

Cambridge International AS & A Level

CANDIDATE
NAME

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

1 Solve the inequality $|2x - 1| < 3|x + 1|$.

[4]

[illegible]

- 2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 1 - i| \leq 1$ and $\arg(z - 1) \leq \frac{3}{4}\pi$. [4]

3 The variables x and y satisfy the equation $x = A(3^{-y})$, where A is a constant.

- (a)** Explain why the graph of y against $\ln x$ is a straight line and state the exact value of the gradient of the line. [3]

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It is given that the line intersects the y -axis at the point where $y = 1.3$.

- (b)** Calculate the value of A , giving your answer correct to 2 decimal places. [2]

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- 4** Using integration by parts, find the exact value of $\int_0^2 \tan^{-1}(\frac{1}{2}x) \, dx$. [5]

[illegible]

- 5** The complex number u is given by $u = 10 - 4\sqrt{6}i$.

Find the two square roots of u , giving your answers in the form $a + ib$, where a and b are real and exact. [5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6 (a) Prove that $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan \theta$.

[3]

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- (b) Hence show that $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} (\operatorname{cosec} 2\theta - \cot 2\theta) \, d\theta = \frac{1}{2} \ln 2$.

[4]

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- 7 A curve is such that the gradient at a general point with coordinates (x, y) is proportional to $\frac{y}{\sqrt{x+1}}$. The curve passes through the points with coordinates $(0, 1)$ and $(3, e)$.

By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x . [7]

This image shows a full page of a worksheet designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, and there are no margins, text, or other markings present.

- 8** The equation of a curve is $y = e^{-5x} \tan^2 x$ for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

Find the x -coordinates of the stationary points of the curve. Give your answers correct to 3 decimal places where appropriate. [8]

This image shows a full page of primary-ruled paper. It features multiple sets of horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The background is white, and there are no margins or other markings present.

9 Let $f(x) = \frac{14 - 3x + 2x^2}{(2 + x)(3 + x^2)}$.

(a) Express $f(x)$ in partial fractions.

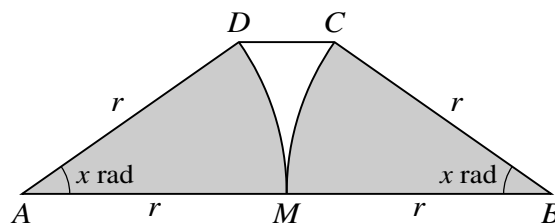
[5]

[illegible]

- (b)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

[illegible]

10



The diagram shows a trapezium $ABCD$ in which $AD = BC = r$ and $AB = 2r$. The acute angles BAD and ABC are both equal to x radians. Circular arcs of radius r with centres A and B meet at M , the midpoint of AB .

- (a) Given that the sum of the areas of the shaded sectors is 90% of the area of the trapezium, show that x satisfies the equation $x = 0.9(2 - \cos x) \sin x$. [3]

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- (b) Verify by calculation that x lies between 0.5 and 0.7. [2]

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- (c) Show that if a sequence of values in the interval $0 < x < \frac{1}{2}\pi$ given by the iterative formula

$$x_{n+1} = \cos^{-1} \left(2 - \frac{x_n}{0.9 \sin x_n} \right)$$

converges, then it converges to the root of the equation in part (a). [2]

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- (d) Use this iterative formula to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- 11** With respect to the origin O , the points A and B have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j}$ and $\overrightarrow{OB} = \mathbf{j} - 2\mathbf{k}$.

- (a) Show that $OA = OB$ and use a scalar product to calculate angle AOB in degrees. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

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