

CANDIDATE
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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3 (P3)

October/November 2019

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.



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- 1** Solve the equation $5 \ln(4 - 3^x) = 6$. Show all necessary working and give the answer correct to 3 decimal places. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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- This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 3** The polynomial $x^4 + 3x^3 + ax + b$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $x^2 + x - 1$ the remainder is $2x + 3$. Find the values of a and b . [5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook paper or a template for handwriting practice. There are no margins, text, or other markings on the page.

- 4 (i) Express $(\sqrt{6}) \sin x + \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 3 decimal places. [3]

[illegible]

(ii) Hence solve the equation $(\sqrt{6}) \sin 2\theta + \cos 2\theta = 2$, for $0^\circ < \theta < 180^\circ$.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

- 5** The equation of a curve is $2x^2y - xy^2 = a^3$, where a is a positive constant. Show that there is only one point on the curve at which the tangent is parallel to the x -axis and find the y -coordinate of this point. [7]

[illegible]

6 The variables x and θ satisfy the differential equation

$$\sin \frac{1}{2} \theta \frac{dx}{d\theta} = (x + 2) \cos \frac{1}{2} \theta$$

for $0 < \theta < \pi$. It is given that $x = 1$ when $\theta = \frac{1}{3}\pi$. Solve the differential equation and obtain an expression for x in terms of $\cos \theta$. [8]

[illegible]

where z^* denotes the complex conjugate of z . Give your answer in the form $x + iy$, where x and y are real. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) (i) On a single Argand diagram sketch the loci given by the equations $|z - 2i| = 2$ and $\text{Im } z = 3$, where $\text{Im } z$ denotes the imaginary part of z . [2]

- (ii) In the first quadrant the two loci intersect at the point P . Find the exact argument of the complex number represented by P . [2]

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8 Let $f(x) = \frac{2x^2 + x + 8}{(2x - 1)(x^2 + 2)}$.

(i) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (ii) Hence, showing full working, find $\int_1^5 f(x) dx$, giving the answer in the form $\ln c$, where c is an integer. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 9 It is given that $\int_0^a x \cos \frac{1}{3}x \, dx = 3$, where the constant a is such that $0 < a < \frac{3}{2}\pi$.

(i) Show that a satisfies the equation

$$a = \frac{4 - 3 \cos \frac{1}{3}a}{\sin \frac{1}{3}a}. \quad [5]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (ii) Verify by calculation that a lies between 2.5 and 3.

[2]

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- (iii) Use an iterative formula based on the equation in part (i) to calculate a correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

[3]

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10 The line l has equation $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$. The plane p has equation $2x + y - 3z = 5$.

(i) Find the position vector of the point of intersection of l and p . [3]

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(ii) Calculate the acute angle between l and p . [3]

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- (iii) A second plane q is perpendicular to the plane p and contains the line l . Find the equation of q , giving your answer in the form $ax + by + cz = d$. [5]

[illegible]

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