Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **17** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\left[\mathrm{f}(x)=\right]\ 2x^3+\dfrac{8}{x}\ [+c]$	**B1**	Allow any correct form
	$7 = 16 + 4 + c$	**M1**	Substitute f(2) = 7 into an integral. c must be present. Expect $c = -13$
	$\mathrm{f}(x)=\ 2x^3+\dfrac{8}{x}-13$	**A1**	Allow $y =$, $\mathrm{f}(x)$ or y can appear earlier in answer
		3	

Question	Answer	Marks	Guidance
2	$\left[\mathrm{f}^{-1}(x)=\right]\left((2x-1)^{1/2}\right)\times\left(\dfrac{1}{3}\times 2\times\dfrac{3}{2}\right)(-2)$	**B2, 1, 0**	Expect $(2x-1)^{1/2}-2$
	$(2x-1)^{1/2}-2\leqslant 0 \quad\rightarrow\quad 2x-1\leqslant 4$ or $2x-1<4$	**M1**	SOI. Rearranging and then squaring, must have power of ½ not present Allow '=0'at this stage but do not allow '≥ 0' or '>0' If '–2' missed then must see $\leqslant$ or $<$ for the M1
	Value [of a] is 2½ or $a = 2½$	**A1**	WWW, OE e.g. $\dfrac{5}{2}$, 2.5 Do not allow from '=0' unless some reference to negative gradient.
		4	

Question	Answer	Marks	Guidance
3	$x^2-4x+3=mx-6$ leading to $x^2-x(4+m)+9$	*M1	Equating and gathering terms. May be implied on the next line.
	b^2-4ac leading to $(4+m)^2-4\times 9$	DM1	SOI. Use of the discriminant with *their a*, *b* and *c*
	$4+m=\pm 6$ or $(m-2)(m+10)=0$ leading to $m=2$ or -10	A1	Must come from $b^2-4ac=0$ SOI
	Substitute both *their m* values into *their* equation in line 1	DM1	
	m = 2 leading to x =3 ; m = –10 leading to x = –3	A1	
	(3, 0), (–3, 24)	A1	Accept 'when x = 3, y = 0; when x = –3, y = 24' If final A0A0 scored, **SC B1** for one point correct WWW
	Alternative method for Question 3		
	$\frac{dy}{dx}=2x-4 \rightarrow 2x-4=m$	*M1	
	$x^2-4x+3=(2x-4)x-6$	DM1	
	$x^2-4x+3=2x^2-4x-6 \rightarrow 9=x^2 \rightarrow x=\pm 3$	A1	
	$y=0,\ 24$ or (3, 0), (–3, 24)	A1	
	Substitute both *their x* values into *their* equation in line 1	DM1	Or substitute both *their* (x,y) into $y=mx-6$
	When x = 3, m = 2; when x = –3, m = –10	A1	If A0, DM1, A0 scored, **SC B1** for one point correct WWW
		6	

Question	Answer	Marks	Guidance
4(a)	$\dfrac{\tan x+\sin x}{\tan x-\sin x}[=k]$ leading to $\dfrac{\sin x+\sin x\cos x}{\sin x-\sin x\cos x}[=k]$ or $\dfrac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}$ [=k] or $\dfrac{\tan x+\tan x\cos x}{\tan x-\tan x\cos x}$ [=k]	**M1**	Multiply numerator and denominator by $\cos x$, or divide numerator and denominator by tan x or sin x
	$\dfrac{\sin x(1+\cos x)}{\sin x(1-\cos x)}$ or $\dfrac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}.\dfrac{\cos x}{\cos x}$ or $\dfrac{\tan x(1+\cos x)}{\tan x(1-\cos x)}$ leading to $\dfrac{1+\cos x}{1-\cos x}[=k]$	**A1**	AG, WWW
		2	
4(b)	$k-k\cos x=1+\cos x$ leading to $k-1=k\cos x+\cos x$	**M1**	Gather like terms on LHS and RHS
	$k-1=(k+1)\cos x$ leading to $\cos x=\dfrac{k-1}{k+1}$	**A1**	WWW, OE
		2	
4(c)	Obtaining $\cos x$ from *their* **(b)** or **(a)**	**M1**	Expect $\cos x=\dfrac{3}{5}$
	±0.927 (only solutions in the given range)	**A1**	AWRT. Accept ±0.295π
		2	

Question	Answer	Marks	Guidance
5(a)	$\frac{1}{2} \times 4^2 \times \text{angle BAD} = 10$	**M1**	Use of sector area formula
	$\text{Angle BAD} = 1.25$	**A1**	OE. Accept 0.398π, 71.6^o for **SC B1** only
		2	
5(b)	$\text{Arc } BD = 4 \times \textit{their}\, 1.25$	**M1**	Use of arc length formula. Expect 5.
	$BC = 4\tan(\textit{their}\, 1.25)$	**M1**	Expect 12.0(4). May use *ACB*=0.321 or 18.4º
	$CD = \dfrac{4}{\cos(\textit{their}\, 1.25)} - 4$ or $\sqrt{4^2 + (\textit{their } BC)^2} - 4$	**M1**	Expect 12.69 – 4 = 8.69. May use *ACB*.
	Perimeter = 5 + 12.0(4) + 8.69 = 25.7 (cm)	**A1**	AWRT
		4	

Question	Answer	Marks	Guidance
6(a)	$\mathrm{f}(x) = (x-1)^2 + 4$	**B1**	
	$\mathrm{g}(x) = (x+2)^2 + 9$	**B1**	
	$g(x) = f(x+3) + 5$	**B1 B1**	B1 for each correct element. Accept $p = 3, q = 5$
		4	

Question	Answer	Marks	Guidance
6(b)	Translation or Shift	**B1**	
	$\begin{pmatrix}-3\\5\end{pmatrix}$ or acceptable explanation	**B1 FT**	If given as 2 single translations both must be described correctly e.g. $\begin{pmatrix}-3\\0\end{pmatrix}$ & $\begin{pmatrix}0\\5\end{pmatrix}$ FT from *their* $\mathrm{f}(x+p)+q$ *or their* $\mathrm{f}(x)\to\mathrm{g}(x)$ Do not accept $\begin{pmatrix}1\\4\end{pmatrix}$ or $\begin{pmatrix}-2\\9\end{pmatrix}$
		2	

Question	Answer	Marks	Guidance
7(a)	$(a-x)^6 = a^6 - 6a^5x + 15a^4x^2 - 20a^3x^3 + \ldots$	**B2, 1, 0**	Allow extra terms. Terms may be listed. Allow a^6x^0.
		2	
7(b)	$\left(1+\frac{2}{ax}\right)\left(\ldots 15a^4x^2 - 20a^3x^3 + \ldots\right)$ leading to $\left[x^2\right]\left(15a^4 - 40a^2\right)$	**M1**	Attempting to find 2 terms in x^2
	$15a^4 - 40a^2 = -20$ leading to $15a^4 - 40a^2 + 20[=0]$	**A1**	Terms on one side of the equation
	$\left(5a^2 - 10\right)\left(3a^2 - 2\right)$ $[=0]$	**M1**	OE. M1 for attempted factorisation or solving for a^2 or u ($=a^2$) using e.g. formula or completing the square
	$a = \pm\sqrt{2},\ \pm\sqrt{\frac{2}{3}}$	**B1 B1**	OE exact form only If B0B0 scored then **SC B1** for $\sqrt{2}, \sqrt{\frac{2}{3}}$ WWW or ±1.41,±0,816 WWW
		5	

Question	Answer	Marks	Guidance
8(a)	$\left[\mathrm{fg}(x)=\right]1/(2x+1)^2-1$	**B1**	SOI
	$1/(2x+1)^2-1=3$ leading to $4(2x+1)^2=1$ or $\dfrac{1}{(2x+1)}=[\pm]2$ or $16x^2+16x+3=0$	**M1**	Setting $\mathrm{fg}(x)=3$ and reaching a stage before $2x+1=\pm½$ or reaching a 3 term quadratic in x
	$2x+1=\pm½$ or $2x+1=-½$ or $(4x+1)(4x+3)[=0]$	**A1**	Or formula or completing square on quadratic
	$x=-\dfrac{3}{4}$ only	**A1**	
	Alternative method for Question 8(a)		
	$x^2-1=3$	**M1**	
	$\mathrm{g}(x)=-2$	**A1**	
	$\dfrac{1}{(2x+1)}=-2$	**M1**	
	$x=-\dfrac{3}{4}$ only	**A1**	
		4	

Question	Answer	Marks	Guidance
8(b)	$y=\frac{1}{(2x+1)^2}-1$ leading to $(2x+1)^2=\frac{1}{y+1}$ leading to $2x+1=[\pm]\frac{1}{\sqrt{y+1}}$	*M1	Obtain $2x+1$ or $2y+1$ as the subject
	$x=[\pm]\frac{1}{2\sqrt{y+1}}-\frac{1}{2}$	DM1	Make $x(\text{or } y)$ the subject
	$-\frac{1}{2\sqrt{x+1}}-\frac{1}{2}$	A1	OE e.g. $-\frac{\sqrt{x+1}}{2x+2}-\frac{1}{2}$, $-\left(\sqrt{\frac{-x}{4x+4}+\frac{1}{4}}+\frac{1}{2}\right)$
		3	

Question	Answer	Marks	Guidance
9(a)	$ar=\frac{24}{100}\times\frac{a}{1-r}$	M1	Form an equation using a numerical form of the percentage and correct formula for u_2 and S_∞
	$100r^2-100r+24[=0]$	A1	OE. All 3 terms on one side of an equation.
	$(20r-8)(5r-3)[=0]\to\ r=\frac{2}{5},\frac{3}{5}$	A1	Dependent on factors or formula seen from their quadratic.
		3	

Question	Answer	Marks	Guidance
9(b)	$3\times\{(a+4d)\}=\{(2(a+1)+11(d+1))\}$	*M1	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $a+d=13$	A1	
	$\left[\frac{5}{2}\right]\times 3\{(2a+4d)\}=\left[\frac{5}{2}\right]\times 2\{(4(a+1)+4(d+1))\}$	*M1	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $-a+2d=8$	A1	
	Solve 2 linear equations simultaneously	DM1	Elimination or substitution expected
	$d=7,\ a=6$	A1	**SC B1** for a=6, d=7 without complete working
		6	

Question	Answer	Marks	Guidance
10(a)	Gradient of $AB = -\frac{3}{5}$, gradient of $BC = \frac{5}{3}$ or lengths of all 3 sides or vectors	M1	Attempting to find required gradients, sides or vectors
	$m_{ab}m_{bc}=-1$ or Pythagoras or $\overrightarrow{AB}.\overrightarrow{BC}$=0 or $\cos ABC=0$ from cosine rule	A1	WWW
		2	
10(b)	Centre = mid-point of AC = (2,4)	B1	
		1	

Question	Answer	Marks	Guidance
10(c)	$(x - \mathit{their}\ x_c)^2 + (y - \mathit{their}\ y_c)^2 \left[= r^2\right] or \left(\mathit{their}\ x_c - x\right)^2 + \left(\mathit{their}\ y_c - y\right)^2 = \left[r^2\right]$	**M1**	Use of circle equation with *their* centre
	$(x-2)^2 + (y-4)^2 = 17$	**A1**	Accept $x^2 - 4x + y^2 - 8y + 3 = 0$ OE
		2	
10(d)	$\left(\frac{x+3}{2}, \frac{y+0}{2}\right) = (2,4)$ or **BE** = 2**BD** = $2\begin{pmatrix}-1\\4\end{pmatrix}$ Or Equation of *BE* is $y = -4(x-3)$ or $y - 4 = -4(x-2)$ leading to $y = -4x + 12$ Substitute equation of *BE* into circle and form a 3-term quadratic.	**M1**	Use of mid-point formula, vectors, steps on a diagram May be seen to find *x* coordinate at *E*
	$(x, y) = (1, 8)$ or **OE** = $\begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}-2\\8\end{pmatrix} = \begin{pmatrix}1\\8\end{pmatrix}$	**A1**	$E = (1, 8)$ Accept without working for both marks **SC B2**
	Gradient of *BD*, *m*, = –4 or gradient $AC = \frac{1}{4}$ = gradient of tangent	**B1**	Or gradient of *BE* = -4
	Equation of tangent is $y - 8 = ¼(x - 1)$ OE	**M1 A1**	For M1, equation through *their* E or (1, 8) (not, *A*, *B* or *C*) and with gradient $\frac{-1}{\mathit{their} - 4}$
		5	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{2}k^2x^{-3/2}$	B1 B1	Allow any correct unsimplified form
	$\frac{1}{2}x^{-1/2} - \frac{1}{2}k^2x^{-3/2} = 0$ leading to $\frac{1}{2}x^{-1/2} = \frac{1}{2}k^2x^{-3/2}$	M1	OE. Set to zero and one correct algebraic step towards the solutions. $\frac{dy}{dx}$ must only have 2 terms.
	$(k^2, 2k)$	A1	
		4	
11(b)	When $x = 4k^2$, $\frac{dy}{dx} = \left[\frac{1}{4k} - \frac{1}{16k} =\right]\frac{3}{16k}$	B1	OE
	$y = \left[2k + k^2 \times \frac{1}{2k}\right] = \frac{5k}{2}$	B1	OE. Accept $2k + \frac{k}{2}$
	Equation of tangent is $y - \frac{5k}{2} = \frac{3}{16k}(x - 4k^2)$ or $y = mx + c \rightarrow \frac{5k}{2} = \frac{3}{16k}(4k^2) + c$	M1	Use of line equation with *their* gradient and $(4k^2, \textit{their } y)$,
	When $x = 0$, $y = \left[\frac{5k}{2} - \frac{3k}{4} =\right]\frac{7k}{4}$ or from $y = mx + c$, $c = \frac{7k}{4}$	A1	OE
		4	

Question	Answer	Marks	Guidance
11(c)	$\int\left(x^{\frac{1}{2}}+k^2x^{-\frac{1}{2}}\right)\mathrm{d}x=\dfrac{2x^{\frac{3}{2}}}{3}+2k^2x^{\frac{1}{2}}$	**B1**	Any unsimplified form
	$\left(\dfrac{16k^3}{3}+4k^3\right)-\left(\dfrac{9k^3}{4}+3k^3\right)$	**M1**	Apply limits $\dfrac{9}{4}k^2 \to 4k^2$ to an integration of y. M0 if volume attempted.
	$\dfrac{49k^3}{12}$	**A1**	OE. Accept $4.08\,k^3$
		3	