

### Question 7

- (i) The majority of candidates attempted to form two simultaneous equations and to solve them to find  $a$  and  $T$ . Many were successful in finding  $a$  and  $T$  accurately. However, some treated the system as if there were no inclined plane, solving  $4.9 - T = 0.49a$  and  $T - 7.6 = 0.76a$  to obtain  $a = -2.16 \text{ ms}^{-2}$ . A few believed that  $P$  was moving down the plane rather than up.
- (ii) Most candidates applied  $v^2 = u^2 + 2as$  or equivalent but frequently used  $s = 30$  instead of  $s = 0.3$ . There was also some inconsistent use of signs if  $a < 0$  in part (i) to overcome the problem of  $v^2 < 0$ . A few candidates believed  $a = g$  rather than using the acceleration calculated in part (i).
- (iii) This part of the question was found to be challenging. Many persisted with the acceleration found in part (i) even after the string became slack rather than considering  $a = -g \sin \theta$  with  $\sin \theta = 40/160$ . Some used  $a = g/4$  rather than  $a = -g/4$ . Others correctly calculated the distance travelled up the plane by  $P$  to be 0.288 m but then gave the total distance travelled as  $0.3 + 2 \times 0.288$  instead of  $0.3 + 0.288$ .

Answer:  $2.4 \text{ ms}^{-2}$     3.72 N     $1.2 \text{ ms}^{-1}$     0.588 m

# MATHEMATICS

Paper 9709/51

Paper 51

## General Comments

Most candidates produced work that was neat, well presented and clearly argued but there were a few exceptions to this.

Only a few candidates used  $g = 9.8$  or  $9.81$  instead of the value of  $10$  recommended in the rubric.

Candidates should be reminded to refer to the formula booklet for certain formulae which they may need in answering questions.

Candidates found questions **1**, **6** and **7** to be easiest and questions **3(ii)**, **(iii)**, **4(ii)** and **5** to be hardest.

## Comments on Specific Questions

### Question 1

This question was generally well done with many candidates scoring both marks. Several different approaches were possible, all of which were seen.

*Answer:* Angle of projection is  $41.8^\circ$

### Question 2

- (i) This part of the question involved taking moments about the point A. Some candidates resolved the  $10\text{ N}$  force horizontally and vertically before taking moments and then omitted to use one of them. The correct equation should be  $10\cos 30 \times 1.2\sin\theta - 10\sin 30 \times 1.2\cos\theta = 6 \times 0.8\sin\theta$ .

The candidates who firstly found the perpendicular distance from A to the  $10\text{ N}$  force more often than not went on to find  $\theta$  correctly. The correct equation this time was  $10 \times 1.2\sin(\theta - 30) = 6 \times 0.8\sin\theta$ .

- (ii) This part of the question required the candidates to find the friction force  $F$  and the normal reaction  $R$ . They then needed to use  $F = \mu R$ , where  $\mu$  is the coefficient of friction.

*Answers:* (i)  $\theta = 47.0^\circ$  (ii)  $\mu = 0.532$

### Question 3

- (i) This part of the question was generally well done.
- (ii) A 4 term energy equation was needed in this part of the question. Too many candidates had only 3 terms or failed to find the correct potential energy. The correct equation should be  $0.4u^2/2 + 16 \times 0.2^2/(2 \times 0.8) + 0.4g(1.4 - 1.0) = 16 \times 0.6^2/(2 \times 0.8)$ .
- (iii) Here a 3 term energy equation is required. Often the EE term was omitted and only a PE term and a KE term appeared.

*Answers:* (i) Extension =  $0.2\text{ m}$  (ii) Speed of projection of P =  $2.83\text{ ms}^{-1}$  (iii) Speed of P when the string first becomes slack =  $2.45\text{ ms}^{-1}$

#### Question 4

- (i) There were many correct methods possible for this part of the question. The quickest and neatest method was to use the energy equation  $mx20^2 - mx18^2 = mgh$ . Some candidates simply said  $18^2 = 20^2 - 2gh$  which is not acceptable.
- (ii) Again here there were many correct methods possible. This part of the question proved to be very difficult for many candidates. A good neat solution was to use  $s = 3.8$  from part (i) and substitute into  $s = ut + at^2/2$ . This gave  $3.8 = (20\sin 40^\circ)T - gT^2/2$ ,  $T = 2.230$  and  $0.341$ .

Hence time required =  $2.230 - 0.341 = 1.89$ s. Horizontal distance =  $(20\cos 40^\circ) \times 1.89 = 29.0$ m.

Answers: (i) Height of P above the ground = 3.8m. (ii) Time speed of P is less than  $18\text{ms}^{-1} = 1.8$  s and horizontal distance = 29.0m.

#### Question 5

This question proved to be the most difficult question on the paper.

- (i) Some candidates used the incorrect formula for the centre of mass of the semi-circular arc. Too many candidates used 1.8m as the diameter and not the radius. Often the value found for the centre of mass of the semi-circular arc was left as the final answer.
- (ii) Here again a wrong formula was often used for the centre of mass of the semi-circular lamina. Not many candidates managed to set up a correct moment equation and, of those who did, the accuracy was often not very good. Too many terms were prematurely approximated.

Answers: (i) Distance of centre of mass of frame from O = 0.700m. (ii) Weight of frame = 37.3 N.

#### Question 6

This question was well done by many candidates. Occasionally candidates tried to use rectilinear equations of motion with constant acceleration.

- (i) On a number of occasions  $\int \frac{dv}{10-5v} = -5\ln(10-5v)$  was seen instead of  $-\frac{1}{5}\ln(10-5v)$ . Some candidates assumed that c was zero.
- (ii) This part of the question was well done by most candidates. Some candidates used  $0.6dv/dt = -3v$  and then tried to find  $t$ . They then went on to try to find  $x$ . This was not possible.

Answers: (i) Time for P to reach the ground = 0.738s. (ii) Distance travelled by P before coming to rest = 0.39m.

#### Question 7

Good marks were scored on this question by many candidates.

- (i) A few candidates used 42 not  $42m$  for the coefficient of elasticity.
- (ii) Some candidates found the velocity instead of the angular speed.
- (iii) Speed = 2.04 and -2.04 or 1.6 and -1.6 was seen quite often.
- (iv) The majority of candidates scored all 3 marks for this part of the question.

Answers: (i) Vertical component of contact force = 3.7mN. (ii) Angular speed of B =  $4.58\text{ rad s}^{-1}$  (iii) The two possible speeds of B are  $2.04\text{ ms}^{-1}$  and  $1.6\text{ ms}^{-1}$ . (iv) Speed of B =  $2\text{ ms}^{-1}$ .

# MATHEMATICS

Paper 9709/52

Paper 52

## General Comments

Most candidates produced work that was neat, well presented and clearly argued but there were a few exceptions to this.

Only a few candidates used  $g = 9.8$  or  $9.81$  instead of the value of  $10$  recommended in the rubric.

Candidates should be reminded to refer to the formula booklet for certain formulae which they may need in answering questions.

Candidates found questions **1**, **6** and **7** to be easiest and questions **3(ii)**, **(iii)**, **4(ii)** and **5** to be hardest.

## Comments on Specific Questions

### Question 1

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The candidates who firstly found the perpendicular distance from A to the  $10\text{ N}$  force more often than not went on to find  $\theta$  correctly. The correct equation this time was  $10 \times 1.2\sin(\theta - 30) = 6 \times 0.8\sin \theta$ .

- (ii) This part of the question required the candidates to find the friction force  $F$  and the normal reaction  $R$ . They then needed to use  $F = \mu R$ , where  $\mu$  is the coefficient of friction.

*Answers:* (i)  $\theta = 47.0^\circ$  (ii)  $\mu = 0.532$

### Question 3

- (i) This part of the question was generally well done.
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- (iii) Speed = 2.04 and -2.04 or 1.6 and -1.6 was seen quite often.
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Answers: (i) Vertical component of contact force = 3.7mN. (ii) Angular speed of B =  $4.58\text{ rad s}^{-1}$  (iii) The two possible speeds of B are  $2.04\text{ ms}^{-1}$  and  $1.6\text{ ms}^{-1}$ . (iv) Speed of B =  $2\text{ ms}^{-1}$ .

# MATHEMATICS

Paper 9709/53

Paper 53

## General Comments

Generally candidates' answers to the questions on this paper were neat and well presented with very few exceptions.

Very few candidates are now using  $g = 9.8$  or  $9.81$ . The rubric asks that  $g = 10$  be used throughout.

Candidates found questions **1**, **2** and **3(i)** to be easiest and Q. **5(ii)**, **6** and **7** hardest.

## Comments on Specific Questions

### Question 1

This question was generally well done with the majority of candidates scoring all 3 marks. An energy equation was attempted which required 3 terms. Sign errors did occur from time to time.

*Answer:* Speed of P is  $3 \text{ ms}^{-1}$

### Question 2

This question was well done by the majority of candidates with full marks being scored by many. Some candidates were unable to find a correct moment equation. Some candidates did not know where the centre of mass of a triangle was.

*Answer:*  $F = 8$

### Question 3

- (i) This part of the question was well done.
- (ii) In this part of the question too often  $0.2\text{d}v/\text{d}t = 0.42 + 0.32t$  was seen instead of  $0.2\text{d}v/\text{d}t = 0.42 - 0.32t$ . Many candidates attempted to integrate and applied the limits correctly. Weaker candidates failed to score marks on this part of the question.
- (iii) Again sign errors occurred. The correct equation should have been  $0.2\text{d}v/\text{d}t = 0.42 - 0.32t + 0.06t^2$ . Integration was used and limits were substituted. Incorrect limits were applied on too many occasions.

*Answers:* (i)  $v = 2.1$  (ii)  $v = 1.8$  (iii)  $v = 1.8$ . Same for  $t = 2$  and  $t = 3$ .

### Question 4

Full marks were scored by many candidates on this question.

- (i) By resolving vertically the tension in the string could be found. By applying Newton's Second Law horizontally the speed was then found.
- (ii) By using Newton's Second Law horizontally and using  $a = \omega^2 r$ , the tension in the string could be calculated. The angle was then found by resolving vertically using the new tension.

*Answers:* (i) Speed of P =  $6 \text{ ms}^{-1}$  (ii) Required angle =  $58.6^\circ$

### Question 5

- (i) Quite a number of candidates did not obey the instructions given in the question and used the trajectory equation to find the connection between  $x$  and  $y$ .
- (ii) The use of Pythagoras gave  $x^2 + y^2 = 18^2$ . If  $y = 0.2x^2$  was then substituted this gave a quartic equation in  $x$  and, when solved, gave the correct value of  $x$ . To find the speed of the ball as it strikes the ground, its horizontal and vertical components were needed. Pythagoras's theorem was then used to give the required speed.

Answers: (i)  $x = 5t$ ,  $y = gt^2/2$ ,  $y = 0.2x^2$  (ii)  $x = 8.85$  Speed immediately before it hits the ground is  $18.4 \text{ ms}^{-1}$

### Question 6

Quite number of candidates failed to score marks on this question.

- (i)  $T = \lambda x/L$  should be used here to give  $T$  in terms of  $\theta$ . When forces are resolved vertically for P then the normal reaction force can be calculated.  
 $T = 9 \times (0.4/\sin\theta)/1.5$ ,  $R = 0.6g - T \sin \theta$ . This leads to  $R = 3.6 \text{ N}$  where  $R$  is the normal reaction.
- (ii) Here the candidate was required to find the elastic energy at both A and B. An energy equation was then set up to find the work done against friction. Many candidates failed to score marks on this part of the question.
- (iii) Candidates needed to use  $F = \mu R$ . The friction force,  $F$ , comes from  $F = \text{Work done/distance}$ .  
 $F = 0.615/(0.4/\tan\theta)$  and  $R = 3.6$  from part (i).

Answers: (i) Normal force on P is  $3.6 \text{ N}$  (ii) Work done against friction is  $0.615 \text{ J}$  (iii) Coefficient of friction is  $0.247$

### Question 7

This question proved to be the hardest question on the paper.

- (i) Some candidates used the wrong formulae for the centre of mass of the handle. Note the correct formula can be found in the formula booklet. By using a moment equation the required centre of mass can now be found.
- (ii) This part of the question required the candidate to find both the horizontal and vertical distances of the centre of mass from O. Pythagoras's theorem is then used.
- (iii) The required angle  $\theta$  can be found by using  $\tan \theta = \text{horizontal distance/vertical distance}$ . These are the distances found in part (ii).

Answers: (i) Distance of the centre of mass from O is  $0.026 \text{ m}$  (ii) Distance of the centre of mass from O is  $0.118 \text{ m}$  (iii) Angle which the plane makes with the horizontal is  $36.0^\circ$

# MATHEMATICS

Paper 9709/61

Paper 61

## General comments

A considerable number of candidates found the paper too challenging. Many were unaware of both the Binomial and Normal Distributions and lacked the basic concepts of Statistics. There was no evidence of candidates having insufficient time. Many scripts lost marks through careless and untidy presentation; examples included reading off 0.516 instead of 0.5106 in tables, misreading their own writing, not using a ruler to draw lines and not working to at least 4 significant figures prior to an answer corrected to 3 significant figures.

## Comments on specific questions

### Question 1

Those candidates who were familiar with the Normal Distribution were comfortable with this standard type of question. It was pleasing to find very few instances of confusion between standard deviation/variance and the non-use of a continuity correction. The common errors were in the reading off of  $z = -0.5106$  for the probability of 0.6952 and in the establishment of the required probability. The latter is made easier with the help of a simple diagram.

Answer: 0.537

### Question 2

Again, the comments for the previous question are valid here. A diagram would clearly demonstrate the need to obtain the  $z$ -values corresponding to 0.96 and 0.68 as +1.751 and +0.468 respectively. The many good attempts at solving the simultaneous equations were all too often marred by substituting the first value obtained, corrected to 3 significant figures, and thus producing an error in the second value. It cannot be stressed enough that working should be at least to 4 significant figures if answers are required to 3 significant figures.

Answers:  $\mu = 7.91$  and  $\sigma = 2.34$

### Question 3

Many candidates were unfamiliar with the Binomial Distribution. The response from those who were familiar with it revealed inconsistencies and errors.

- (i) There was confusion, with the requirements for approximating the Binomial Distribution to the Normal Distribution often being cited. There were many lengthy presentations but all that was required were 3 of the answers stated below.
- (ii) The most common errors were interpreting 'probability at least 3' as 'probability of 3 exactly', adding the value of  $P(3)$ , omitting the value  $P(0)$  and truncating the values of the 3 constituent probabilities before adding.

Answers: (i) Constant / given probability,  
Trials are independent,  
Fixed / given number of trials,  
Only two outcomes.

(ii) 0.520

#### Question 4

- (i) The given answer of  $3/14$  was arrived at correctly in several ways with no particular preference, using combinations, multiplying a series of fractions or a tree diagram. Many of the latter were spoilt by untidy presentation and careless enumerating of probabilities. When an answer is given, the solution must be fully correct with all the explanation detailed. It is not sufficient to just simply to add 'x 6' without either  ${}^4C_2$  being quoted, the 6 relevant branches being identified on the tree diagram, or the 6 possible outcomes stated.
- (ii) A large minority of candidates did not realise that there were only 3 possible values of  $X$ . A few used  $E(X) = 3$  to realise symmetry and thus easily found the probabilities.
- (iii) Despite the reminder in the question that  $E(X) = 3$ , a surprising number of scripts omitted  $\{E(X)\}^2$  in the calculation.

Answers: (i)  $3/14$  **AG**

(ii)

|             |        |        |        |
|-------------|--------|--------|--------|
| x           | 2      | 3      | 4      |
| Probability | $3/14$ | $8/14$ | $3/14$ |

(iii)  $3/7$  (0.429)

#### Question 5

It was pleasing to see the many correct responses to this question and that conditional probability is becoming increasingly understood. The tree diagram approach was popular, but again some responses were marred by untidy presentation and careless enumerations.

- (i) The final mark was often lost by converting fractions into decimals and approximating prematurely, e.g.  $1/9$  as 0.11 instead of at least 0.1111
- (ii) A common error was in simplifying the denominator of  $\frac{1}{4} \times \frac{1}{12} + \frac{1}{2} \times \frac{4}{16}$

Those who resorted to a calculator invariably obtained the wrong answer.

Answers: (i)  $53/288$  (0.184) (ii)  $1/7$  (0.143)

#### Question 6

Parts (i) and (ii) were well understood by candidates but very few grasped that in the remainder of the question the constraints placed on  $N$  and  $A$  reduced the choices available.

- (i) The presence of the two 'N's was often overlooked and consequently division by  $2!$  was not seen.
- (ii) Again the two 'N's and the three 'A's were overlooked and  $4! \times 4!$  was given as the final answer.
- (iii) Only a handful of candidates recognised the importance of 'exactly 1 N and 1 A' meant that there were only 3 letters remaining to fill 2 places. Hence  ${}^3C_2$  was the required number of ways.
- (iv) Similarly, 'exactly 1 N' required the 4 separate cases of 0, 1, 2 or 3 'A's to be considered, i.e. filling 3, 2, 1 or 0 places with the 3 letters (T, Z and I).

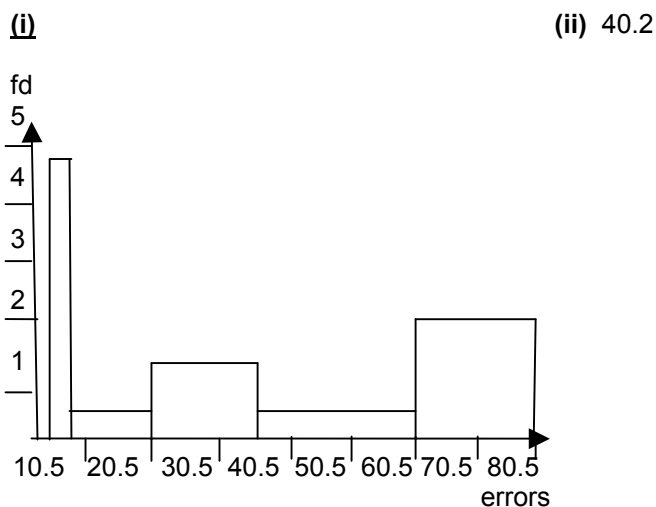
Answers: (i) 360 (ii) 48 (iii) 3 (iv) 8

### Question 7

Surprisingly, the majority of candidates scored less than half marks on this basic question on grouped data.

- (i) Very few Centres were familiar with 'frequency density' and most candidates produced a frequency histogram or a cumulative frequency curve instead. The class boundaries of 0.5, 5.5, 20.5, etc were often not realised and marks were lost on the graph through untidy presentation. A line across the top of a rectangle indicating the height (value) gets no credit if it is not horizontal or several mm thick.
- (ii) A very large minority of candidates did not use the 'mid class values', preferring to use wrongly the class widths or various class boundaries instead.
- (iii) Frequently 27.75 and 83.25 (or 28 and 84) were given as the quartile values and the question of which class was ignored.

Answers:



(iii) (6-20) Class, (61-80) Class, 41.

# MATHEMATICS

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Paper 9709/62

Paper 62

## Key Messages

To do well in this paper, candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working so that, in the event of a mistake being made, credit can be given for method; a wrong answer with no working shown scores no marks. Candidates should label graphs and axes including units, and choose sensible scales. It is not necessary to fill the entire page.

## General comments

This paper was well attempted by those candidates who had worked and prepared for it. The marks ranged from 0 to 50 with many candidates from certain Centres scoring good marks. Many candidates appeared to have covered all the topics and were able to make a start on the questions. There were some discriminating questions, in particular **Question 5**, which tested whether candidates could actually apply their mathematical knowledge of what a multiple of 5 was, to finding how many ways this could be done in the context of the question. Good candidates managed to answer all parts successfully.

There were however, too many untidy and poorly presented papers and some Centres where the candidates had clearly done no work and consequently only scored in single figures.

## Comments on specific questions

### **Question 1**

This question was well done by the majority of candidates, who realised that the binomial distribution had to be used. Some were unsure whether to use 19 or 12 for  $n$  but managed to pick up some method marks by showing their working using a binomial expression and finding  $P(0, 1, 2, 3)$  in a binomial situation. Many lost a mark at the end because they wrote the answer as 0.81 to 2 significant figures instead of 0.813. Those who used the normal approximation scored no marks as it is invalid.

*Answer:* 0.813

### **Question 2**

This question was also well done by many candidates. However, a surprising number misread the question and took it that there were three members in Year 3. Provided working was shown, credit was given for the correct methods.

*Answer:* 231

### **Question 3**

Most candidates could calculate the probability of Roger winning in 2 sets, but not Andy winning in 2 sets. Generous 'follow through' marks meant that candidates could still gain credit for the conditional probability in part (ii). This was generally well done although the correct answer of 0.417 was often seen as 0.42 which is only to 2 significant figures and hence lost a mark.

*Answers:* 0.72, 0.417

#### Question 4

When a question says 'Show that' it means that **more** explanation is required than might normally be needed, so that to gain the full 3 marks candidates had to show that there were three options and sum these. There were plenty of candidates who used the fact that

$$\frac{1}{9} + \frac{1}{4} = \frac{13}{36}$$

but no credit was given for this fortuitous result. The distribution table usually gained candidates a few marks and, even if they did not have all the probabilities correct, credit was given for knowing how to find the expected value.

$$\text{Answers: } P(0) = \frac{1}{12}, \quad P(1) = \frac{13}{36}, \quad P(2) = \frac{4}{9}, \quad P(3) = \frac{1}{9}; \frac{19}{12}$$

#### Question 5

This was the question which caused most candidates difficulty and succeeded in discriminating between the candidates. They could often cope with part (i) but were unable to analyse the situations in parts (ii) and (iii). Candidates did not appreciate the significance of digits being repeated or not repeated, nor did they realise that if a number is a multiple of 5 then the last digit has to be 5 as there is no zero to choose. Even for those who did realise that a number less than 1000 can have  $5^*$ , they did not appreciate that the number can also be 5 or  $5^*$  as well.

Answers: 720, 60, 57

#### Question 6

Part (i) should have been very straightforward but many candidates failed to read the question carefully and were unable to score the mark. Part (i) should have helped candidates to realise that frequencies were needed before they could find frequency densities to draw the histogram in part (ii). Generally the graph plotting was not good, with candidates drawing lines freehand, poor scales going up in 3s, no labelling of axes.

Answers: 6, 11.7, 0.547

#### Question 7

This question was well done. Even candidates who struggled with earlier questions often managed to gain 7 marks or more. In part (iii), candidates were told to use an approximation and this meant the normal approximation to the binomial. There were some candidates who could not find the correct parameters for the binomial distribution and so were unable to use the normal approximation. They instead used the mean and standard deviation from part (i). Credit was given for using the continuity correction but nothing more.

Answers: 0.653, 0.561, 0.321

# MATHEMATICS

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**Paper 9709/63**

**Paper 63**

## **Key Messages**

Candidates should be encouraged to show all necessary workings. A significant number of candidates did not show sufficient working to make their approach clear.

To do well in this paper, candidates must work to 4 significant figures or more to achieve the accuracy required in their final answers.

Candidates should be encouraged to sketch normal distribution graphs where appropriate.

## **General comments**

Answers to **Questions 2, 3 and 4** were generally stronger than answers to other questions.

The majority of candidates used the answer booklets provided effectively, however a number failed to utilise the available space appropriately, for example by answering the entire paper on a single page. Candidates should be reminded that, if they use supplemental answer booklets or graph paper, it is important to secure them appropriately to their main answer booklet.

A number of candidates made more than one attempt at a question and then did not indicate which their submitted solution was by crossing out other attempts.

## **Comments on Specific Questions**

### **Question 1**

Most candidates understood the concept of a stem-and-leaf diagram.

- (i) Good solutions had the data in order and the terms evenly spaced. Better candidates initially produced an unordered stem-and-leaf diagram. Many did not realise that the diagram enabled the distributions to be compared by the shape produced. Most candidates provided two separate keys for the diagram rather than the required single back-to-back key. Good candidates included the units that were required for the correct interpretation of the key.
- (ii) Most candidates made comparisons about the range and median of the data, although weaker candidates made statements without a comparison. A number of candidates commented on the shape of the distribution. A significant number of candidates commented twice on a single aspect of the data.

*Answer:* (ii) children's estimates more spread out, adults' estimates lower

### **Question 2**

Most candidates used the normal distribution formula correctly.

- (i) A very small number of candidates incorrectly used a continuity correction. Good candidates sketched the normal distribution to identify the areas required. A number of candidates evaluated the probabilities separately but failed to sum their answers.

- (ii) Few candidates provided the appropriate justification of both  $np > 5$  and  $nq > 5$ . Many candidates commented inappropriately on the variance here. A number of candidates who confirmed that the normal approximation was appropriate in part (i) did not interpret their answer at this stage and provided incorrect justifications.

Answer: (i) 0.184

### Question 3

Most candidates attempted this question successfully.

- (i) The majority of candidates used a combination approach to the probability, although a significant number also considered the 5 separate outcomes and then identified that there were  ${}^5C_2$  ways these could be arranged. A few candidates did not provide sufficient evidence to justify the final probability as it was given. Weaker candidates did not appreciate that the chosen pet was removed from the available pool.
- (ii) Almost all candidates produced a probability distribution table for  $x = 0, 1, 2$  and 3 only. It was common for candidates to use the same method as in part (i). Many candidates used the property that the sum of the probabilities was 1 to calculate the final entry. Good solutions ensured accuracy by working fully in fractions rather than introducing decimal rounding errors.

Answers: (ii)  $P(0) = \frac{2}{42}$ ,  $P(1) = \frac{15}{42}$ ,  $P(3) = \frac{5}{42}$

### Question 4

Many candidates gained full credit for this question. Good solutions showed clear algebraic manipulation throughout.

- (i) Almost all candidates attempted this question successfully. A few candidates did not use the correct number of people in the new group.
- (ii) Most candidates rearranged the standard deviation formula accurately to find  $\Sigma x^2$  of the original group. Many candidates stated this to 3 significant figures. Good solutions then used their accurate answer to calculate the removal of the person identified in (i) before using the formula again. Weaker candidates often used either their rounded interim answer or failed to remove the additional person before calculating.

Answers: (i) 173 (ii)  $\Sigma x^2 = 834700$ ,  $\sigma = 4.16$

### Question 5

Most candidates attempted to use the Normal Distribution and Binomial distributions appropriately. However, many candidates could benefit from additional focus on the correct interpretation of the Normal Distribution Function table.

- (a) Most candidates were able to standardise correctly, although a number were unable to find the required range. Good solutions had the algebraic manipulations clearly stated.
- (b) Good candidates appreciated that being within 1 standard deviation of the mean required  $z = 1$  to be used to calculate the probability of the identified phone calls. Most candidates used the Binomial Distribution correctly to evaluate the probability that more than 7 phone calls meet the time requirement. Weaker candidates included  $P(7)$  within their answer.

Answers: (a) 4.24 (b) 0.167

### Question 6

Most candidates had a clear understanding of the requirements of parts (i) and (ii). Good solutions often contained tree diagrams to identify the outcomes and statements to clarify the method being followed. Some candidates consistently reversed Ben and Tom in this question.

- (i) Almost all candidates multiplied 2 identical probabilities.
- (ii) Good solutions summed 3 correct options. Weaker solutions assumed that if Ben won the first 2 games he then played again and lost the third game to become champion.
- (iii) Few good solutions were seen for this part. Many candidates did not accurately state the conditional probability formula. A number of candidates assumed that the question still related to Ben. Good solutions utilised their answer from (ii) for the denominator and used a tree diagram to identify the correct options for the numerator. Poor solutions identified only one potential option for Tom to become champion if he won the second game.

Answers: (i) 0.49 (ii) 0.784 (iii) 0.708

### Question 7

Most candidates used the correct technique to evaluate part (i) and then failed to recognise the alternative approach required for part (ii). Some candidates were penalised because they assumed that the digits were chosen and replaced.

- (i) (a) Good solutions displayed workings that ensured the information provided was evaluated in a logical manner. Most solutions identified the number of ways the even digits could be arranged, and then removed the repeated solutions. The possible options for the placing of the remaining odd digits were then considered. A number of solutions did not consider the impact of having the repeated 3 at this stage.
- (b) Good solutions identified the 3 possible combinations that fulfilled the criteria and then evaluated the number of possible combinations that the remaining 7 digits could make. A few solutions failed to sum the results obtained. Weaker candidates often considered only two options, often excluding the 3...3 possibility.
- (ii) (a) Good solutions considered the number of ways each of the 3 digits could be chosen if there were no repeats, without explicitly stating that there were only 5 possible digits. A few candidates considered  ${}^5C_3$  without recognising that the order of the digits is essential.
- (b) Several different methods were possible for this part. As only 2 digits could be altered for the 3-digit number to be within the required range, many candidates listed all the possible solutions and then stated the number they had identified. This same method was applied in a more theoretical manner by considering the possible combinations available for each potential value of the second digit, or the considering permutations possible and adjusting for the repeated digits. Common errors were to consider  ${}^8P_2$  but without then subtracting the repeated solutions.

Answers: (i)(a) 720 (b) 1260 (ii)(a) 60 (b) 22

# MATHEMATICS

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Paper 9709/71

Paper 71

## General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a complete range of scripts from good ones to poor ones. Questions particularly well attempted were **Question 4** and **Question 6**, whilst **Question 7** proved to be quite demanding for candidates.

Candidates in general kept to the level of accuracy required, but presentation of answers, in particular the legibility of numbers, could have been better. It is important that candidates write numbers in a clear and unambiguous way. Examiners reported cases where legibility was poor and it was difficult to tell which digit had been written. If candidates wish to change a digit that they have written, it must be neatly crossed out and re-written. Over-writing could result in the loss of the mark.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

## Comments on specific questions

### **Question 1**

This question gave candidates a straightforward start to the paper, with most candidates successfully finding the unbiased estimate for the population mean. The variance was not so well attempted; there were a few cases where candidates found the biased variance, thinking that this was the answer required, and some candidates confused the two alternative formulae for the unbiased estimate of the population variance.

*Answers:* 250.75      38.5

### **Question 2**

This question was reasonably well done. Many candidates attempted to find the correct Normal Distribution  $N(8, 9.49)$  but errors were often made when finding the variance. Standardising and finding the correct tail for the probability was usually correctly done.

*Answer:* 0.0047

### **Question 3**

Many candidates realised that a Normal approximation was required and correctly used  $N(50, 50)$  or  $N(5, 5)$ . Candidates mostly remembered to state their Null and Alternative Hypotheses, and most used a correct two tail test, though weaker candidates still struggled to include all the necessary steps for the hypothesis test. The main error that prevented candidates from scoring full marks was either to apply an incorrect continuity correction or to omit one. It was pleasing to see many candidates clearly showing the required comparison (either as an inequality statement or on a clearly labelled diagram) and in most cases a valid comparison was made. A suitable conclusion with no contradictory statements, and preferably in the context of the question, was then required.

*Answer:* No evidence that the mean has changed

#### Question 4

This question was generally well attempted, with parts (ii) and (iii) in particular being a good source of marks even for weaker candidates. In part (i) a common error was to use  $\lambda$  as 1.5 rather than the correct value of 4.5. The method for the Poisson calculation was usually correctly applied, though there was the usual confusion caused by the statement 'greater than 2' which led to errors from some candidates.

Most candidates realised the need to use natural logarithms in part (ii) and successfully found the correct value for  $\lambda$ . In part (iii), the correct initial statement was usually given, with only careless algebraic errors causing loss of marks.

Answers: 0.826  
 $\lambda=0.648$   
 $\mu=12$

#### Question 5

Many candidates used the correct value for  $p$  (0.46), though an incorrect value of  $p$  (0.5) was occasionally seen. It was pleasing to note that most candidates were able to find, and use, the correct  $z$  value of 1.96 in their confidence interval formula. Many candidates used an expression of the correct form for the confidence interval. It should be noted that the answer here must be given as an interval; some weaker candidates gave their final answer as "0.411 or 0.509", or as "0.411 and 0.509", rather than an interval 0.411 to 0.509.

Part (ii) required candidates to compare 0.5 with the confidence interval found in part (i) and to draw the relevant conclusion depending on whether 0.5 was within their confidence interval or not. Many candidates did not realise what was required.

In part (iii) many candidates successfully found the correct  $z$  value (2.006), but were unable to convert this to the correct percentage confidence interval. Most candidates merely found  $\Phi(2.006)$  ( $=0.9775$ ) and incorrectly left their answer as 97.75%. Very few candidates used a correct method to find the required percentage.

Answers: 0.411 to 0.509  
Claim not supported  
95.5%

#### Question 6

Questions on probability density functions are usually well attempted. On the whole this was the case with this question though, in a proportion of responses, part (ii) was not always fully correct.

Part (i) was well attempted; but, as has been highlighted on occasions in the past, candidates must realise that when an answer is given it is important that all necessary steps in working are clearly shown. There were some cases here where marks were withheld due to lack of essential working, but the majority of candidates gave a convincing demonstration that  $k$  was  $1/64$ . In part (ii) candidates were able to find the probability of one person spending less than one hour on a visit, but often doubled this answer rather than squaring as required. Part (iii) was generally well attempted, though some candidates confused 'mean' and 'median'.

Answers: 0.0147  
2.13

#### Question 7

This was not a well answered question, the calculation parts being slightly better attempted than the parts that were testing statistical understanding. In part (i), whilst some candidates realised that the second method was more representative and were able to give suggestions as to why this was so, others suggested that the second method gave a random selection of appointments.

In part (ii) candidates were required to give their answer in the context of the question, not to merely quote text book definitions. This was not always successfully done.

In part (iii) a one tail test was required; not all candidates remembered to state their hypotheses, and the calculation of the  $z$  value caused problems for some candidates. However, a valid comparison was often successfully completed and followed by a suitable conclusion (preferably non-definite and in context).

Answers to part (iv) often indicated that candidates did not fully understand the Central Limit theorem. It was important that candidates appreciated that it was because the population was Normal that the theorem was not needed. Answers such as 'it' is Normal were not sufficient.

Answers:  $2^{\text{nd}}$  is more representative of *all* appointments  
0.01 Concluding that times spent are too long when they are not  
No reason to believe that appointments are too long  
Normal population

# MATHEMATICS

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**Paper 9709/72**

**Paper 72**

## General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a complete range of scripts from good ones to poor ones. Questions particularly well attempted were **Question 4** and **Question 6**, whilst **Question 7** proved to be quite demanding for candidates.

Candidates in general kept to the level of accuracy required, but presentation of answers, in particular the legibility of numbers, could have been better. It is important that candidates write numbers in a clear and unambiguous way. Examiners reported cases where legibility was poor and it was difficult to tell which digit had been written. If candidates wish to change a digit that they have written, it must be neatly crossed out and re-written. Over-writing could result in the loss of the mark.

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*Answers:* 250.75      38.5

### **Question 2**

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*Answer:* 0.0047

### **Question 3**

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*Answer:* No evidence that the mean has changed

#### Question 4

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Answers: 0.826  
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#### Question 5

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Answers: 0.411 to 0.509  
Claim not supported  
95.5%

#### Question 6

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Answers: 0.0147  
2.13

#### Question 7

This was not a well answered question, the calculation parts being slightly better attempted than the parts that were testing statistical understanding. In part (i), whilst some candidates realised that the second method was more representative and were able to give suggestions as to why this was so, others suggested that the second method gave a random selection of appointments.

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In part (iii) a one tail test was required; not all candidates remembered to state their hypotheses, and the calculation of the  $z$  value caused problems for some candidates. However, a valid comparison was often successfully completed and followed by a suitable conclusion (preferably non-definite and in context).

Answers to part (iv) often indicated that candidates did not fully understand the Central Limit theorem. It was important that candidates appreciated that it was because the population was Normal that the theorem was not needed. Answers such as 'it' is Normal were not sufficient.

*Answers:* 2<sup>nd</sup> is more representative of *all* appointments  
0.01 Concluding that times spent are too long when they are not  
No reason to believe that appointments are too long  
Normal population

# MATHEMATICS

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Paper 9709/73

Paper 73

## Key messages

- Candidates should not use the Normal distribution approximation unless it is appropriate.
- Candidates need to check if probabilities are conditional.
- When significance testing a discrete distribution the tail probability must be calculated.

## General Comments

Many candidates were able to demonstrate a good understanding of the topics covered by this paper. **Questions 1, 3, 4, 5, and 7(iii) and 8** were very well done in the main, **Question 8** which tested sums and multiples of Normal variables particularly so. The most demanding questions were **Questions 2, 6 and 7(iv)**. Solutions were generally well presented, and answers were given to an appropriate level of accuracy. A minority of candidates worked with too few significant figures to allow them to give their final answers correct to 3 significant figures.

## Comments on Specific Questions

### **Question 1**

Candidates were required to identify that the appropriate approximation was to the Poisson distribution and those who did went on to calculate  $P(0) + P(1)$  with the correct mean. A number of candidates did not use an approximation, instead using the underlying distribution, the Binomial. Partial credit was given for this. A number of candidates used the Normal approximation which was not appropriate. Nearly all candidates met the three significant figure accuracy requirement.

*Answer:* 0.0916

### **Question 2**

This proved to be a demanding question. Candidates needed either to find the height of the triangle or the equation of the line. The majority of candidates chose to try to find the equation of the line. Most wrongly assumed that the height of the triangle was 1, and so worked with  $y = x/4$  rather than making use of the area of the triangle being 1 to find the correct equation  $y = x/8$ . Nearly all candidates integrated their linear function using appropriate limits and equated to 0.5. A minority of the candidates obtained the correct value for the median, since many had not obtained the equation of the given line.

*Answer:*  $\sqrt{8}$  or 2.83 or equivalent.

### **Question 3**

Many candidates were able to calculate the required confidence interval making use of the Approximate distribution of sample proportion formula given in the formula book. Some candidates confused proportion with the number of occurrences using  $p$  as 56, whilst others used  $n$  as 1 rather than 100. Nearly all candidates used the normal tables appropriately to find the correct  $z$  value, and most then correctly found the upper and lower bounds of the confidence interval.

*Answer:* 0.452 to 0.668

#### Question 4

There were many excellent solutions to this question which examined the significance testing of a mean. Candidates were required to find unbiased estimates of the mean and variance using the given summary statistics. A small number of candidates used the biased estimator of the variance. This was then used to find the test statistic either as a  $z$  value or a probability. Some candidates did not use the sample size in the calculation of the test statistic. The final stage was to perform a significance test and draw a conclusion. The test was performed well by most candidates, the majority comparing their figure with 1.96 or, if a probability, with 0.025. Some candidates did not show the test sufficiently well, the majority of these using diagrams without annotation or stating that the critical value was  $\pm 1.96$  with no further explanation. A small number of candidates compared their figure with 1.645.

*Answer:* Reject  $H_0$ . Evidence that  $\mu \neq 1.6$

#### Question 5

This question on continuous random variables was answered well by the majority of candidates. Part (i) required that the upper limit of the pdf be put into context, i.e. the longest time a battery would work for. A number of candidates did not put their answer into context. In part (ii) the majority of candidates showed sufficient working to demonstrate the given answer. Nearly all candidates equated the integral to 1, then integrated correctly and applied the correct limits. There were some instances where candidates did not show sufficient detail to score full marks. In questions where an answer is given candidates need to show all stages of the required manipulation. In part (iii) candidates now needed to recognise that the value of  $a$  was 2.5 and to use this to find a numerical value for the mean lifetime. The vast majority of candidates correctly integrated  $xf(x)$  and obtained the correct mean. A very small number of candidates confused calculation of mean with median or did not multiply out  $xf(x)$  before attempting integration

*Answers:* (i) The longest lifetime of a battery (ii)  $k = \frac{a}{a-1}$  **AG** (iii)  $5/3 \ln 2.5$  or 1.53

#### Question 6

This question was the most demanding on the paper. Part (i) required a significance test to be applied to a binomial variable. Most candidates correctly stated the hypotheses and the correct distribution. However since this is a significance test of a discrete distribution the whole tail probability needed to be calculated, in this case  $P(0) + P(1)$ . Many candidates calculated only  $P(1)$ , rendering any test invalid. Those who did calculate the correct probability usually compared correctly with 0.025, and drew the correct conclusion that there was insufficient evidence of fewer fives. A number of candidates attempted to use the Normal distribution, with very few of these including the essential continuity correction. The question was designed to examine significance testing of a discrete distribution. As  $np = 5$  the minimal condition for the approximation of the Binomial by a Normal was almost met so partial credit was given. Candidates should be aware that the approximation is rather inaccurate in these conditions yielding 0.0401 compared with the true figure of 0.0274. In part (ii) most candidates correctly identified that the Normal distribution would be appropriate. A common error in this part was to use the observed outcome rather than the theoretical outcome to calculate the parameters of the Normal distribution, the mean and variance being calculated with  $p$  as 0.18 rather than 0.2. In part (iii) candidates were required to explain the meaning of a Type II error in context. Many candidates correctly identified that this would be a situation in which it was concluded that the number 5 was produced as often as the other digits when in fact it was being produced less often.

*Answers:* (i) Accept  $H_0$ . No evidence of fewer 5s (ii)  $N(200, 180)$  (iii) Conclude that the machine produces an equal number of 5s when it actually produces fewer

### Question 7

Part (i) required candidates to identify that there was a constant mean rate (of items being handed in) and many identified this. A number of candidates incorrectly stated that  $\lambda > 15$  was required. In part (ii) many correctly identified  $\lambda$  as  $4/7$  (the daily rate) to calculate  $P(2)$ , although a common error was to use the weekly mean  $\lambda = 4$ . Part (iii) was very well done, with nearly all candidates using the correct  $\lambda$  ( $40/7$ ) and finding  $1 - P(X \leq 3)$ . Part (iv) proved much more challenging. Candidates needed to identify that for 5 items to be handed in during a week with none on the first day then that would mean none on 1 day and then 5 in a six day period. The required calculation was  $P(0 \mid \lambda = 4/7) \times P(5 \mid \lambda = 24/7)$ . Many candidates were unable to identify the 5 items in a 6 day period part of this calculation. A number who did then applied the conditional probability formula inappropriately. A common error was to calculate the probability of 5 items in a 7 day period.

Answers: (i) Constant mean rate (ii) 0.0921/2 (iii) 0.821 (iv) 0.0723

### Question 8

This question was very well done by many candidates. Part (i) required identifying the distribution of  $X + 2.5Y$  and standardising to find  $p(X + 2.5Y > 140)$ . A small number of candidates did not use  $2.5^2$  in the variance calculation. In part (ii) candidates who scored full marks could work with the distribution of  $X - Y$  and find  $P(X - Y) > 20$ , or with the distribution of  $X - Y - 20$  and find  $P(X - Y - 20) > 0$ . A common error was to work with  $2.5X$  from part (i). A number of candidates incorrectly interpreted '20 more' as 20 less or 20 times, leading to very unrealistic answers. Nearly all candidates who standardised in both parts used tables correctly.

Answers: (i) 0.0254 (ii) 0.983