

Cambridge International AS & A Level

CANDIDATE
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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

October/November 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1** Find the quotient and remainder when $2x^4 + 1$ is divided by $x^2 - x + 2$. [3]

[illegible]

- 2 (a) Sketch the graph of $y = |2x - 3|$.

[1]

- (b) Solve the inequality $|2x - 3| < 3x + 2$.

[3]

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- 3** Solve the equation $4^{x-2} = 4^x - 4^2$, giving your answer correct to 3 decimal places. [4]

[illegible]

4 Find the exact value of $\int_{\frac{1}{3}\pi}^{\pi} x \sin \frac{1}{2}x \, dx$.

[5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

5 Solve the equation $\sin \theta = 3 \cos 2\theta + 2$, for $0^\circ \leq \theta \leq 360^\circ$.

[5]

[illegible]

- 6 (a)** By first expanding $\cos(x - 60^\circ)$, show that the expression

$$2 \cos(x - 60^\circ) + \cos x$$

can be written in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

[illegible]

- (b)** Hence find the value of x in the interval $0^\circ < x < 360^\circ$ for which $2 \cos(x - 60^\circ) + \cos x$ takes its least possible value. [2]

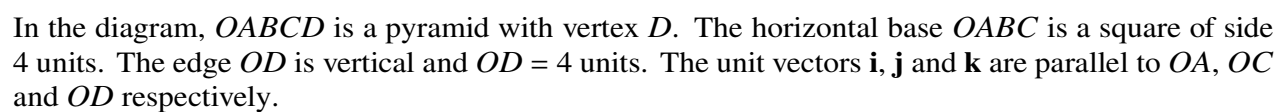
[illegible]

7 The equation of a curve is $\ln(x + y) = x - 2y$.

(a) Show that $\frac{dy}{dx} = \frac{x+y-1}{2(x+y)+1}$. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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(a) Find a vector equation for the line through M and N . [5]

[illegible]

[illegible]

9 Let $f(x) = \frac{1}{(9-x)\sqrt{x}}$.

- (a) Find the x -coordinate of the stationary point of the curve with equation $y = f(x)$. [4]

[illegible]

(b) Using the substitution $u = \sqrt{x}$, show that $\int_0^4 f(x) \, dx = \frac{1}{3} \ln 5$. [6]

- 10** A large plantation of area 20 km^2 is becoming infected with a plant disease. At time t years the area infected is $x \text{ km}^2$ and the rate of increase of x is proportional to the ratio of the area infected to the area not yet infected.

When $t = 0$, $x = 1$ and $\frac{dx}{dt} = 1$.

- (a)** Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{19x}{20 - x}. \quad [2]$$

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- (b)** Solve the differential equation and show that when $t = 1$ the value of x satisfies the equation $x = e^{0.9+0.05x}$. [5]

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- (c) Use an iterative formula based on the equation in part (b), with an initial value of 2, to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- (d) Calculate the value of t at which the entire plantation becomes infected. [1]

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11 The complex number $-\sqrt{3} + i$ is denoted by u .

- (a) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$, giving the exact values of r and θ . [2]

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- (b) Hence show that u^6 is real and state its value. [2]

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- (c) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $0 \leq \arg(z - u) \leq \frac{1}{4}\pi$ and $\operatorname{Re} z \leq 2$. [4]

- (ii) Find the greatest value of $|z|$ for points in the shaded region. Give your answer correct to 3 significant figures. [2]

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[illegible]

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