

Cambridge International AS & A Level

CANDIDATE
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MATHEMATICS

9709/22

Paper 2 Pure Mathematics 2

October/November 2021

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

- 1 The polynomial $p(x)$ is defined by

$$p(x) = ax^3 + bx - 10,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $p(x)$ and that the remainder is -55 when $p(x)$ is divided by $(x + 3)$.

- (a) Find the values of a and b .

[5]

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(b) Hence factorise $p(x)$ completely.

[3]

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- 2 (a) Sketch, on the same diagram, the graphs of $y = x + 3$ and $y = |2x - 1|$. [2]

- (b) Solve the equation $x + 3 = |2x - 1|$. [3]

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- (c) Find the value of y such that $5^{\frac{1}{2}y} + 3 = |2 \times 5^{\frac{1}{2}y} - 1|$. Give your answer correct to 3 significant figures. [2]

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3 The curve with equation

$$y = 5x - 2 \tan 2x$$

has exactly one stationary point in the interval $0 \leq x < \frac{1}{4}\pi$.

Find the coordinates of this stationary point, giving each coordinate correct to 3 significant figures. [6]

[illegible]

- 4 Given that $\int_a^{a+14} \frac{1}{3x} \, dx = \ln 2$, find the value of the positive constant a . [5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

5 A curve has equation $x^2 + 4x \cos 3y = 6$.

Find the exact value of the gradient of the normal to the curve at the point $(\sqrt{2}, \frac{1}{12}\pi)$. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6 (a)** By sketching a suitable pair of graphs on the same diagram, show that the equation

$$\ln x = 2e^{-x}$$

has exactly one root.

[2]

- (b)** Verify by calculation that the root lies between 1.5 and 1.6.

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- (c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = e^{2e^{-x_n}}$$

converges, then it converges to the root of the equation in part (a).

[1]

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- (d) Use the iterative formula in part (c) to determine the root correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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- 7 (a) Prove that $4 \sin x \sin\left(x + \frac{1}{6}\pi\right) \equiv \sqrt{3} - \sqrt{3} \cos 2x + \sin 2x$. [3]

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- (b) Find the exact value of $\int_0^{\frac{5}{6}\pi} 4 \sin x \sin\left(x + \frac{1}{6}\pi\right) dx$. [4]

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- (c) Find the smallest positive value of y satisfying the equation

$$4 \sin(2y) \sin\left(2y + \frac{1}{6}\pi\right) = \sqrt{3}.$$

Give your answer in an exact form.

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[illegible]

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