



Cambridge International AS & A Level

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



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- 1 Find, in terms of a , the set of values of x satisfying the inequality

$$2|3x + a| < |2x + 3a|,$$

where a is a positive constant.

[4]

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2 Solve the equation $\cos(\theta - 60^\circ) = 3 \sin \theta$, for $0^\circ \leq \theta \leq 360^\circ$.

[5]

[illegible]

- 3 (a)** Show that the equation $\log_3(2x + 1) = 1 + 2\log_3(x - 1)$ can be written as a quadratic equation in x . [3]

[illegible]

- (b) Hence solve the equation $\log_3(4y + 1) = 1 + 2 \log_3(2y - 1)$, giving your answer correct to 2 decimal places. [2]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

- 4** The curve $y = e^{-4x} \tan x$ has two stationary points in the interval $0 \leq x < \frac{1}{2}\pi$.

- (a) Obtain an expression for $\frac{dy}{dx}$ and show it can be written in the form $\sec^2 x(a + b \sin 2x)e^{-4x}$, where a and b are constants. [4]

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(b) Hence find the exact x -coordinates of the two stationary points.

[3]

[illegible]

5 The complex number $3 - i$ is denoted by u .

- (a) Show, on an Argand diagram with origin O , the points A , B and C representing the complex numbers u , u^* and $u^* - u$ respectively.

State the type of quadrilateral formed by the points O , A , B and C .

[3]

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- (b) Express $\frac{u^*}{u}$ in the form $x + iy$, where x and y are real.

[3]

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[Turn over

- 6** The parametric equations of a curve are $x = \frac{1}{\cos t}$, $y = \ln \tan t$, where $0 < t < \frac{1}{2}\pi$.

(a) Show that $\frac{dy}{dx} = \frac{\cos t}{\sin^2 t}$. [5]

[illegible]

[3]

[illegible]

7 Let $f(x) = \frac{5x^2 + 8x - 3}{(x - 2)(2x^2 + 3)}$.

(a) Express $f(x)$ in partial fractions.

[5]

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- (b)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

[illegible]

- 8** At time t days after the start of observations, the number of insects in a population is N . The variation in the number of insects is modelled by a differential equation of the form $\frac{dN}{dt} = kN^{\frac{3}{2}} \cos 0.02t$, where k is a constant and N is a continuous variable. It is given that when $t = 0$, $N = 100$.

(a) Solve the differential equation, obtaining a relation between N , k and t . [5]

[illegible]

- (b) Given also that $N = 625$ when $t = 50$, find the value of k . [2]

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- (c) Obtain an expression for N in terms of t , and find the greatest value of N predicted by this model. [2]

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- 9** With respect to the origin O , the point A has position vector given by $\overrightarrow{OA} = \mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$. The line l has vector equation $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

- (a) Find in degrees the acute angle between the directions of OA and l . [3]

[illegible]

- (b) Find the position vector of the foot of the perpendicular from A to l . [4]

[illegible]

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- (c) Hence find the position vector of the reflection of A in l . [2]

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10 The constant a is such that $\int_1^a x^2 \ln x \, dx = 4$.

(a) Show that $a = \left(\frac{35}{3 \ln a - 1} \right)^{\frac{1}{3}}$. [5]

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- (b) Verify by calculation that a lies between 2.4 and 2.8.

[2]

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- (c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

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