

Cambridge International AS & A Level

CANDIDATE
NAME

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CENTRE
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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

1 Solve the inequality $2|3x - 1| < |x + 1|$.

[4]

[illegible]

- 2 Find the real root of the equation $\frac{2e^x + e^{-x}}{2 + e^x} = 3$, giving your answer correct to 3 decimal places.
Your working should show clearly that the equation has only one real root. [5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 3** (a) Given that $\cos(x - 30^\circ) = 2 \sin(x + 30^\circ)$, show that $\tan x = \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$. [4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b)** Hence solve the equation

$$\cos(x - 30^\circ) = 2 \sin(x + 30^\circ),$$

for $0^\circ < x < 360^\circ$. [2]

[illegible]

- 4 (a) Prove that $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} \equiv \tan^2 \theta$. [2]

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- (b) Hence find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{1 - \cos 2\theta}{1 + \cos 2\theta} d\theta$. [4]

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- 5 (a) Solve the equation $z^2 - 2piz - q = 0$, where p and q are real constants. [2]

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In an Argand diagram with origin O , the roots of this equation are represented by the distinct points A and B .

- (b) Given that A and B lie on the imaginary axis, find a relation between p and q . [2]

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[3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6 The parametric equations of a curve are

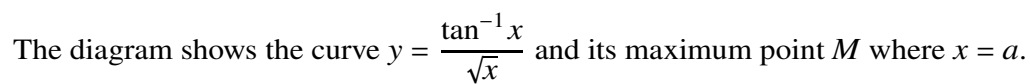
$$x = \ln(2 + 3t), \quad y = \frac{t}{2 + 3t}.$$

(a) Show that the gradient of the curve is always positive.

[5]

[illegible]

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$$a = \tan\left(\frac{2a}{1+a^2}\right). \quad [4]$$
This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- (b) Verify by calculation that a lies between 1.3 and 1.5.

[2]

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- (c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

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- 8** With respect to the origin O , the points A and B have position vectors given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. The line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

- (a) Find the acute angle between the directions of AB and l . [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

- (b)** Find the position vector of the point P on l such that $AP = BP$.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

9 The equation of a curve is $y = x^{-\frac{2}{3}} \ln x$ for $x > 0$. The curve has one stationary point.

(a) Find the exact coordinates of the stationary point.

[5]

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(b) Show that $\int_1^8 y \, dx = 18 \ln 2 - 9$.

[5]

[illegible]

- 10** The variables x and t satisfy the differential equation $\frac{dx}{dt} = x^2(1 + 2x)$, and $x = 1$ when $t = 0$.

Using partial fractions, solve the differential equation, obtaining an expression for t in terms of x .

[11]

[illegible]

[illegible]

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