

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/22

Paper 2, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1	(i) State indefinite integral of the form $k \ln (4x - 1)$, where $k = 2, 4$, or $\frac{1}{2}$	M1	
	State correct integral $\frac{1}{2} \ln (4x - 1)$	A1	[2]
	(ii) Substitute limits correctly	M1	
	Use law for the logarithm of a power or a quotient	M1	
	Obtain $\ln 3$ correctly	A1	[3]
2	Use quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate (numerator) of derivative to zero and solve for x	DM1	
	Obtain $x = \frac{1}{3}$	A1	
	Obtain $y = \frac{3}{2}$	A1	[5]
3	Use trig identity correctly to obtain a quadratic in $\operatorname{cosec} \theta$ or $\sin \theta$	M1	
	Solve the quadratic correctly	M1	
	Obtain $\sin \theta = \frac{1}{4}$ or $-\frac{2}{3}$	A1	
	Obtain one correct answer	A1	
	Carry out correct method for second answer from either root	DM1	
	Obtain remaining 3 answers from 14.5, 165.5, 221.8, 318.2 and no others in the range	A1	
	[Ignore answers outside the given range]		[6]
4	(i) Substitute $x = 3$ or $x = -2$ and equate to zero	M1	
	Obtain a correct equation in any form	A1	
	Obtain a second correct equation in any form	A1	
	Solve a relevant pair of equations for a or for b	M1	
	Obtain $a = 4$ and $b = -3$	A1	[5]
	(ii) Attempt division by $x + 2$ (or $x - 3$) and obtain partial quotient of $ax^2 + kx$	M1	
	Obtain linear factors $4x + 1$, $x + 2$ and $x - 3$	A1	
	[If linear factor $4x + 1$ obtained by remainder theorem or inspection, award B2]		
	[If linear factor $4x + 1$ obtained by division by $x^2 - x - 6$, award M1 A1]		[2]
	Alternative Method:		
	Attempt to form identity $(x^2 - x - 6)(rx + s) \equiv ax^3 + bx^3 - 25x - 6$	M1	
	Attempt to equate like terms	M1	
	Leads to $s = 1$ B1, $r = 4$ A1, $b = -3$ A1, $a = 4$	A1	
	Obtain linear factors $4x + 1$, $x + 2$ and $x - 3$	A1	

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- 5 (i) State $\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}$ or $\frac{dy}{dt} = \frac{3}{t}$ B1
- Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1
- Use $y = 6$ to find t M1
- Obtain $t = e^2$ A1
- Obtain $\frac{dy}{dx} = \frac{6}{e}$ A1 [5]
- (ii) Obtain x and form equation of the tangent at their point M1
- Obtain correct equation for tangent $\left(y - 6 = \frac{6}{e}(x - (1 + e))\right)$ A1
- Show that tangent passes through $(1, 0)$ by substitution A1 [3]
- 6 (a) Expand brackets and use $\sin^2 x + \cos^2 x = 1$ M1
- Obtain $1 - \sin 2x$ A1
- Integrate and obtain term of form $\pm k \cos 2x$, where $k = \frac{1}{2}, 1$ or 2 M1
- State correct integral $x + \frac{\cos 2x}{2} (+c)$ A1 [4]
- (b) (i) State or imply correct ordinates $1.4142\dots, 1.0823\dots, 1$ B1
- Use correct formula, or equivalent, correctly with $h = \frac{\pi}{8}$ and three ordinates M1
- Obtain answer 0.899 with no errors seen A1 [3]
- (ii) Make a recognisable sketch of $y = \operatorname{cosec} x$ for $0 < x \leq \frac{1}{2}\pi$ B1
- Justify statement that the trapezium rule gives an over-estimate B1 [2]

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- 7 (i) Integrate to obtain terms $4x^2$ and $\frac{1}{2}e^x$ B1 + B1
Substitute limits correctly M1
Obtain correct equation in any form $4a^2 + \frac{1}{2}e^a - \frac{1}{2} = \frac{1}{2}$ A1
Rearrange to given answer correctly A1 [5]

- (ii) Consider sign of $\sqrt{\frac{2-e^a}{8}} - a$, or equivalent M1
Complete the argument correctly with appropriate calculations (f(0.2)=0.112, f(0.3)=−0.015) A1 [2]

- (iii) Use the iterative formula correctly at least once M1
Obtain final answer 0.29 A1
Show sufficient iterations to justify its accuracy to 2 d.p. B1

$x_0 = 0.2$	$x_0 = 0.25$	$x_0 = 0.3$
0.3120	0.2992	0.2851
0.2815	0.2853	0.2894
0.2905	0.2894	
0.2879		

or show there is a sign change in the interval (0.285, 0.295) [3]