

Cambridge IGCSE · Additional Mathematics 0606 · Notes

Applications of differentiation

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Read first: [Differentiation](#), [Straight-line graphs](#)

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What you need to know

By the end of this topic you should be able to:

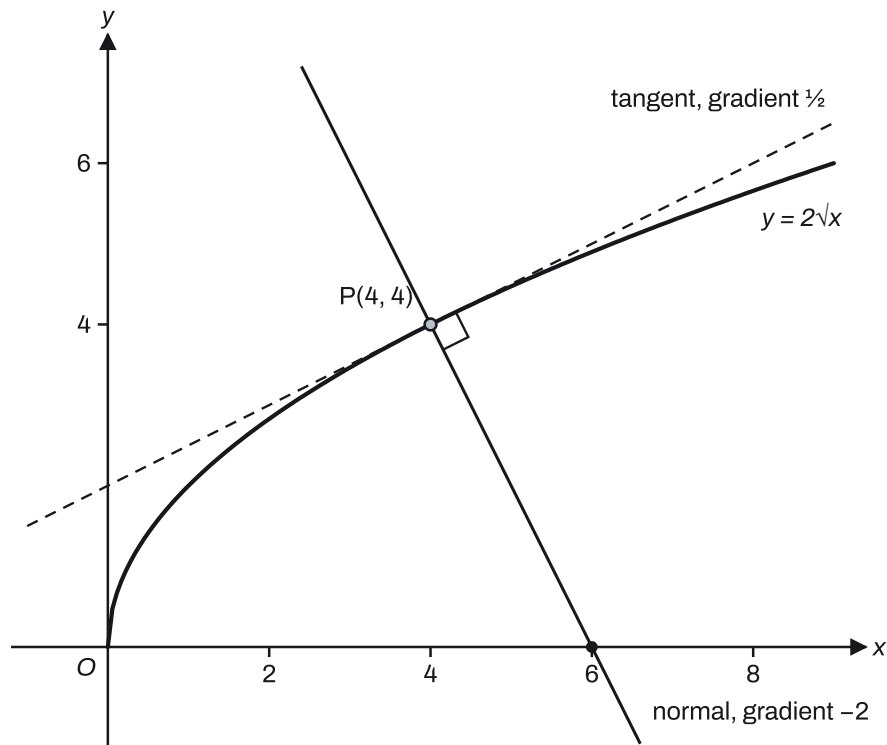
1. **Find gradients, tangents and normals.** Use $\frac{dy}{dx}$ to get the gradient of a curve at a point, then write the equation of the tangent or the normal there.
2. **Find stationary points**, the points where $\frac{dy}{dx} = 0$. Points of inflexion are not in the syllabus.
3. **Use differentiation for connected rates of change** (how fast one quantity changes when you know how fast a linked quantity changes) and for **small increments and approximations** ($\delta y \approx \frac{dy}{dx} \delta x$).
4. **Solve practical maximum and minimum problems:** build a formula for a quantity, then make it as large or as small as possible.
5. **Decide whether a stationary point is a maximum or a minimum** with the first derivative test or the second derivative test, and justify the conclusion fully. You may be asked to explain how to tell them apart. Any valid method is allowed unless the question names one.

The differentiation itself (standard derivatives, chain, product and quotient rules) is in the Differentiation note; this note is about using the result.

Understand it

The derivative is a gradient

Substituting a value of x into $\frac{dy}{dx}$ gives the gradient of the curve at that point, which is the gradient of the **tangent** there. The **normal** is the line through the same point at right angles to the tangent. Two perpendicular gradients multiply to -1 , so if the tangent has gradient m , the normal has gradient $-\frac{1}{m}$.



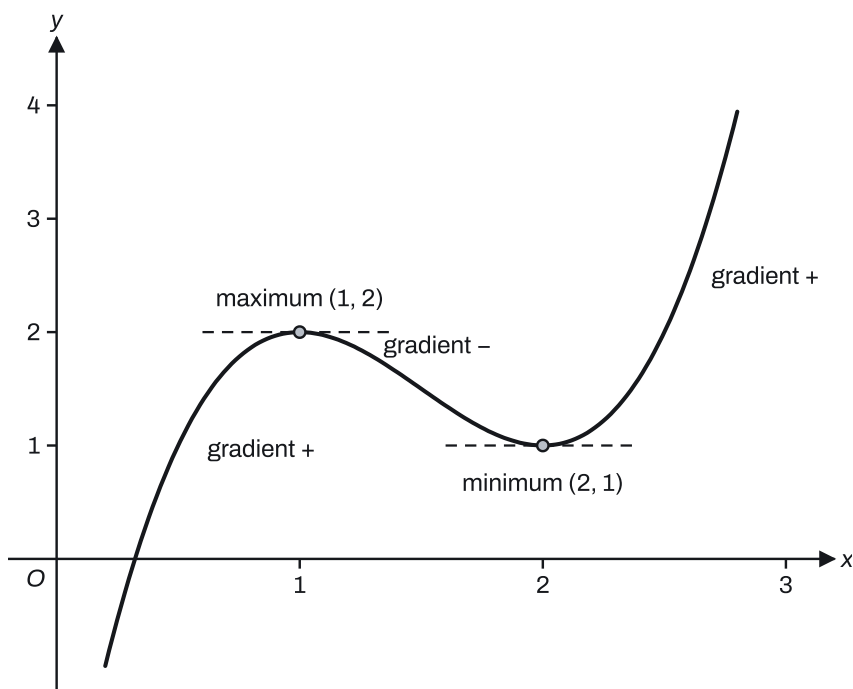
For $y = 2\sqrt{x} = 2x^{1/2}$: $\frac{dy}{dx} = x^{-1/2}$, which is $\frac{1}{2}$ at $x = 4$. The point is $P(4, 4)$, so

- tangent: $y - 4 = \frac{1}{2}(x - 4)$, that is $y = \frac{1}{2}x + 2$;
- normal: gradient $-1 \div \frac{1}{2} = -2$, so $y - 4 = -2(x - 4)$, that is $y = -2x + 12$.

A line needs **a gradient from calculus and a point on the curve**. The y -coordinate always comes from the curve's equation, never from $\frac{dy}{dx}$.

Stationary points and their nature

Where a curve turns, at the top of a hump or the bottom of a dip, the tangent is horizontal and the gradient is 0. These are **stationary points**: find them by solving $\frac{dy}{dx} = 0$.



For $y = 2x^3 - 9x^2 + 12x - 3$: $\frac{dy}{dx} = 6x^2 - 18x + 12 = 6(x - 1)(x - 2)$, which is 0 at $x = 1$ and $x = 2$. Substituting into the curve gives the points (1, 2) and (2, 1). There are two ways to tell which is which.

First derivative test. Look at the sign of $\frac{dy}{dx}$ just before and just after the point.

x	0.5	1	1.5	2	2.5
$\frac{dy}{dx}$	4.5	0	-1.5	0	4.5
sign	+	0	-	0	+

Going +, 0, - means uphill, flat, downhill: a **maximum** at (1, 2). Going -, 0, + means a **minimum** at (2, 1). Choose test values close to the point, with no other stationary point in between.

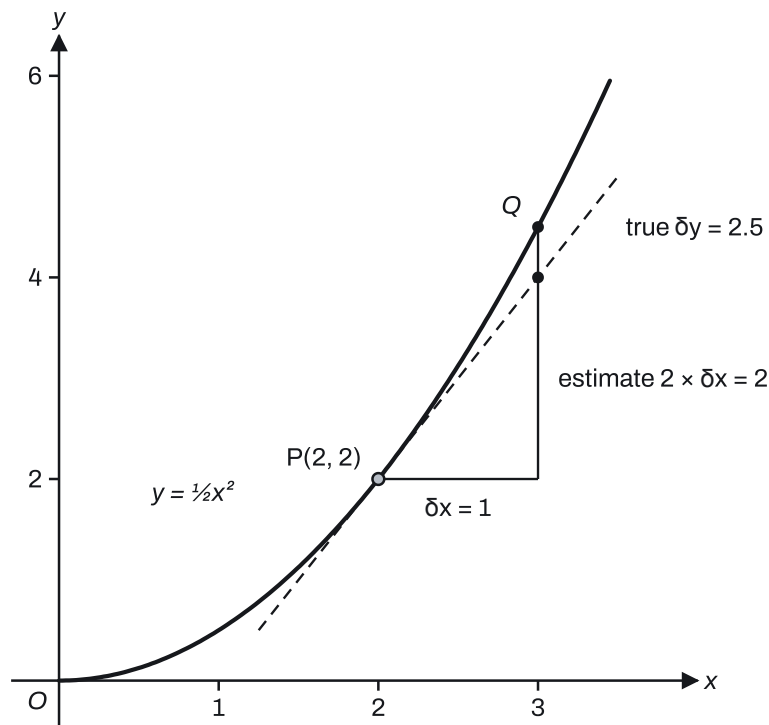
Second derivative test. $\frac{d^2y}{dx^2}$ says whether the gradient is growing or shrinking. At a maximum the gradient goes from positive to negative, so it is decreasing: $\frac{d^2y}{dx^2} < 0$. At a minimum it goes from negative to positive: $\frac{d^2y}{dx^2} > 0$. Here $\frac{d^2y}{dx^2} = 12x - 18$, which is -6 at $x = 1$ (maximum) and 6 at $x = 2$ (minimum).

The second derivative test is usually quicker. The first derivative test is better when $\frac{d^2y}{dx^2}$ would be messy (a quotient, for example), and you must use it when the question asks for it. If $\frac{d^2y}{dx^2} = 0$ the second derivative test tells you nothing, so switch to the first derivative test.

Small changes

Close to a point, a curve and its tangent are almost the same. So if x changes by a small amount δx , the change in y is almost what the tangent predicts:

$$\delta y \approx \frac{dy}{dx} \times \delta x, \quad \text{with } \frac{dy}{dx} \text{ worked out at the starting value of } x.$$



For $y = \frac{1}{2}x^2$ at $x = 2$, $\frac{dy}{dx} = 2$. With the large step $\delta x = 1$ in the picture the estimate 2 is poor (the true change is 2.5), but with $\delta x = 0.1$ the estimate 0.2 is very close to the true 0.205. The smaller δx , the better the estimate. It works backwards too: $\delta x \approx \delta y \div \frac{dy}{dx}$.

" x increases from 2 to $2 + h$, where h is small" means $\delta x = h$, so here $\delta y \approx 2h$.

Connected rates of change

A rate of change is a derivative with respect to time, t : $\frac{dV}{dt}$ is how fast V changes per second. When two quantities are linked by a formula, their rates are linked by the chain rule:

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}.$$

Write the rates so the letters "cancel" (dr on the bottom of one and the top of the next). $\frac{dV}{dr}$ comes from differentiating the formula that links V and r ; the other rate is given. A **decreasing** quantity has a **negative** rate.

Maxima and minima in real problems

Questions ask for the largest volume, the least material, the shortest perimeter. The method is always the same: write the quantity as a formula in **one** variable (using the fixed information to remove the second variable), differentiate, set the derivative to 0, then check it really is a maximum or a minimum. Example 4 does this for a window.

Key facts and formulas

The syllabus gives no calculus formulas in the List of formulas, so every calculus result is "Learn it". The List of formulas does give the sphere and cone formulas below.

Formula	Status
Gradient of the curve at $x = a$: the value of $\frac{dy}{dx}$ at $x = a$	Learn it
Tangent at (x_1, y_1) with gradient m : $y - y_1 = m(x - x_1)$	Learn it
Normal gradient = $-\frac{1}{m}$ (perpendicular gradients multiply to -1)	Learn it
Stationary point: $\frac{dy}{dx} = 0$	Learn it
First derivative test: sign of $\frac{dy}{dx}$ goes $+, 0, -$ for a maximum and $-, 0, +$ for a minimum	Learn it
Second derivative test: $\frac{d^2y}{dx^2} < 0$ maximum, $\frac{d^2y}{dx^2} > 0$ minimum	Learn it
Small changes: $\delta y \approx \frac{dy}{dx} \times \delta x$, and $\delta x \approx \delta y \div \frac{dy}{dx}$	Learn it
Connected rates: $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$, and $\frac{dx}{dt} = \frac{dy}{dt} \div \frac{dy}{dx}$	Learn it
Sphere: $V = \frac{4}{3}\pi r^3$, surface area $A = 4\pi r^2$	Given
Cone: $V = \frac{1}{3}\pi r^2 h$, curved surface area $A = \pi r l$	Given
Cylinder: $V = \pi r^2 h$, curved surface $2\pi r h$; circle: area πr^2 , circumference $2\pi r$; sector: arc $r\theta$, area $\frac{1}{2}r^2\theta$	Learn it

How to do it

In the Paper 1 and Paper 2 questions we have tagged for this topic (113 questions, 2021–2025), the types came up this often. 7 of the 113 mix more than one type, so the counts add up to 121.

Question type	Questions
Find stationary points and their nature	24
Find the equation of a normal and use it	22
Use small changes to find an approximate change	21
Find the equation of a tangent and use it	19

Question type	Questions
Practical optimisation (form an expression, then find its maximum or minimum)	14
Connected rates of change	13
Stationary points of a cubic, then k for the number of roots of $f(x) = k$	8

1. Find stationary points and their nature (24 questions)

- Differentiate** and write $\frac{dy}{dx}$ down. Often an earlier part has already asked you to show $\frac{dy}{dx}$ in a given form: use that form.
- Set $\frac{dy}{dx} = 0$ and solve.** A factor $e^{\dots}$ is never 0; a fraction is 0 only when its numerator is 0; factorise trigonometric expressions and solve each factor in the given domain, keeping every root (and no extras).
- Find y** from the curve's equation, exactly if asked.
- Determine the nature** of each point: work out $\frac{d^2y}{dx^2}$ there and state its sign, or give the sign of $\frac{dy}{dx}$ either side with the x -values you used. Then write "maximum" or "minimum".

Variations you will meet:

- "Show that there is exactly one stationary point"** (or none). Solve $\frac{dy}{dx} = 0$ completely, then reject any root that is outside the domain, **giving the reason**. For $y = x^2 - 8 \ln x$: $\frac{dy}{dx} = 2x - \frac{8}{x} = 0$ gives $x^2 = 4$, so $x = \pm 2$; but $\ln x$ needs $x > 0$, so $x = -2$ is rejected and $x = 2$ is the only stationary point. For "no stationary points", show the numerator can never be 0 (for example a squared term plus a positive number, or a negative discriminant).
- "Use the first derivative test"**: a second derivative answer scores nothing. Make a small table of x values either side, the value (or sign) of $\frac{dy}{dx}$ and the conclusion.
- "Explain how you could determine the nature"**: describe a test in words, for example "find $\frac{d^2y}{dx^2}$ at the point: if it is negative the point is a maximum, if positive a minimum".
- Unknown coefficients**. If you are told the x -values of the stationary points, each makes $\frac{dy}{dx} = 0$, giving one equation each. For $y = x^3 + ax^2 + bx$ with stationary points at $x = 1$ and $x = 3$: $\frac{dy}{dx} = 3x^2 + 2ax + b$ must equal $3(x - 1)(x - 3) = 3x^2 - 12x + 9$, so $a = -6$ and $b = 9$. Another fact (a point on the curve, a remainder) gives the constant term.
- Only $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ is given**. Integrate to get back to y , using the given point to find the constant (see the Integration note), then proceed as usual. You can still find the stationary x -values straight from $\frac{dy}{dx} = 0$.
- No calculator (Paper 1)** with surds: solve $\frac{dy}{dx} = 0$ and rationalise the denominator to reach the form $a + b\sqrt{3}$.

2. Find the equation of a normal and use it (22 questions)

- Differentiate and substitute the x -coordinate to get the tangent gradient m . Find y from the curve if it isn't given.
- Normal gradient $= -\frac{1}{m}$.
- $y - y_1 = -\frac{1}{m}(x - x_1)$, then rearrange into the form asked for.

4. **Use it**, as the question asks:

- meets an axis: put $y = 0$ (or $x = 0$);
- meets a line: solve the two equations together;
- meets the curve again: substitute the normal into the curve's equation. The known point's x is a root, so for a cubic take out that factor and solve the quadratic left over (exactly, on Paper 1);
- a triangle's area, a midpoint or the gradient of another line: coordinate geometry from there.

Variations: unknown constants in the curve or the normal: the gradient condition gives one equation and "the point lies on both" gives another. The point is sometimes given by its y -value, as in " $(a, 0)$ ", where $0 < a < \pi$ ": solve the curve's equation for x first. Answers are usually exact (π , $\ln$, surds), so keep them exact throughout.

3. Use small changes to find an approximate change (21 questions)

1. **Differentiate** (product, quotient and chain rules as needed) and write $\frac{dy}{dx}$ down.
2. **Evaluate** $\frac{dy}{dx}$ **at the starting value** of x (in radians for trigonometric functions).
3. **Write the relationship with the numbers in:** $\delta y \approx \left(\frac{dy}{dx}\right) \times h$.
4. Give the answer as asked: " $1.35h$ ", or a number if δx is a number.

Variations:

- **Change in x from a change in y :** $\delta x \approx \delta y \div \frac{dy}{dx}$. If you are given the starting y , first find the matching x (often an exact trigonometric value).
- **"Write down"** after an earlier part: one line, using the value already found.
- **Kinematics:** the change in velocity is $\delta v \approx \frac{dv}{dt} \delta t$.

4. Find the equation of a tangent and use it (19 questions)

1. Gradient m from $\frac{dy}{dx}$ at the point; y from the curve.
2. $y - y_1 = m(x - x_1)$, then the form asked for (for example $y = mx + c$ with exact m and c).
3. Use it: intercepts with the axes (then a midpoint, a length or a perpendicular bisector), where it meets a given line, or an area between the curve, the tangent and an axis (with integration).

Variations:

- **Constants from a given tangent**, such as "the tangent at $x = 2$ is $y = 7x + b$ ": the gradient gives one equation, and the point lying on the curve and on the line gives another.
- **A point where the tangent has a given gradient** (or is parallel to a given line): set $\frac{dy}{dx}$ equal to that gradient and solve for x .
- **Angle between a tangent and a line:** a gradient m makes angle $\tan^{-1} m$ with the x -axis; subtract the angles and give the acute one.
- **Tangent to the inverse function:** at the reflected point (b, a) the gradient is $\frac{1}{m}$, the reciprocal of the original gradient.

5. Practical optimisation (14 questions)

1. **Write the formula** for the quantity to be made largest or smallest. It usually has two variables. Count the faces or edges carefully: a closed cylinder has surface $2\pi r^2 + 2\pi rh$; an open-topped one has $\pi r^2 + 2\pi rh$; a sector's perimeter is $2r + r\theta$.
2. **Use the fixed quantity** (volume, area, perimeter) to write one variable in terms of the other, and substitute. This is usually a "show that": work forwards to the printed formula, every step visible.
3. **Differentiate, set to 0, solve**, rejecting negative lengths.
4. **Justify the nature**: $\frac{d^2A}{dr^2}$ with its value or sign ("as $r > 0$, $\frac{d^2A}{dr^2} > 0$, so minimum"), or the first derivative test.
5. **Answer the question asked**: the value of the variable, the other dimension, or the maximum or minimum value itself.

Shapes seen include closed and open cylinders, open tanks, half-cylinders, cones, a hemisphere on a cylinder, sectors of fixed area, and shapes with parts cut out. Similar shapes (a prism inside a pyramid, say) give a ratio that links the two variables.

6. Connected rates of change (13 questions)

1. **Write down the rate given and the rate wanted**, such as $\frac{dV}{dt} = 5$ and $\frac{dh}{dt} = ?$.
2. **Find the linking derivative** by differentiating the formula that joins the two quantities. If you are told V (not r) at the instant, find r first.
3. **Write the chain rule with the actual letters** of the question, then substitute and solve.
4. Give the units and the sign (negative for decreasing).

On a curve. A point moves on $y = x^3 + x$ with x increasing at 0.2 units per second. At $x = 1$, $\frac{dy}{dx} = 3x^2 + 1 = 4$, so $\frac{dy}{dt} = 4 \times 0.2 = 0.8$ units per second. If instead y is **decreasing** at 2 units per second there, $\frac{dx}{dt} = -2 \div 4 = -0.5$ units per second.

Variations: a two-step chain such as $\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$ after finding $\frac{dr}{dt}$ from $\frac{dV}{dt}$; liquid filling a container where similar shapes give r in terms of h , so write V in terms of h alone before differentiating; a trigonometric curve where you are given y , so solve for x first.

7. Stationary points of a cubic, then k for the number of roots (8 questions)

1. Expand the cubic (or use the product rule), differentiate, and solve $\frac{dy}{dx} = 0$.
2. Find **both coordinates** of each stationary point. The y -values are what the later part needs.
3. After the sketch (see Equations, inequalities and graphs), use horizontal lines $y = k$: the number of solutions of $f(x) = k$ is the number of times the line meets the curve.

For $y = x^3 - 12x + 5$: $\frac{dy}{dx} = 3x^2 - 12 = 0$ gives $x = \pm 2$, with a maximum at $(-2, 21)$ and a minimum at $(2, -11)$. So $f(x) = k$ has three solutions for $-11 < k < 21$, two for $k = -11$ or $k = 21$, and one for $k < -11$ or $k > 21$. For $|f(x)| = k$, reflect the part below the x -axis upwards first, then count again.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

It is given that $y = (x^2 - 5)e^{x-2}$.

(a) Use differentiation to find the approximate change in y as x increases from 2 to $2 + h$, where h is small. **[4]**

(b) Hence find the approximate change in x when y increases from -1 by the small amount 0.06. **[1]**

(a) Product rule with $u = x^2 - 5$ and $v = e^{x-2}$:

$$\frac{dy}{dx} = 2x e^{x-2} + (x^2 - 5)e^{x-2} = e^{x-2}(x^2 + 2x - 5).$$

B1 for $\frac{d}{dx}e^{x-2} = e^{x-2}$, **M1** for a product rule with the right structure, **A1** for the whole derivative.

At $x = 2$: $\frac{dy}{dx} = e^0(4 + 4 - 5) = 3$, so $\delta y \approx 3 \times h = 3h$. **A1**

(b) At $x = 2$, $y = (4 - 5)e^0 = -1$, so this is the same point. $\delta x \approx \frac{\delta y}{dy/dx} = \frac{0.06}{3} = 0.02$. **B1**

Example 2

A curve has equation $y = e^{2x} - 3x$. The point P on the curve has x -coordinate $\ln 2$.

(a) Find the equation of the tangent to the curve at P , in the form $y = mx + c$, where m and c are exact. **[4]**

(b) The normal to the curve at P meets the y -axis at A , and the tangent at P meets the y -axis at B . Find the exact area of triangle PAB . **[4]**

(a) $\frac{dy}{dx} = 2e^{2x} - 3$. **B1**

At $x = \ln 2$: $e^{2 \ln 2} = e^{\ln 4} = 4$, so $y = 4 - 3 \ln 2$ and $\frac{dy}{dx} = 8 - 3 = 5$. **B1** for both.

$$y - (4 - 3 \ln 2) = 5(x - \ln 2) \Rightarrow y = 5x + 4 - 8 \ln 2.$$

M1 for a line through P with the gradient from calculus, **A1**.

(b) Normal gradient $= -\frac{1}{5}$. **M1**

$$y - (4 - 3 \ln 2) = -\frac{1}{5}(x - \ln 2) \Rightarrow y = -\frac{1}{5}x + 4 - \frac{14}{5} \ln 2.$$

A1

So $A = (0, 4 - \frac{14}{5} \ln 2)$ and $B = (0, 4 - 8 \ln 2)$. Take AB as the base, along the y -axis:

$$AB = (4 - \frac{14}{5} \ln 2) - (4 - 8 \ln 2) = \frac{26}{5} \ln 2.$$

The height is the distance of P from the y -axis, $\ln 2$. **M1** for $\frac{1}{2} \times \text{base} \times \text{height}$ with their values.

$$\text{Area} = \frac{1}{2} \times \frac{26}{5} \ln 2 \times \ln 2 = \frac{13}{5} (\ln 2)^2 (\approx 1.25).$$

A1**Example 3**

A curve has equation $y = \frac{x-2}{x^2+5}$.

(a) Show that $\frac{dy}{dx} = \frac{5+4x-x^2}{(x^2+5)^2}$. **[3]**

(b) Find the coordinates of the stationary points on the curve. **[3]**

(c) Use the first derivative test to determine the nature of each stationary point. **[3]**

(a) Quotient rule with $u = x - 2$, $v = x^2 + 5$: $\frac{du}{dx} = 1$, $\frac{dv}{dx} = 2x$. **M1** for the right structure.

$$\frac{dy}{dx} = \frac{(x^2+5)(1) - (x-2)(2x)}{(x^2+5)^2} = \frac{x^2+5-2x^2+4x}{(x^2+5)^2} = \frac{5+4x-x^2}{(x^2+5)^2}.$$

A1 for the correct unsimplified derivative, **A1** for reaching the printed form with the expansion shown.

(b) The denominator is never 0, so $5 + 4x - x^2 = 0$, that is $(5-x)(1+x) = 0$. **M1**

$$x = 5: y = \frac{3}{30} = \frac{1}{10}. \quad x = -1: y = \frac{-3}{6} = -\frac{1}{2}. \quad \mathbf{A1 \ A1}$$

The stationary points are $(-1, -\frac{1}{2})$ and $(5, \frac{1}{10})$.

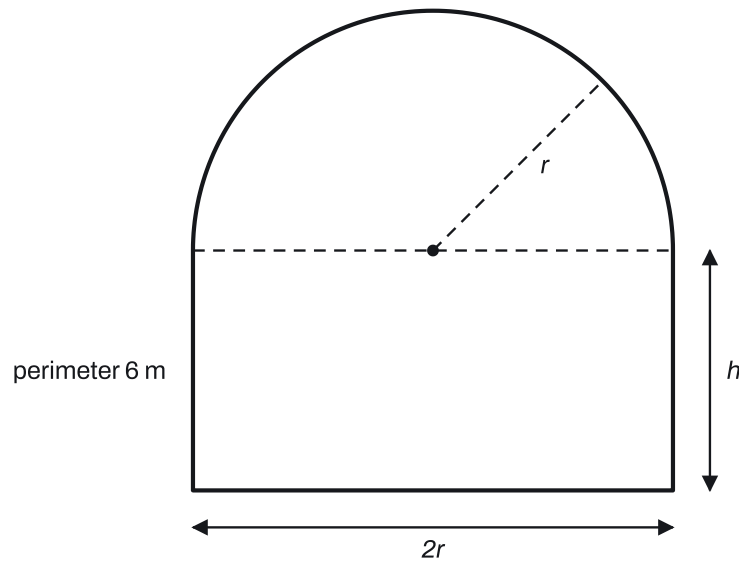
(c) $(x^2+5)^2 > 0$, so the sign of $\frac{dy}{dx}$ is the sign of $(5-x)(1+x)$.

x	-2	-1	0	5	6
$(5-x)(1+x)$	-7	0	5	0	-7
sign of $\frac{dy}{dx}$	-	0	+	0	-

M1 for the signs either side of a point, with the values used.

At $x = -1$ the gradient goes $-, 0, +$, so $(-1, -\frac{1}{2})$ is a **minimum**. **A1** At $x = 5$ it goes $+, 0, -$, so $(5, \frac{1}{10})$ is a **maximum**. **A1**

Example 4



A window is a rectangle of width $2r$ metres and height h metres, with a semicircle of radius r metres on top, as shown. The perimeter of the whole window is 6 m.

(a) Show that the area, $A \text{ m}^2$, of the window is given by $A = 6r - 2r^2 - \frac{\pi r^2}{2}$. [3]

(b) Given that r can vary, find the exact value of r for which A is stationary, and show that this gives a maximum. [4]

(c) Find the maximum area and the height h of the rectangle for this window. [2]

(a) Perimeter: two sides, the bottom and the semicircular arc: $2h + 2r + \pi r = 6$, so $h = \frac{6 - 2r - \pi r}{2}$. **B1** for the perimeter, **M1** for making h the subject and substituting:

$$A = 2r \times h + \frac{1}{2}\pi r^2 = r(6 - 2r - \pi r) + \frac{1}{2}\pi r^2 = 6r - 2r^2 - \frac{\pi r^2}{2}.$$

A1, the given answer reached with every step shown.

(b) $\frac{dA}{dr} = 6 - 4r - \pi r$. **B1**

$$6 - (4 + \pi)r = 0 \Rightarrow r = \frac{6}{4 + \pi} (\approx 0.840). \text{ **M1 A1**}$$

$\frac{d^2A}{dr^2} = -(4 + \pi) < 0$ for every r , so this is a **maximum**. **B1**

(c) Substitute $r = \frac{6}{4 + \pi}$: $A = 6r - \frac{4 + \pi}{2}r^2 = \frac{36}{4 + \pi} - \frac{18}{4 + \pi} = \frac{18}{4 + \pi} \approx 2.52 \text{ m}^2$. **B1**

$$h = \frac{6 - (2 + \pi)r}{2} = \frac{6(4 + \pi) - 6(2 + \pi)}{2(4 + \pi)} = \frac{6}{4 + \pi}, \text{ the same as } r \text{ (about 0.840 m). B1}$$

Common mistakes

- **Using the tangent gradient for the normal, or the other way round.** Examiners see both every year. Read the question again before writing the line, and use $-\frac{1}{m}$ for a normal.
- **Taking y from the wrong place.** The point's y comes from the curve; the gradient comes from $\frac{dy}{dx}$. Using a different point (such as another labelled point) in the line equation loses the marks.
- **Finding a gradient without calculus.** Working out a second point and the gradient of a chord, or using a calculator's numerical derivative, is not accepted. Write the derivative out.
- **Getting the small-changes relationship upside down.** To find δx from $\delta y = 0.02$ and $\frac{dy}{dx} = 40$, divide: $\delta x \approx 0.02 \div 40 = 0.0005$. Multiplying the two, or dividing the other way round, are common wrong answers.
- **Skipping calculus in a small-changes question.** Working out y at $x = 0.1$ and at $x = 0.1 + h$ (or x at two values of y) and subtracting scores nothing when the question says "use differentiation". Also, evaluate $\frac{dy}{dx}$ at the start value, not at $0.1 + h$, and don't drop the h from the answer.
- **Losing the minus sign of a decreasing rate.** " y is decreasing at 0.5 units per second" means $\frac{dy}{dt} = -0.5$, and the answer is then negative too.
- **Finding an average rate instead of an instantaneous one.** A rate "at the instant when..." needs differentiation and the chain rule, not a change divided by a time.
- **Stating a general chain rule only.** $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ earns credit only when written with the question's own variables and values.
- **Mixing up the stationary-point conditions.** Some candidates set $\frac{d^2y}{dx^2} = 0$ to find stationary points; it is $\frac{dy}{dx} = 0$. And "the minimum value" is a y -value, while "the minimum point" needs both coordinates.
- **Wrong surface-area formulas in optimisation.** Terms like $2\pi r^2 + \pi r^2 h$ mix areas with volumes. Check every term is a length times a length, count which faces exist (open top? two ends?), and use the constraint the question gives.
- **Degree mode.** Every calculus answer with trigonometric functions uses radians.
- **Rounding.** $0.08487h$ to 3 significant figures is $0.0849h$, not $0.085h$: counting starts at the first non-zero digit. Keep exact values until the last line.

How to get the marks

- **Show the derivative first.** The first marks are usually **B1** or **M1** for a correct $\frac{dy}{dx}$; every later method mark needs it. On "use differentiation" or "use calculus" questions, an answer without a derivative scores nothing, even if correct.
- **Tangents and normals:** show the gradient you used and $y - y_1 = m(x - x_1)$ with numbers in. Examiners note that this form causes fewer sign errors than $y = mx + c$. Follow-through marks are given for using a wrong gradient correctly, so keep going.

- **Stationary points:** "find the stationary points" means both coordinates. "Determine the nature" means a calculation **and** a conclusion for each point. The syllabus expects full justification: state the value or sign of $\frac{d^2y}{dx^2}$, or the signs of $\frac{dy}{dx}$ either side with the x -values.
- **Small changes:** write the relationship with values, such as $\delta y \approx 1.94 \times h$; that line earns the method mark.
- **Rates:** state a chain rule with the question's variables, find any needed value first (for example r from V), then substitute.
- **"Show that"** gives you the answer: every step must be visible, working forwards from the facts, not backwards from the printed result.
- **"Hence"** means use the previous part, such as the derivative just shown.
- **"Exact"** means π , e , $\ln$, surds and fractions, no decimals. Otherwise give 3 significant figures, keeping more in the working.
- **Paper 1 has no calculator.** Show exact trigonometric values, $e^0 = 1$, $e^{\ln a} = a$ and surd simplification step by step.

Quick check

1. The area of a regular hexagon of side x cm is $A = \frac{3\sqrt{3}}{2}x^2$. Use differentiation to find the approximate increase in the area when the side increases from 4 cm to 4.02 cm.
2. A ladder 5 m long leans against a vertical wall. The foot of the ladder is x m from the wall and the top is $y = \sqrt{25 - x^2}$ m above the ground. The foot slides away from the wall at 0.3 m per second. Find the rate at which the top of the ladder is moving when the foot is 3 m from the wall.
3. Find the coordinates of the point on the curve $y = x \ln x$ at which the tangent is parallel to the line $y = 3x + 1$, and find the equation of this tangent.
4. Two positive numbers x and y have $x + y = 12$. Find the greatest value of $P = x^2y$, showing that it is a maximum.
5. A curve has equation $y = x^3 - 3x^2 - 9x + 2$. Find the coordinates of its stationary points and their nature, and find the values of k for which the equation $x^3 - 3x^2 - 9x + 2 = k$ has three distinct solutions.

Answers

1. $\frac{dA}{dx} = 3\sqrt{3}x = 12\sqrt{3}$ at $x = 4$. $\delta A \approx 12\sqrt{3} \times 0.02 = 0.24\sqrt{3} \approx 0.416 \text{ cm}^2$.
2. $\frac{dy}{dx} = \frac{-x}{\sqrt{25-x^2}} = -\frac{3}{4}$ at $x = 3$. $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = -\frac{3}{4} \times 0.3 = -0.225$, so the top slides down the wall at 0.225 m per second.
3. $\frac{dy}{dx} = \ln x + 1 = 3$, so $\ln x = 2$ and $x = e^2$; $y = e^2 \times 2 = 2e^2$. The point is $(e^2, 2e^2)$ and the tangent is $y - 2e^2 = 3(x - e^2)$, that is $y = 3x - e^2$.
4. $P = x^2(12 - x) = 12x^2 - x^3$. $\frac{dP}{dx} = 24x - 3x^2 = 3x(8 - x) = 0$ gives $x = 8$ (as $x > 0$), so $y = 4$ and $P = 256$. $\frac{d^2P}{dx^2} = 24 - 6x = -24 < 0$ at $x = 8$, so 256 is the greatest value.
5. $\frac{dy}{dx} = 3x^2 - 6x - 9 = 3(x+1)(x-3) = 0$ gives $x = -1$ and $x = 3$; the points are $(-1, 7)$ and $(3, -25)$. $\frac{d^2y}{dx^2} = 6x - 6$ is -12 at $x = -1$ (maximum) and 12 at $x = 3$ (minimum). The line $y = k$ meets the curve three times when $-25 < k < 7$.

Past questions to try

- **0606/22/O/N/24 Q12**: least metal for an open tank with a square base.
- **0606/21/O/N/24 Q7**: the normal to a trigonometric product and the triangle it makes with the axes.
- **0606/22/M/J/23 Q4**: connected rates on a tangent curve, finding x from y first.
- **0606/12/O/N/24 Q2**: stationary points of a cubic, then the values of k for one solution.