Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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1087265794

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Sketch the graph of $y = |2x + 1|$. [1]

(b) Solve the inequality $3x + 5 < |2x + 1|$. [3]

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[Turn over

2 On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z| \leqslant 3$, $\mathrm{Re}\, z \geqslant -2$ and $\frac{1}{4}\pi \leqslant \arg z \leqslant \pi$. [4]

3 Solve the equation $2^{3x-1} = 5(3^{-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [4]

4 Solve the equation $\tan(x + 45°) = 2\cot x$ for $0° < x < 180°$. [5]

5 The complex numbers u and w are defined by $u = 2e^{\frac{1}{4}\pi i}$ and $w = 3e^{\frac{1}{3}\pi i}$.

(a) Find $\dfrac{u^2}{w}$, giving your answer in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the exact values of r and θ. [3]

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(b) State the least positive integer n such that both $\text{Im}\, w^n = 0$ and $\text{Re}\, w^n > 0$. [1]

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6 **(a)** Prove the identity $\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4 \theta$. [4]

(b) Hence solve the equation $\cos 4\theta + 4\cos 2\theta = 4$ for $0° \leqslant \theta \leqslant 180°$. [3]

7 The equation of a curve is $y = \dfrac{x}{\cos^2 x}$, for $0 \leqslant x < \frac{1}{2}\pi$. At the point where $x = a$, the tangent to the curve has gradient equal to 12.

(a) Show that $a = \cos^{-1}\left(\sqrt[3]{\dfrac{\cos a + 2a \sin a}{12}}\right)$. [3]

(b) Verify by calculation that a lies between 0.9 and 1. [2]

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(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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8 In a certain chemical reaction the amount, x grams, of a substance is increasing. The differential equation satisfied by x and t, the time in seconds since the reaction began, is

$$\frac{\mathrm{d}x}{\mathrm{d}t} = kx\mathrm{e}^{-0.1t},$$

where k is a positive constant. It is given that $x = 20$ at the start of the reaction.

(a) Solve the differential equation, obtaining a relation between x, t and k. [5]

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(b) Given that $x = 40$ when $t = 10$, find the value of k and find the value approached by x as t becomes large. [3]

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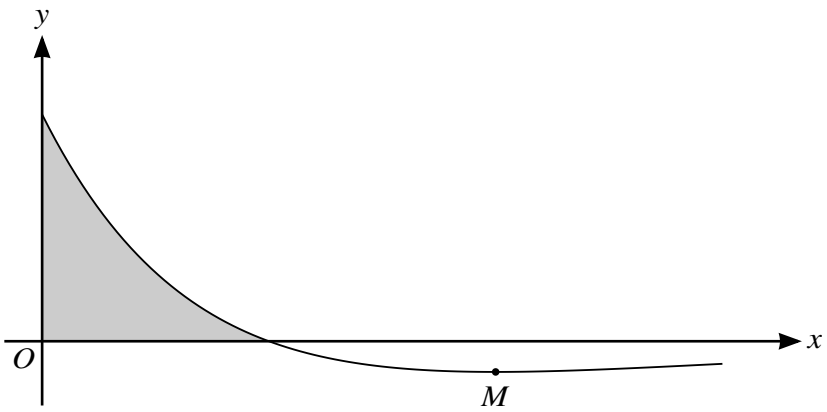

The diagram shows part of the curve $y = (3 - x)e^{-\frac{1}{3}x}$ for $x \geqslant 0$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

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(b) Find the area of the shaded region bounded by the curve and the axes, giving your answer in terms of e. [5]

10 Let $\mathrm{f}(x) = \dfrac{2x^2 + 7x + 8}{(1 + x)(2 + x)^2}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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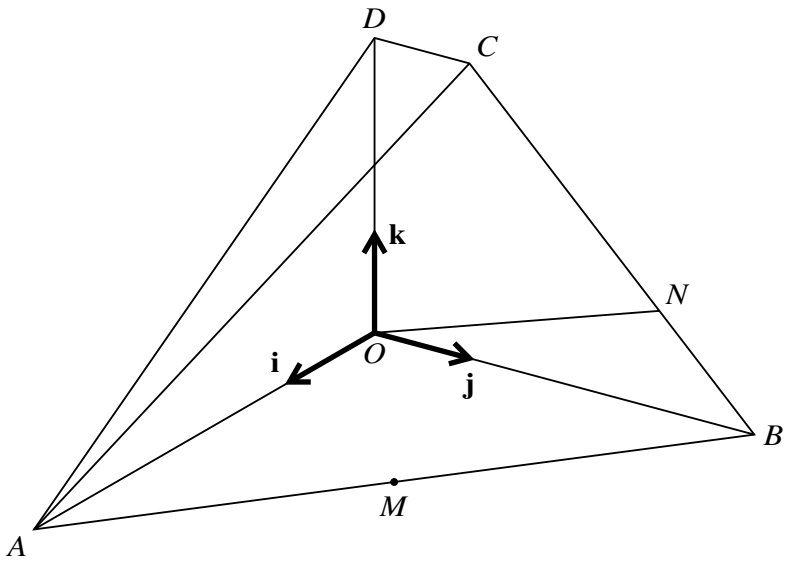

In the diagram, $OABCD$ is a solid figure in which $OA = OB = 4$ units and $OD = 3$ units. The edge OD is vertical, DC is parallel to OB and $DC = 1$ unit. The base, OAB, is horizontal and angle $AOB = 90°$. Unit vectors **i**, **j** and **k** are parallel to OA, OB and OD respectively. The midpoint of AB is M and the point N on BC is such that $CN = 2NB$.

(a) Express vectors $\overrightarrow{MD}$ and $\overrightarrow{ON}$ in terms of **i**, **j** and **k**. [4]

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(b) Calculate the angle in degrees between the directions of $\overrightarrow{MD}$ and $\overrightarrow{ON}$. [3]

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(c) Show that the length of the perpendicular from M to ON is $\sqrt{\frac{22}{5}}$. [4]

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Additional Page

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