

Cambridge International AS & A Level

CANDIDATE
NAME

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CENTRE
NUMBER

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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

October/November 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages.

- 1** The coefficient of x^3 in the expansion of $(3 + 2ax)^5$ is six times the coefficient of x^2 in the expansion of $(2 + ax)^6$.

Find the value of the constant a .

[4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

$$\frac{1}{6}\pi + \tan^{-1}(4x) = -\cos^{-1}(\frac{1}{2}\sqrt{3}). \quad [2]$$

[illegible]

- 3 The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{2}x + \frac{72}{x^4}$. The curve passes through the point $P(2, 8)$.

(a) Find the equation of the normal to the curve at P . [2]

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(b) Find the equation of the curve. [4]

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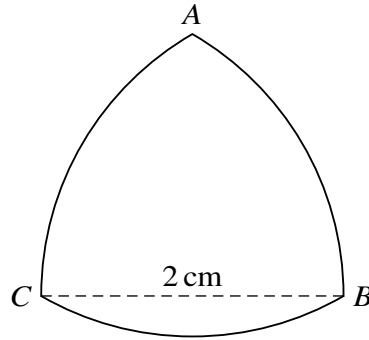
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The diagram shows the shape of a coin. The three arcs AB , BC and CA are parts of circles with centres C , A and B respectively. ABC is an equilateral triangle with sides of length 2 cm.

- (a) Find the perimeter of the coin. [2]

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- (b) Find the area of the face ABC of the coin, giving the answer in terms of π and $\sqrt{3}$. [4]

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- 5** The first, second and third terms of a geometric progression are $\sin \theta$, $\cos \theta$ and $2 - \sin \theta$ respectively, where θ radians is an acute angle.

(a) Find the value of θ .

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) Using this value of θ , find the sum of the first 10 terms of the progression. Give the answer in the form $\frac{b}{\sqrt{c}-1}$, where b and c are integers to be found. [3]

[illegible]

6 The equation of a curve is $y = x^2 - 8x + 5$.

- (a)** Find the coordinates of the minimum point of the curve. [2]

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The curve is stretched by a factor of 2 parallel to the y -axis and then translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

- (b)** Find the coordinates of the minimum point of the transformed curve. [2]

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[Turn over

- 7 (a) Verify the identity $(2x - 1)(4x^2 + 2x - 1) \equiv 8x^3 - 4x + 1$. [1]

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- (b)** Prove the identity $\frac{\tan^2 \theta + 1}{\tan^2 \theta - 1} \equiv \frac{1}{1 - 2 \cos^2 \theta}$. [3]

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(c) Using the results of (a) and (b), solve the equation

$$\frac{\tan^2 \theta + 1}{\tan^2 \theta - 1} = 4 \cos \theta,$$

for $0^\circ \leq \theta \leq 180^\circ$.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

8 Functions f and g are defined by

$$f(x) = (x + a)^2 - a \text{ for } x \leq -a,$$

$$g(x) = 2x - 1 \text{ for } x \in \mathbb{R},$$

where a is a positive constant.

(a) Find an expression for $f^{-1}(x)$. [3]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

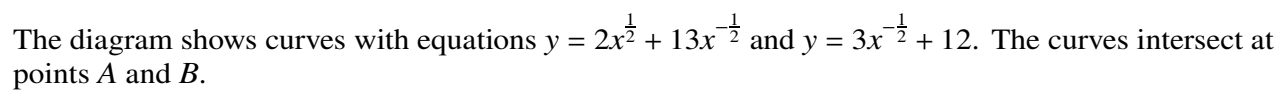
(b) (i) State the domain of the function f^{-1} . [1]

(ii) State the range of the function f^{-1} . [1]

- (c) Given that $a = \frac{7}{2}$, solve the equation $gf(x) = 0$.

[3]

[illegible]



- [illegible]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.

- 10** The equation of a curve is $y = f(x)$, where $f(x) = (4x - 3)^{\frac{5}{3}} - \frac{20}{3}x$.

- (a) Find the x -coordinates of the stationary points of the curve and determine their nature. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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(b) State the set of values for which the function f is increasing. [1]

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- 11** The coordinates of points A , B and C are $(6, 4)$, $(p, 7)$ and $(14, 18)$ respectively, where p is a constant. The line AB is perpendicular to the line BC .

(a) Given that $p < 10$, find the value of p .

[4]

[illegible]

A circle passes through the points A , B and C .

- (b) Find the equation of the circle. [3]

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- (c) Find the equation of the tangent to the circle at C , giving the answer in the form $dx + ey + f = 0$, where d , e and f are integers. [3]

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[illegible]

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