

Cambridge International AS & A Level

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MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

October/November 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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- 1** A curve is such that its gradient at a point (x, y) is given by $\frac{dy}{dx} = x - 3x^{-\frac{1}{2}}$. It is given that the curve passes through the point $(4, 1)$.

Find the equation of the curve.

[4]

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2 The circle with equation $(x - 3)^2 + (y - 5)^2 = 40$ intersects the y -axis at points A and B .

(a) Find the y -coordinates of A and B , expressing your answers in terms of surds. [2]

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(b) Find the equation of the circle which has AB as its diameter. [2]

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- 3 (a) Show that the equation

$$5 \cos \theta - \sin \theta \tan \theta + 1 = 0$$

may be expressed in the form $a \cos^2 \theta + b \cos \theta + c = 0$, where a , b and c are constants to be found. [3]

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- (b) Hence solve the equation $5 \cos \theta - \sin \theta \tan \theta + 1 = 0$ for $0 < \theta < 2\pi$. [4]

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- 4 (a)** Expand the following in ascending powers of x up to and including the term in x^2 .

(i) $(1 + 2x)^5$. [1]

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(ii) $(1 - ax)^6$, where a is a constant. [2]

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In the expansion of $(1 + 2x)^5(1 - ax)^6$, the coefficient of x^2 is -5 .

- (b)** Find the possible values of a . [4]

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5 The first, second and third terms of a geometric progression are $2p + 6$, $5p$ and $8p + 2$ respectively.

(a) Find the possible values of the constant p . [3]

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(b) One of the values of p found in (a) is a negative fraction.

Use this value of p to find the sum to infinity of this progression. [4]

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- 6** A line has equation $y = 6x - c$ and a curve has equation $y = cx^2 + 2x - 3$, where c is a constant. The line is a tangent to the curve at point P .

Find the possible values of c and the corresponding coordinates of P .

[7]

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7 The function f is defined by $f(x) = 1 + \frac{3}{x-2}$ for $x > 2$.

(a) State the range of f .

[1]

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(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} .

[4]

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The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $gf(x)$.

[2]

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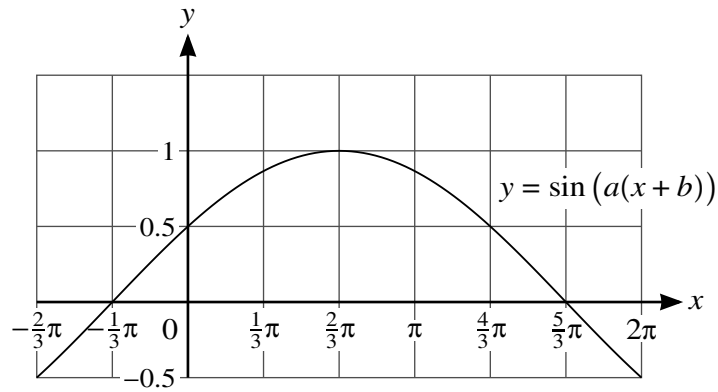
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The diagram shows part of the graph of $y = \sin(a(x+b))$, where a and b are positive constants.

- (a) State the value of a and one possible value of b . [2]

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Another curve, with equation $y = f(x)$, has a single stationary point at the point (p, q) , where p and q are constants. This curve is transformed to a curve with equation

$$y = -3f\left(\frac{1}{4}(x+8)\right).$$

- (b) For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of p and q . [3]

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9 A curve has equation $y = 2x^{\frac{1}{2}} - 1$.

- (a) Find the equation of the normal to the curve at the point $A(4, 3)$, giving your answer in the form $y = mx + c$. [3]

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A point is moving along the curve $y = 2x^{\frac{1}{2}} - 1$ in such a way that at A the rate of increase of the x -coordinate is 3 cm s^{-1} .

- (b) Find the rate of increase of the y -coordinate at A . [2]

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At A the moving point suddenly changes direction and speed, and moves down the normal in such a way that the rate of decrease of the y -coordinate is constant at 5 cm s^{-1} .

- (c) As the point moves down the normal, find the rate of change of its x -coordinate. [3]

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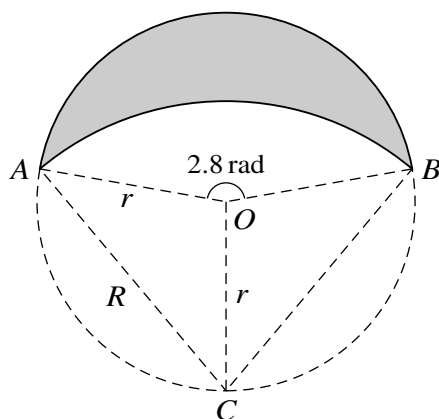
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The diagram shows points A , B and C lying on a circle with centre O and radius r . Angle AOB is 2.8 radians. The shaded region is bounded by two arcs. The upper arc is part of the circle with centre O and radius r . The lower arc is part of a circle with centre C and radius R .

- (a) State the size of angle ACO in radians. [1]

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- (b) Find R in terms of r . [1]

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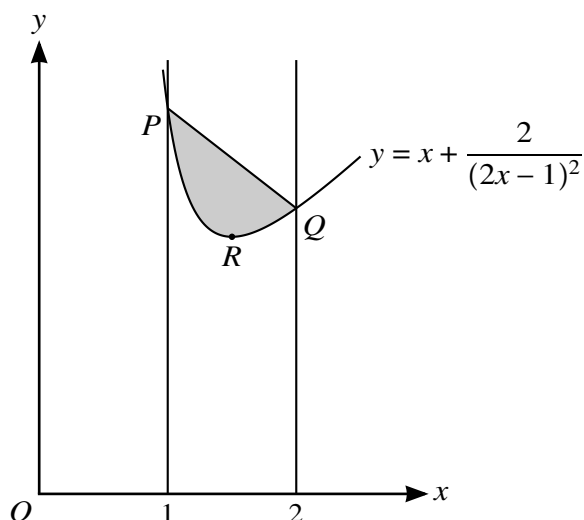
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- (c) Find the area of the shaded region in terms of r .

[7]

[illegible]



The diagram shows part of the curve with equation $y = x + \frac{2}{(2x-1)^2}$. The lines $x = 1$ and $x = 2$ intersect the curve at P and Q respectively and R is the stationary point on the curve.

- (a) Verify that the x -coordinate of R is $\frac{3}{2}$ and find the y -coordinate of R . [4]

[illegible]

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This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, which is otherwise blank.

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