



Cambridge International AS & A Level

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



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- 1** Solve the inequality $|5x - 3| < 2|3x - 7|$.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 2** Solve the equation $\ln(2x^2 - 3) = 2 \ln x - \ln 2$, giving your answer in an exact form. [3]

[illegible]

- 3 (a) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying $|z + 3 - 2i| = 2$. [2]

- (b) Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [2]

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- 4** Solve the equation $2 \cos x - \cos \frac{1}{2}x = 1$ for $0 \leq x \leq 2\pi$.

[5]

[illegible]

- 5 The complex number $2 + yi$ is denoted by a , where y is a real number and $y < 0$. It is given that $f(a) = a^3 - a^2 - 2a$.

(a) Find a simplified expression for $f(a)$ in terms of y . [3]

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(b) Given that $\operatorname{Re}(f(a)) = -20$, find $\arg a$. [3]

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6 The equation $\cot \frac{1}{2}x = 3x$ has one root in the interval $0 < x < \pi$, denoted by α .

(a) Show by calculation that α lies between 0.5 and 1.

[2]

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(b) Show that, if a sequence of positive values given by the iterative formula

$$x_{n+1} = \frac{1}{3} \left(x_n + 4 \tan^{-1} \left(\frac{1}{3x_n} \right) \right)$$

converges, then it converges to α .

[2]

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- (c) Use this iterative formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

[illegible]

7 The equation of a curve is $3x^2 + 4xy + 3y^2 = 5$.

(a) Show that $\frac{dy}{dx} = -\frac{3x+2y}{2x+3y}$. [4]

[illegible]

- (b)** Hence find the exact coordinates of the two points on the curve at which the tangent is parallel to $y + 2x = 0$. [5]

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- 8 (a) The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{4 + 9y^2}{e^{2x+1}}.$$

It is given that $y = 0$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x .

[7]

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- (b) State what happens to the value of y as x tends to infinity. Give your answer in an exact form. [1]

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9 Let $f(x) = \frac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}$.

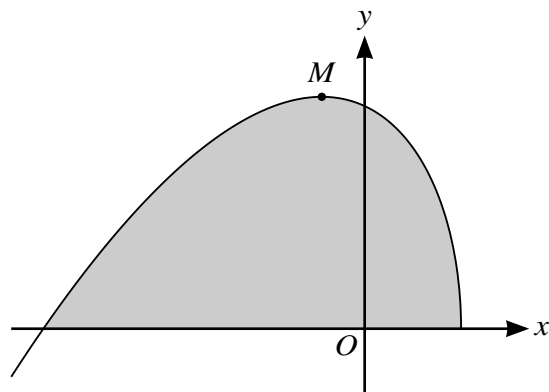
(a) Express $f(x)$ in partial fractions.

[5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

(b) Hence show that $\int_0^1 f(x) \, dx = \frac{5}{2} - \ln 72$. [5]

[illegible]



The diagram shows the curve $y = (x + 5)\sqrt{3 - 2x}$ and its maximum point M .

- (a) Find the exact coordinates of M .

[5]

[illegible]

- (b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x -axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]

[illegible]

- 11** The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$.

(a) Show that l does not intersect the line passing through A and B . [5]

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[illegible]

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