

Cambridge International AS & A Level

CANDIDATE
NAME

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CENTRE
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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

May/June 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

- 1 (a)** Find the coefficient of x^2 in the expansion of $\left(x - \frac{2}{x}\right)^6$. [2]

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- (b)** Find the coefficient of x^2 in the expansion of $(2 + 3x^2)\left(x - \frac{2}{x}\right)^6$. [3]

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- 2 (a) Express the equation $3 \cos \theta = 8 \tan \theta$ as a quadratic equation in $\sin \theta$. [3]

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- (b) Hence find the acute angle, in degrees, for which $3 \cos \theta = 8 \tan \theta$. [2]

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- 3** A weather balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of 600 cm^3 per second. The balloon was empty at the start of pumping.

(a) Find the radius of the balloon after 30 seconds. [2]

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(b) Find the rate of increase of the radius after 30 seconds. [3]

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- 4** The n th term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84.

[5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

5 The function f is defined for $x \in \mathbb{R}$ by

$$f : x \mapsto a - 2x,$$

where a is a constant.

(a) Express $ff(x)$ and $f^{-1}(x)$ in terms of a and x . [4]

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(b) Given that $ff(x) = f^{-1}(x)$, find x in terms of a . [2]

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6 The equation of a curve is $y = 2x^2 + kx + k - 1$, where k is a constant.

(a) Given that the line $y = 2x + 3$ is a tangent to the curve, find the value of k . [3]

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It is now given that $k = 2$.

(b) Express the equation of the curve in the form $y = 2(x + a)^2 + b$, where a and b are constants, and hence state the coordinates of the vertex of the curve. [3]

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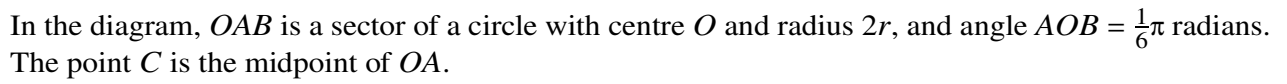
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- This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find the exact perimeter of the shaded region.

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(c) Find the exact area of the shaded region.

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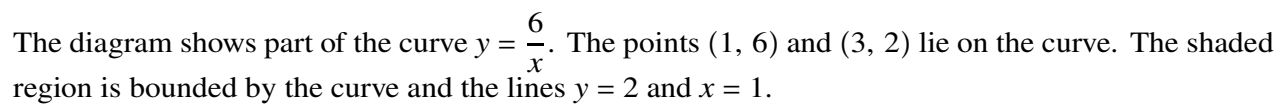
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- [illegible]

- (b)** The tangent to the curve at a point X is parallel to the line $y + 2x = 0$. Show that X lies on the line $y = 2x$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 9 Functions f and g are such that

$$f(x) = 2 - 3 \sin 2x \quad \text{for } 0 \leq x \leq \pi,$$

$$g(x) = -2f(x) \quad \text{for } 0 \leq x \leq \pi.$$

- (a) State the ranges of f and g .

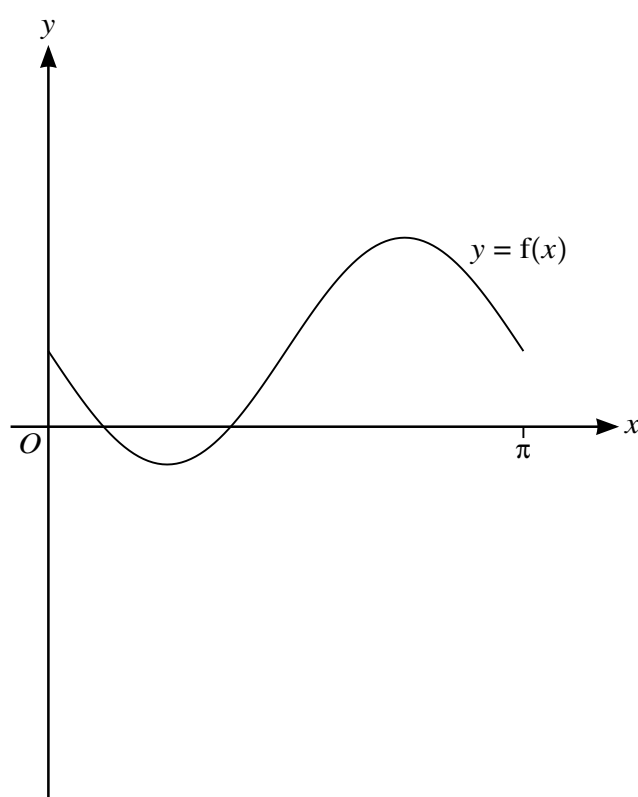
[3]

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The diagram below shows the graph of $y = f(x)$.



- (b) Sketch, on this diagram, the graph of $y = g(x)$.

[2]

The function h is such that

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leq x \leq 0.$$

- (c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$.

[3]

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10 The equation of a curve is $y = 54x - (2x - 7)^3$.

- (a)** Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

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- (b)** Find the coordinates of each of the stationary points on the curve. [3]

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- (c)** Determine the nature of each of the stationary points. [2]

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11 The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

- (a)** Find the radius of the circle and the coordinates of C . [3]

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The point $P(1, 2)$ lies on the circle.

- (b)** Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]

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The point Q also lies on the circle and PQ is parallel to the x -axis.

- (c) Write down the coordinates of Q . [2]

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The tangents to the circle at P and Q meet at T .

- (d) Find the coordinates of T . [3]

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