

Cambridge IGCSE · Additional Mathematics 0606 · Notes

Coordinate geometry of the circle

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Read first: [Straight-line graphs](#), [Quadratic functions](#)

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What you need to know

By the end of this topic you should be able to:

1. **Know and use the equation of a circle** with centre (a, b) and radius r : $(x - a)^2 + (y - b)^2 = r^2$. Read off the centre and radius whatever form the equation is written in, including the expanded form $x^2 + y^2 + 2gx + 2fy + c = 0$, and write the equation of a circle from facts about it (its centre and a point, the ends of a diameter, three points on it, lines it touches).
2. **Solve problems where a straight line meets a circle**. Find the points where they cross, and decide whether a line is a **tangent** (it touches the circle once), a **chord** (it cuts the circle twice) or **misses** the circle altogether.
3. **Solve problems with tangents to a circle**, including finding the equation of a tangent. This is done with the radius and perpendicular gradients: no calculus is expected.
4. **Solve problems where two circles meet**. Find their points of intersection and the equation of their **common chord**, and decide whether two circles cross, touch or do not meet at all.

This topic joined the 0606 syllabus in 2025, so every past question on it so far is from 2025.

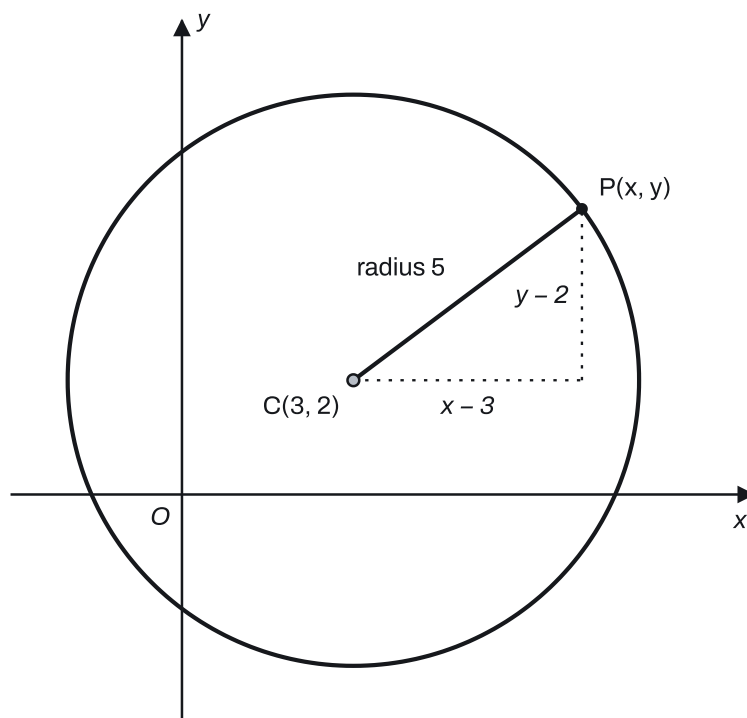
Understand it

The equation comes from Pythagoras

A circle is every point that is the same distance, the **radius** r , from one fixed point, the **centre** $C(a, b)$. Take any point $P(x, y)$ on the circle. Going from C to P you move $x - a$ across and $y - b$ up, and the straight distance CP is r . These three lengths make a right-angled triangle, so Pythagoras gives

$$(x - a)^2 + (y - b)^2 = r^2.$$

Every point on the circle fits this equation, and no point off the circle does.



A different example: for centre $(3, -2)$ and radius 5: $(x - 3)^2 + (y + 2)^2 = 25$. Watch the signs: the equation has **minus** the centre's coordinates, so $y + 2$ means $b = -2$.

To test a point, substitute it into the left-hand side and compare with r^2 :

- $(6, 2)$: $3^2 + 4^2 = 25$, so it is **on** the circle;
- $(1, 0)$: $(-2)^2 + 2^2 = 8 < 25$, so it is **inside**;
- $(8, 1)$: $5^2 + 3^2 = 34 > 25$, so it is **outside**.

The expanded form

Multiply out $(x - 3)^2 + (y + 2)^2 = 25$ and you get $x^2 + y^2 - 6x + 4y - 12 = 0$. Any circle can be written like this:

$$x^2 + y^2 + 2gx + 2fy + c = 0, \quad \text{centre } (-g, -f), \quad r = \sqrt{g^2 + f^2 - c}.$$

You spot a circle's equation because x^2 and y^2 have the **same** coefficient and there is no xy term. If the coefficient isn't 1, divide through by it first.

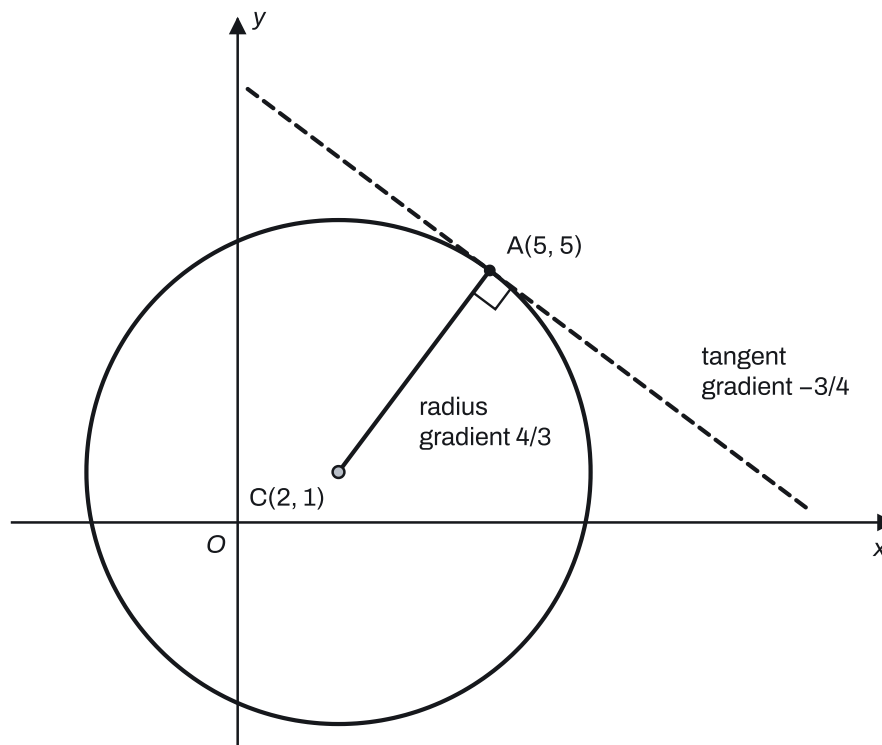
To find the centre and radius, **complete the square** in x and in y separately. For $x^2 + y^2 - 4x + 10y + 20 = 0$:

$$(x - 2)^2 - 4 + (y + 5)^2 - 25 + 20 = 0 \Rightarrow (x - 2)^2 + (y + 5)^2 = 9,$$

so the centre is $(2, -5)$ and the radius is 3. The formula gives the same: $g = -2$, $f = 5$, $r = \sqrt{4 + 25 - 20} = 3$. If $g^2 + f^2 - c$ comes out negative (as it does for $x^2 + y^2 - 2x + 6y + 12 = 0$, where it is -2), the equation is not a circle at all.

Three facts about circles you will use

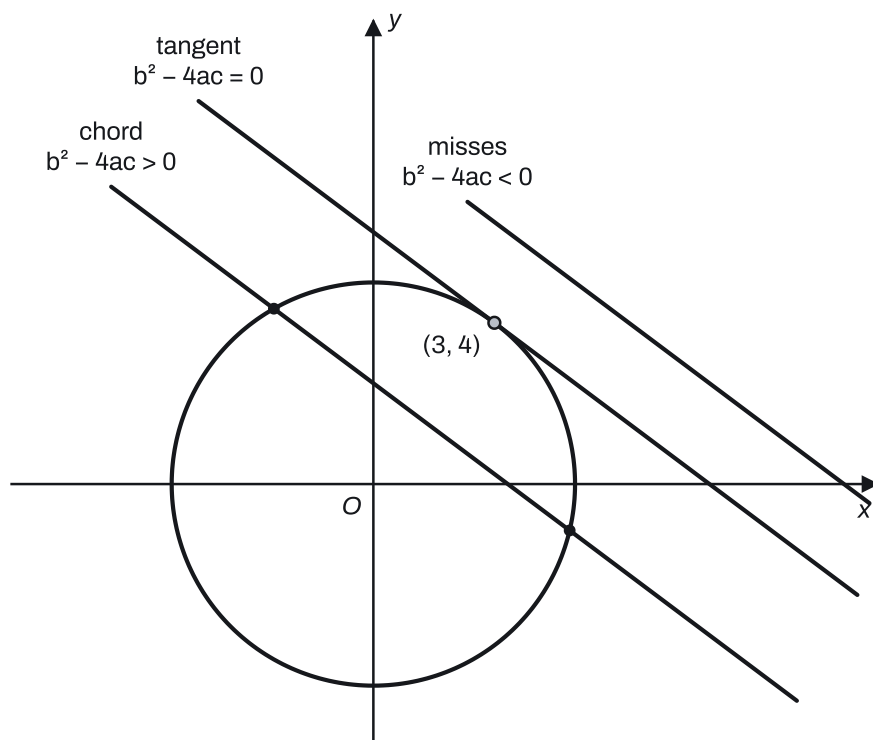
- **The tangent is perpendicular to the radius** at the point where it touches. So the tangent's gradient is $-\frac{1}{m}$, where m is the gradient of the radius to that point.
- **The perpendicular bisector of any chord passes through the centre.** Two chords' perpendicular bisectors meet at the centre.
- **The angle in a semicircle is a right angle.** If A , B and C are on a circle and angle $ABC = 90^\circ$, then AC is a diameter, so its midpoint is the centre.



Where a line meets a circle

To find where the line and circle meet, solve their equations together: rearrange the line to $y = \dots$ (or $x = \dots$), substitute into the circle and simplify to a quadratic. Each root gives one crossing point. The **discriminant** $b^2 - 4ac$ of that quadratic tells you how many there are:

$b^2 - 4ac$	Roots	The line is...
> 0	two	a chord : it cuts the circle at two points
$= 0$	one (repeated)	a tangent : it touches the circle at one point
< 0	none	outside the circle: it does not meet it

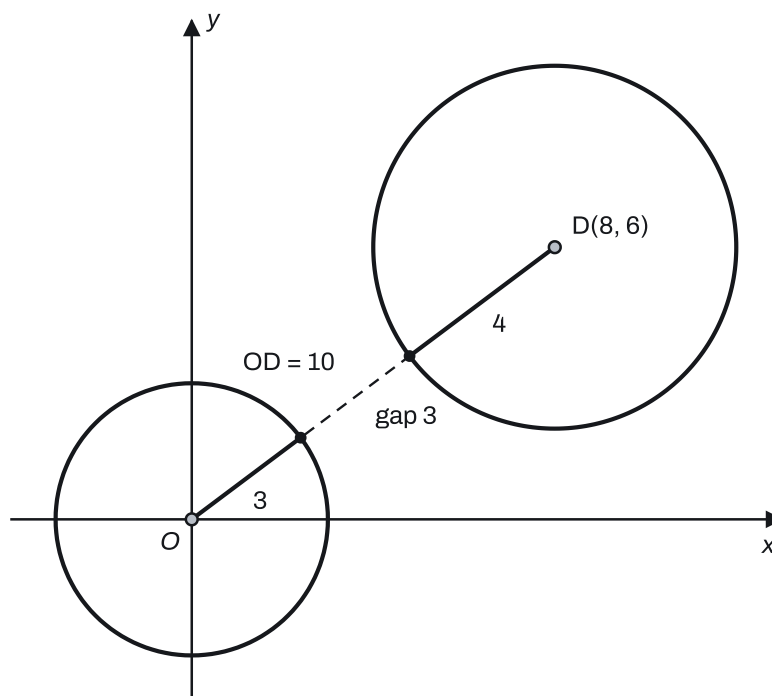


For $x^2 + y^2 = 25$ and $3x + 4y = 25$: $y = \frac{25-3x}{4}$, and substituting gives $25x^2 - 150x + 225 = 0$, that is $25(x - 3)^2 = 0$. One repeated root, $x = 3$, so the line is a tangent at $(3, 4)$.

Two circles

Two circles are compared through the **distance d between their centres** and their radii r_1 and r_2 :

- $d > r_1 + r_2$: they are apart and do not meet;
- $d = r_1 + r_2$: they touch at one point, outside each other;
- between the two: they cross at two points;
- $d = |r_1 - r_2|$: one touches the other from the inside;
- $d < |r_1 - r_2|$: one is inside the other and they do not meet.



A quick sketch like this makes it clear which comparison you need.

Where two circles cross. Write both equations in expanded form and **subtract** one from the other. The x^2 and y^2 terms cancel, leaving a straight line: the line through both crossing points, which is the **common chord**. Then substitute that line into either circle, exactly as for a line and a circle, to find the points.

For $x^2 + y^2 = 25$ and $(x - 5)^2 + (y - 5)^2 = 25$, that is $x^2 + y^2 - 10x - 10y + 25 = 0$: subtracting gives $10x + 10y - 50 = 0$, so the common chord is $x + y = 5$. Substituting $y = 5 - x$ into $x^2 + y^2 = 25$ gives $2x^2 - 10x = 0$, so $x = 0$ or $x = 5$, and the circles cross at $(0, 5)$ and $(5, 0)$. The common chord is always perpendicular to the line joining the centres.

Key facts and formulas

The first formula is in the 0606 list of formulas; the rest must be learnt.

Formula	Status
Circle with centre (a, b) and radius r : $(x - a)^2 + (y - b)^2 = r^2$	Given
$x^2 + y^2 + 2gx + 2fy + c = 0$ has centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$	Learn it
Distance between two points: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Learn it
Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ (centre = midpoint of a diameter)	Learn it
Tangent at a point is perpendicular to the radius: $m_{\text{tangent}} = -\frac{1}{m_{\text{radius}}}$	Learn it
Line meets circle: substitute, then $b^2 - 4ac > 0$ chord, $= 0$ tangent, < 0 no intersection	Learn it

Formula	Status
Perpendicular bisector of a chord passes through the centre	Learn it
Angle in a semicircle is 90°	Learn it
Two circles: compare the distance between centres with $r_1 + r_2$ and $ r_1 - r_2 $	Learn it

How to do it

We have tagged 8 questions on this topic, all from 2025 (Papers 1 and 2), and the types came up this often. 6 of the 8 questions mix two types, so the counts add up to more than 8.

Question type	Questions
Finding the equation of a circle	6
Where a line meets a circle: tangent, chord or no intersection	3
The tangent at a point on the circle	2
Two circles	2
Points on the circle and the ends of a diameter	1

1. Finding the equation of a circle (6 questions)

Every version comes down to the same two things: **find the centre, then find r^2** , and write $(x - a)^2 + (y - b)^2 = r^2$.

1. **Find the centre** from what you are told (see the variations below).
2. **Find r^2** as the squared distance from the centre to any point on the circle. You rarely need r itself, so keep r^2 as a whole number: $r^2 = 40$, not $r = 6.32$.
3. **Write the equation**, and expand it only if the question asks for the form $x^2 + y^2 + ax + by + c = 0$.

Variations you will meet:

- **Centre and a point.** Centre $(4, -1)$ through $(1, 3)$: $r^2 = 3^2 + 4^2 = 25$.
- **The ends of a diameter.** The centre is the **midpoint**; r^2 is the squared distance from the centre to either end (or a quarter of the diameter squared). Don't use the diameter as the radius.
- **Three points.** Find the gradients of the three sides of the triangle. If two sides are perpendicular (gradients multiply to -1), the third side is a diameter: its midpoint is the centre. Otherwise find the perpendicular bisectors of two sides and solve them together to get the centre.
- **Lines it touches.** A circle touching two parallel lines, such as $x = 1$ and $x = 9$, has its centre halfway between them ($x = 5$) and a radius of half the gap (4). A third line, say $y = 2$, puts the centre 4 above or 4 below it, so the centre is $(5, 6)$ or $(5, -2)$: both are possible unless the question rules one out (for example with a fourth line $y = 10$, or by saying the centre is above the x -axis).

- **A given centre, through another point you have found**, for example a point of contact from an earlier part.
- **Two circles with a common chord**. The line joining the centres is the perpendicular bisector of the common chord. Use Pythagoras with half the chord and the radius to find how far each centre is from the chord, then walk that far along the line of centres (Example 4 uses the same idea).

2. Where a line meets a circle: tangent, chord or no intersection (3 questions)

1. **Substitute** the line into the circle. Make y the subject of the line (or x if that is easier) and replace every y in the circle's equation.
2. **Simplify to a quadratic** $ax^2 + bx + c = 0$. Expand brackets carefully; this is where most marks are lost.
3. **Finish** the way the question asks:
 - **find the points**: solve, then substitute each x into the **line** to get y ;
 - **show it is a tangent**: show $b^2 - 4ac = 0$ (or that the quadratic is a perfect square with one repeated root), and **say** "so the line is a tangent";
 - **show it does not meet the circle**: show $b^2 - 4ac < 0$ and say so;
 - **find a constant** (the line is $y = 2x + k$, say) **so that it is a tangent**: set $b^2 - 4ac = 0$ and solve for k . Two answers are normal, one tangent on each side of the circle. For a chord, solve $b^2 - 4ac > 0$ instead.

Variations you will meet:

- **Where a circle meets an axis**. The y -axis is the line $x = 0$, so put $x = 0$ into the circle and solve for y ; for the x -axis put $y = 0$. For example $(x - 4)^2 + (y - 1)^2 = 25$ meets the y -axis where $16 + (y - 1)^2 = 25$, so $y = 4$ or $y = -2$.
- **The point of contact** of a tangent: the repeated root gives x ; the line gives y .
- **Another way to show a tangent**: find the line through the centre perpendicular to the given line, find where the two lines meet, and show that point is on the circle (its distance from the centre is r). For the circle with centre $(2, 1)$ and radius 5 and the line $3x + 4y = 35$, the perpendicular through $(2, 1)$ meets the line at $(5, 5)$, which is 5 from the centre, so the line is a tangent there. This takes longer; the discriminant is usually quicker.

3. The tangent at a point on the circle (2 questions)

1. **Find the centre C** (complete the square if the equation is expanded).
2. **Gradient of the radius** from C to the point.
3. **Tangent gradient** $= -1 \div (\text{radius gradient})$.
4. $y - y_1 = m(x - x_1)$ through the point, then rearrange to the form asked for, such as $ax + by = c$ with integers.

For the circle with centre $(2, 1)$ and the point $(5, 5)$: the radius gradient is $\frac{4}{3}$, so the tangent gradient is $-\frac{3}{4}$ and the tangent is $y - 5 = -\frac{3}{4}(x - 5)$, that is $3x + 4y = 35$. If the radius is horizontal, the tangent is vertical ($x = \dots$), and the other way round.

4. Two circles (2 questions)

1. Find each **centre and radius**.
2. Find the **distance d between the centres**.
3. **Compare d with $r_1 + r_2$ (and $|r_1 - r_2|$ if one circle might be inside the other), then **state the conclusion** in words.**

Finding where they meet, or the common chord:

1. Expand both equations into the form $x^2 + y^2 + \dots = 0$ (divide through if x^2 has a coefficient other than 1).
2. **Subtract** one from the other: the result is the equation of the common chord.
3. **Substitute** the chord's line into either circle and solve the quadratic; each root gives a crossing point (use the line to get y). A repeated root means the circles touch; no real roots means they do not meet.

For example, $(x - 8)^2 + (y - 6)^2 = 16$ and $x^2 + y^2 = 9$: $d = \sqrt{64 + 36} = 10$ and $r_1 + r_2 = 7 < 10$, so they do not meet. Draw a sketch first; it shows at once which comparison matters. If the circles cross, their common chord and the line of centres are perpendicular (type 1, last variation), which gives a check on your chord's gradient.

5. Points on the circle and the ends of a diameter (1 question)

- **Show a point lies on the circle**: substitute it into the equation and show both sides are equal (or show its distance from the centre equals r).
- **The other end of a diameter**: the centre is the midpoint of the diameter, so from end A to the centre is the same step as from the centre to end B . With centre $(2, -1)$ and $A(-1, 3)$: the step is $(+3, -4)$, so B is $(5, -5)$.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
 - **A1** accuracy mark: for a correct answer; it needs the M mark before it.
 - **B1** mark for a correct result on its own, with no method needed.
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Example 1

A circle has equation $x^2 + y^2 - 6x + 8y - 11 = 0$.

- (a) Find the coordinates of the centre and the radius of the circle. **[3]**

(b) Find, in exact form, the x -coordinates of the points where the circle meets the x -axis. [3]

(c) Determine whether the point $(7, 0)$ lies inside or outside the circle. [2]

(a) Complete the square in x and in y :

$$(x - 3)^2 - 9 + (y + 4)^2 - 16 - 11 = 0 \Rightarrow (x - 3)^2 + (y + 4)^2 = 36.$$

M1 for completing both squares. Centre $(3, -4)$ **A1**, radius 6 **A1**.

(b) On the x -axis $y = 0$: $x^2 - 6x - 11 = 0$. **M1** for substituting $y = 0$ into the circle.

$$x = \frac{6 \pm \sqrt{36 + 44}}{2} = \frac{6 \pm \sqrt{80}}{2} = 3 \pm 2\sqrt{5}.$$

M1 for solving, **A1** for $3 \pm 2\sqrt{5}$ exactly.

(c) Distance squared from the centre: $(7 - 3)^2 + (0 + 4)^2 = 32$. **M1**

$32 < 36$, so $(7, 0)$ is closer to the centre than the radius: it is **inside** the circle. **A1**

Example 2

The points $A(-1, 5)$ and $B(7, -1)$ are the ends of a diameter of a circle.

(a) Find the equation of the circle. [3]

(b) Find the equation of the tangent to the circle at A . Give your answer in the form $ax + by = c$, where a , b and c are integers. [4]

(a) Centre = midpoint of $AB = \left(\frac{-1+7}{2}, \frac{5-1}{2}\right) = (3, 2)$. **B1**

$r^2 = (-1 - 3)^2 + (5 - 2)^2 = 16 + 9 = 25$. **M1** (Or $AB^2 = 8^2 + 6^2 = 100$, so $r^2 = \frac{100}{4} = 25$.)

$$(x - 3)^2 + (y - 2)^2 = 25$$

A1

(b) Gradient of the radius from $(3, 2)$ to $A(-1, 5)$: $\frac{5-2}{-1-3} = -\frac{3}{4}$. **M1**

The tangent is perpendicular, so its gradient is $\frac{4}{3}$. **M1**

$$y - 5 = \frac{4}{3}(x + 1) \Rightarrow 3y - 15 = 4x + 4 \Rightarrow 4x - 3y = -19.$$

M1 for a line through A with their perpendicular gradient, **A1** for $4x - 3y = -19$ (or $-4x + 3y = 19$).

Example 3

A circle has equation $(x - 2)^2 + (y - 1)^2 = 8$ and a line has equation $y = x + k$, where k is a constant.

- (a) Show that the x -coordinates of any points where the line meets the circle satisfy $2x^2 + (2k - 6)x + (k - 1)^2 - 4 = 0$. **[2]**
- (b) Find the two values of k for which the line is a tangent to the circle. **[3]**
- (c) For the positive value of k , find the coordinates of the point where the line touches the circle. **[2]**
- (d) Write down the set of values of k for which the line cuts the circle at two points. **[1]**

(a) Substitute $y = x + k$: **M1**

$$(x - 2)^2 + (x + k - 1)^2 = 8 \Rightarrow x^2 - 4x + 4 + x^2 + 2(k - 1)x + (k - 1)^2 - 8 = 0$$

$$\Rightarrow 2x^2 + (2k - 6)x + (k - 1)^2 - 4 = 0.$$

A1, the given answer with the expansion shown.

(b) Tangent means one repeated root, so $b^2 - 4ac = 0$: **M1**

$$(2k - 6)^2 - 4 \times 2 \times ((k - 1)^2 - 4) = 4k^2 - 24k + 36 - 8k^2 + 16k + 24 = -4k^2 - 8k + 60 = 0.$$

A1 for a correct three-term quadratic in k .

$$k^2 + 2k - 15 = 0 \Rightarrow (k + 5)(k - 3) = 0, \text{ so } k = 3 \text{ or } k = -5. \text{ **A1**}$$

(c) With $k = 3$ the quadratic is $2x^2 + 0x + 0 = 0$, so $x = 0$ (a repeated root). **M1** Then $y = 0 + 3 = 3$: the point is $(0, 3)$. **A1**

Check: $(0 - 2)^2 + (3 - 1)^2 = 8$. $\checkmark$ (For $k = -5$ the line touches at $(4, -1)$ on the other side.)

(d) Two points means $b^2 - 4ac > 0$: $-4(k + 5)(k - 3) > 0$, so $-5 < k < 3$. **B1**

Example 4

The line $y = x + 6$ meets the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at the points P and Q , where P has the larger x -coordinate.

- (a) Find the coordinates of P and Q . **[4]**
- (b) Find the exact length of PQ . **[1]**
- (c) A second circle has the same radius as the first, passes through P and Q , and has a different centre. Find its equation. **[4]**

(a) Substitute $y = x + 6$: **M1**

$$x^2 + (x + 6)^2 - 2x - 4(x + 6) - 20 = 0 \Rightarrow 2x^2 + 6x - 8 = 0 \Rightarrow x^2 + 3x - 4 = 0.$$

A1. $(x + 4)(x - 1) = 0$, so $x = 1$ or $x = -4$. **M1**

$P(1, 7)$ and $Q(-4, 2)$. **A1**

(b) $PQ = \sqrt{(1 + 4)^2 + (7 - 2)^2} = \sqrt{50} = 5\sqrt{2}$. **B1**

(c) The first circle: $(x - 1)^2 + (y - 2)^2 = 1 + 4 + 20 = 25$, so centre $(1, 2)$, radius 5. **B1**

Both centres lie on the perpendicular bisector of the chord PQ , and both are 5 from P and Q , so they sit the same distance either side of the midpoint M of PQ . **M1** for using the perpendicular bisector or this symmetry.

$M = \left(\frac{1-4}{2}, \frac{7+2}{2}\right) = \left(-\frac{3}{2}, \frac{9}{2}\right)$. From $(1, 2)$ to M is a step of $\left(-\frac{5}{2}, +\frac{5}{2}\right)$; take the same step again to reach the second centre: $(-4, 7)$. **A1**

Check: from $(-4, 7)$ to $P(1, 7)$ is 5 and to $Q(-4, 2)$ is 5. ✓

$$(x + 4)^2 + (y - 7)^2 = 25, \quad \text{or} \quad x^2 + y^2 + 8x - 14y + 40 = 0.$$

A1

Common mistakes

- **Getting the centre's signs wrong.** $(x + 3)^2$ means the centre has $x = -3$. In the expanded form the centre is $(-g, -f)$: halve the coefficients of x and y , then change their signs.
- **Using the diameter as the radius, or forgetting to square.** The March 2025 report saw the diameter's length used as r ; others wrote r where the equation needs r^2 . Write " $r^2 = \dots$ " before you write the equation.
- **Leaving r^2 as a decimal.** Keep it exact: $r^2 = 58$, not $r = 7.62$ squared back to 58.06.
- **Misremembering the equation or the distance formula**, for example a minus sign between the squares, or no squares at all. The circle's equation is on the list of formulas: check it.
- **Substituting the end points of a diameter into $(x - a)^2 + (y - b)^2 = r^2$** and getting stuck. Find the centre first (the midpoint), then the radius.
- **Rearranging the circle when the question is about a line.** In June 2025 many candidates who had to show a line was a tangent only completed the square and got no further. The line has to be substituted into the circle.
- **Not concluding.** After finding a discriminant of 0 or a repeated root, say "one point of contact, so the line is a tangent". The report says full marks went to answers with that comment.
- **Not giving the line in the form asked.** The June 2025 report noted many correct tangents that lost the last mark because they weren't rearranged to $ax + by = c$ with integers.
- **Two circles without a diagram.** Very few answered the two-circle part correctly in June 2025; those who drew a sketch saw that they needed the distance between the centres and the sum of the radii.
- **Expansion slips** when substituting a line: $(3x + k - 2)^2$ has three terms squared and three cross terms. Bracket the whole line expression before squaring.

How to get the marks

- **Centre and radius** often earn a mark each, even if the final equation is wrong, so write them down clearly ("centre $(5, 3)$, $r^2 = 50$ ").

- **"Find the equation of the circle"**: the form $(x - a)^2 + (y - b)^2 = r^2$ is fine unless another form is asked for; expanding afterwards is ignored if the first form is right. If the question gives a form such as $x^2 + y^2 + ax + by + c = 0$, finish in that form.
- **"Show that"** means the answer is printed: show the substitution, the expansion and the simplified result exactly as given. For "show that the line is a tangent", the conclusion sentence is part of the marks.
- **"Hence"** means use the previous part: for example, the gradients you have just found show a right angle, so the third side is a diameter.
- **"Exact"** means surds, such as $1 \pm \sqrt{12}$ or $3 \pm 2\sqrt{5}$, not decimals. Surds are fine unsimplified.
- **"Solutions by accurate drawing will not be accepted"**: every value must come from working, not from a graph. A sketch to plan is still a good idea.
- **Method marks follow through**. A wrong centre used correctly in the radius and the equation still earns the method marks, so always finish.
- **Tangent at a point**: the marks are for the centre, the radius gradient, the perpendicular gradient with the right point, and the final form. Show each.

Quick check

1. Find the centre and radius of the circle $2x^2 + 2y^2 + 8x - 12y - 6 = 0$.
2. The points $A(-3, 1)$, $B(1, 5)$ and $C(5, 1)$ lie on a circle. Show that angle ABC is 90° and hence find the equation of the circle.
3. Show that the line $2x + y = 12$ does not meet the circle $x^2 + y^2 = 16$.
4. Find the equation of the tangent to the circle $(x - 1)^2 + (y + 2)^2 = 20$ at the point $(5, 0)$, in the form $ax + by = c$.
5. The circles $x^2 + y^2 = 9$ and $(x - 6)^2 + (y - 8)^2 = r^2$ touch at exactly one point. Find the two possible values of r .
6. The circles $x^2 + y^2 = 10$ and $x^2 + y^2 - 8x - 4y + 10 = 0$ meet at two points. Find the equation of their common chord and the coordinates of the points.

Answers

1. Divide by 2: $x^2 + y^2 + 4x - 6y - 3 = 0$, so $(x + 2)^2 + (y - 3)^2 = 4 + 9 + 3 = 16$. Centre $(-2, 3)$, radius 4.
2. Gradient $AB = \frac{5-1}{1+3} = 1$ and gradient $BC = \frac{1-5}{5-1} = -1$. Their product is -1 , so AB is perpendicular to BC and angle $ABC = 90^\circ$. Hence AC is a diameter: centre $(1, 1)$, radius 4, and the circle is $(x - 1)^2 + (y - 1)^2 = 16$.
3. $y = 12 - 2x$, so $x^2 + (12 - 2x)^2 = 16$, giving $5x^2 - 48x + 128 = 0$. The discriminant is $48^2 - 4 \times 5 \times 128 = 2304 - 2560 = -256 < 0$, so there are no real roots and the line does not meet the circle.
4. Centre $(1, -2)$. Radius gradient $= \frac{0+2}{5-1} = \frac{1}{2}$, so the tangent gradient is -2 . $y - 0 = -2(x - 5)$, that is $2x + y = 10$.
5. The distance between the centres $(0, 0)$ and $(6, 8)$ is 10. Touching outside each other: $3 + r = 10$, so $r = 7$. Touching with the first circle inside: $r - 3 = 10$, so $r = 13$.
6. Subtracting the second equation from the first: $8x + 4y - 10 = 10$, so the common chord is $2x + y = 5$. Substituting $y = 5 - 2x$ into $x^2 + y^2 = 10$ gives $5x^2 - 20x + 15 = 0$, that is $x^2 - 4x + 3 = 0$, so $x = 1$ or $x = 3$. The points are $(1, 3)$ and $(3, -1)$.

Past questions to try

- **0606/12/F/M/25 Q3**: the equation of a circle from the ends of a diameter.
- **0606/12/M/J/25 Q3**: showing a line is a tangent, its point of contact, then a new circle.
- **0606/11/M/J/25 Q3**: a point on the circle, the other end of a diameter and the tangent there.