Cambridge IGCSE · Additional Mathematics 0606 · Notes

Factors of polynomials

Read online at **pastopic.com/cambridge/additional-mathematics-0606/notes/factors-of-polynomials**

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Read first: **Quadratic functions**

What you need to know

By the end of this topic you should be able to:

1. **Know and use the remainder theorem and the factor theorem.** Find the remainder when a polynomial is divided by a linear expression such as $x - 2$ or $2x + 1$ without dividing, and decide whether a linear expression is a factor. Use these facts to form equations and find unknown coefficients.
2. **Find the factors of a polynomial.** For a cubic, first write it as a linear factor times a quadratic factor (by observation, comparing coefficients or algebraic long division), then factorise the quadratic if it can be.
3. **Solve cubic equations**, including showing how many real roots a cubic equation has.

Polynomials of degree higher than 3 and dividing by a quadratic are not needed. The factor and remainder theorems are used with linear divisors only.

Understand it

Polynomials and division

A **polynomial** is a sum of whole-number powers of x with number coefficients, such as $p(x) = 2x^3 - 3x^2 + 4x + 3$. Its **degree** is the highest power: this one is a **cubic** (degree 3). A quadratic has degree 2 and a linear expression degree 1.

Dividing a polynomial by a linear expression works like dividing whole numbers: $17 = 5 \times 3 + 2$ has quotient 3 and remainder 2. In the same way,

$$p(x) = (\text{divisor}) \times q(x) + r,$$

where $q(x)$ is the **quotient** and r is the **remainder**. When the divisor is linear, the quotient of a cubic is a quadratic and the remainder is just a number.

The remainder theorem

Put $x = a$ into $p(x) = (x - a)\, q(x) + r$. The bracket becomes 0, so the whole first term vanishes and $p(a) = r$:

The remainder when $p(x)$ is divided by $x - a$ is $p(a)$.

So you can find a remainder without dividing at all. For $p(x) = x^3 - 2x^2 - 5x + 6$, the remainder on dividing by $x - 2$ is $p(2) = 8 - 8 - 10 + 6 = -4$. (Dividing properly gives $p(x) = (x - 2)(x^2 - 5) - 4$, the same -4.)

Watch the sign: dividing by $x + 3$ means $x - (-3)$, so you work out $p(-3)$. Dividing by x means $x - 0$, so the remainder is $p(0)$, the constant term.

For a divisor $ax - b$, use the value of x that makes it zero, $x = \frac{b}{a}$:

The remainder when $p(x)$ is divided by $ax - b$ is $p\left(\frac{b}{a}\right)$.

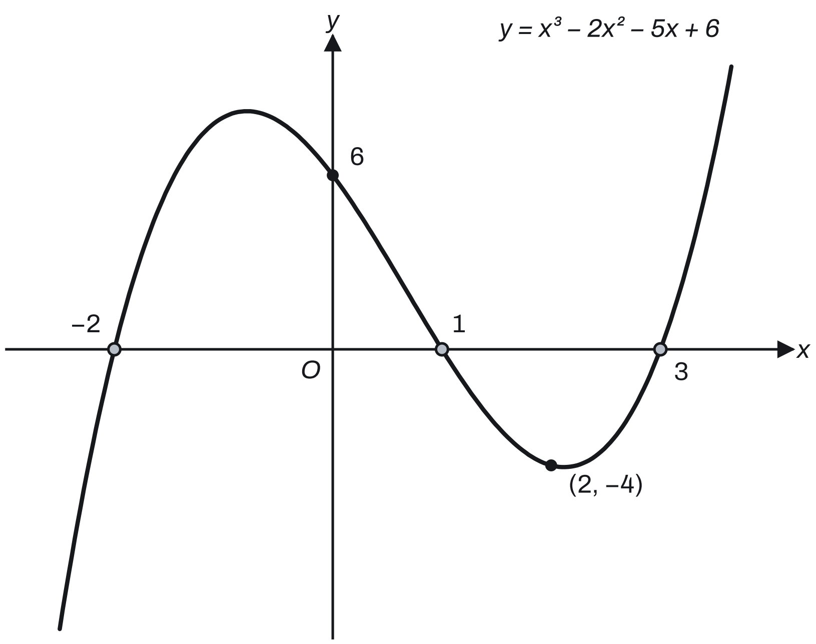
For example, $2x^3 + x^2 - 13x + 6$ divided by $2x + 1$ leaves $p\left(-\frac{1}{2}\right) = -\frac{1}{4} + \frac{1}{4} + \frac{13}{2} + 6 = \frac{25}{2}$.

The factor theorem

A **factor** divides exactly, with remainder 0. Putting that into the remainder theorem:

$x - a$ is a factor of $p(x)$ exactly when $p(a) = 0$. Likewise, $ax - b$ is a factor exactly when $p\left(\frac{b}{a}\right) = 0$.

Each factor gives a root: if $x - a$ is a factor then $x = a$ solves $p(x) = 0$, and that is where the graph $y = p(x)$ crosses the x-axis.



Finding a first factor

To use the factor theorem you need a value to try. If the cubic has integer coefficients and a factor $x - a$ with integer a, then a must divide the **constant term**. So list the factors of the constant, positive and negative, and substitute until one gives 0.

For $p(x) = x^3 - 7x^2 + 7x + 15$, the constant is 15, so try $\pm1, \pm3, \pm5, \pm15$. $p(1) = 16$, but $p(-1) = -1 - 7 - 7 + 15 = 0$, so $x + 1$ is a factor.

If the leading coefficient is not 1, the root might be a fraction: try $\pm\frac{\text{factor of constant}}{\text{factor of leading coefficient}}$, such as $\frac{1}{2}$ or $-\frac{3}{2}$. A root $x = \frac{1}{2}$ gives the factor $2x - 1$; write factors with whole numbers, not $x - \frac{1}{2}$.

Finding the quadratic factor

Once you have one linear factor, the rest is a quadratic. There are three ways to find it.

By observation (comparing coefficients). Write $x^3 - 7x^2 + 7x + 15 = (x + 1)(x^2 + Bx + 15)$. The x^2 term must be x^3's partner: $1 \times x^3$ needs x^2 first. The constant must be 15 because $1 \times 15 = 15$. The middle coefficient comes from matching the x^2 terms: on the right they are $Bx^2 + x^2$, so $B + 1 = -7$ and $B = -8$. Check with the x terms: $15x + Bx = 7x$. ✓ So

$$x^3 - 7x^2 + 7x + 15 = (x + 1)(x^2 - 8x + 15) = (x + 1)(x - 3)(x - 5).$$

By algebraic long division. Divide $2x^3 + x^2 - 13x + 6$ by $x - 2$:

1. $2x^3 \div x = 2x^2$. Multiply back: $2x^2(x - 2) = 2x^3 - 4x^2$. Subtract: $x^2 - (-4x^2) = 5x^2$, then bring down $-13x$.
2. $5x^2 \div x = 5x$. Multiply back: $5x(x - 2) = 5x^2 - 10x$. Subtract: $-13x - (-10x) = -3x$, then bring down $+6$.
3. $-3x \div x = -3$. Multiply back: $-3(x - 2) = -3x + 6$. Subtract: remainder 0.

So the quotient is $2x^2 + 5x - 3$, and

$$2x^3 + x^2 - 13x + 6 = (x - 2)(2x^2 + 5x - 3) = (x - 2)(2x - 1)(x + 3).$$

If the cubic has a missing power, such as $x^3 - 3x + 2$, write it as $x^3 + 0x^2 - 3x + 2$ before dividing, so the columns line up.

By grouping, when the terms pair up: $x^3 - 3x^2 - 4x + 12 = x^2(x - 3) - 4(x - 3) = (x - 3)(x^2 - 4) = (x - 3)(x - 2)(x + 2)$.

Synthetic division also works, but with a divisor like $2x - 1$ it gives a quotient twice too big, so you must divide it by 2. Examiners see this go wrong often; long division or observation is safer.

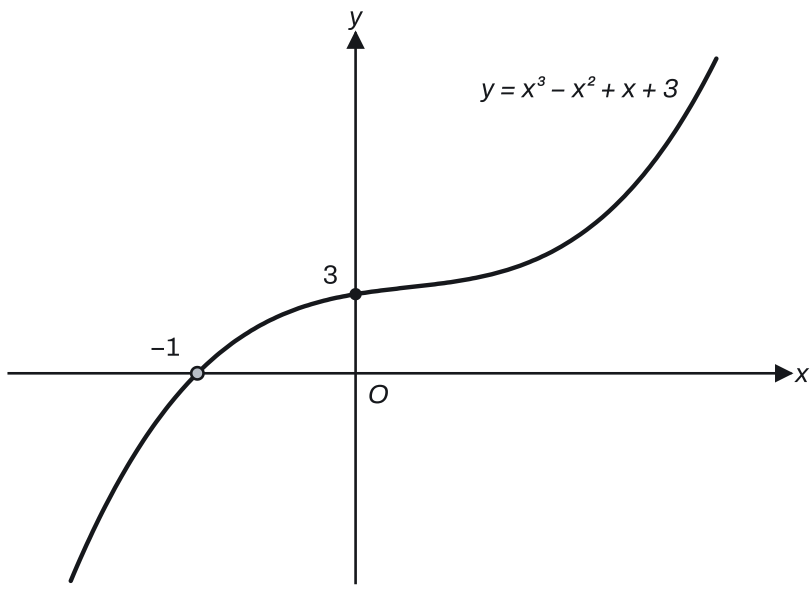
Solving a cubic equation

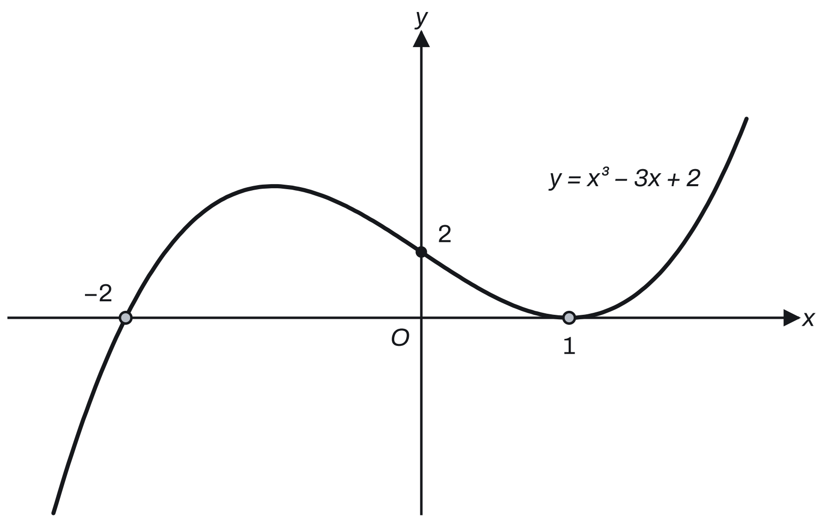
With $p(x)$ written as a product, $p(x) = 0$ when any factor is 0. So

$$(x - 2)(2x - 1)(x + 3) = 0 \;\Rightarrow\; x = 2,\; \tfrac{1}{2},\; -3.$$

A cubic always has at least one real root, and at most three. The quadratic factor decides the rest, through its discriminant $b^2 - 4ac$ (see Quadratic functions):

- $b^2 - 4ac > 0$: two more real roots, so three in all (unless one repeats the linear root);
- $b^2 - 4ac = 0$: the quadratic is a perfect square, giving a **repeated root**, where the graph touches the x-axis;
- $b^2 - 4ac < 0$: no more real roots, so the cubic equation has **only one real root**.





Key facts and formulas

"Given" means it is in the List of formulas on page 2 of the exam paper. "Learn it" means you must remember it.

Formula	Status
$p(x) = (\text{divisor}) \times q(x) + r$	Learn it

Formula	Status
Remainder theorem: dividing $p(x)$ by $x-a$ leaves $p(a)$; by $ax-b$ leaves $p\left(\frac{b}{a}\right)$; by x leaves $p(0)$	Learn it
Factor theorem: $x-a$ is a factor $\iff p(a)=0$; $ax-b$ is a factor $\iff p\left(\frac{b}{a}\right)=0$	Learn it
An integer root of a cubic with integer coefficients divides the constant term	Learn it
Roots α, β, γ and leading coefficient k: $p(x)=k(x-\alpha)(x-\beta)(x-\gamma)$	Learn it
Quadratic formula: $ax^2+bx+c=0 \Rightarrow x=\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$	Given
Quadratic factor with $b^2-4ac<0$: the cubic equation has only one real root; $=0$: a repeated root	Learn it
$\dfrac{d}{dx}(x^n)=nx^{n-1}$, so $p'(0)$ is the coefficient of x	Learn it

How to do it

In the Paper 1 and Paper 2 questions we have tagged for this topic (38 questions, 2021–2025), the types came up this often. 25 of the 38 questions mix types, so the counts add up to more than 38.

Question type	Questions
Find unknown coefficients from factor and remainder facts	23
Factorise a cubic and solve $p(x)=0$	19
Where a curve meets a line or another curve	8
Show there is only one real root (or a repeated root)	8
Find a remainder or a quotient	6
A cubic equation in disguise	4
Build a polynomial from its roots	1

1. Find unknown coefficients from factor and remainder facts (23 questions)

1. **Turn each fact into an equation.**
   - "$x-a$ is a factor" or "$p(x)$ is divisible by $x-a$" or "$x=a$ is a root": $p(a)=0$.
   - "The remainder is R when $p(x)$ is divided by $x-a$": $p(a)=R$.
   - For $ax-b$ or $ax+b$, substitute $x=\frac{b}{a}$ or $x=-\frac{b}{a}$. Multiply the equation through to clear the fractions.
2. **Simplify** each equation to the form $(\ldots)a+(\ldots)b=\text{number}$.
3. **Solve the simultaneous equations** (elimination is usually quickest).
4. **Check**: if the question says a and b are integers and you get a fraction, look for an error.

For example, $p(x)=x^3+ax^2+bx-6$ has a factor $x-2$ and leaves remainder -8 when divided by $x-1$. Then $p(2)=8+4a+2b-6=0$, so $2a+b=-1$, and $p(1)=1+a+b-6=-8$, so $a+b=-3$. Subtracting gives $a=2$, then $b=-5$.

Variations you will meet:

- **"Divided by x"** means substitute $x = 0$: the remainder is the constant term. A question may give this first so that one coefficient is found straight away.
- **A derivative condition**, such as "$p'(0) = 5$" or "$p'(1) = 4$". Differentiate term by term ($x^3 \to 3x^2$, $x^2 \to 2x$, $x \to 1$, constant $\to 0$) and substitute. $p'(0)$ is simply the coefficient of x. "$x - 1$ is a factor of $p'(x)$" means $p'(1) = 0$; "the remainder when $p''(x)$ is divided by x" means $p''(0)$.
- **Stationary points at given x-values** mean $p'(x) = 0$ there, which gives equations too.
- **Three unknowns**: you need three facts; often one (such as $p(0)$ or $p'(0)$) gives a coefficient directly, leaving two simultaneous equations.
- **"Show that $a + 2b = -2$"**: form both equations, eliminate the third unknown, and show every step to the printed result.
- **Then use the result**: "Show that $x - 2$ is also a factor" (substitute and get 0), or "find the remainder when divided by $x - 2$" (substitute and state the number).

2. Factorise a cubic and solve $p(x) = 0$ (19 questions)

1. **Find one factor.** Use a factor you were given or found earlier ("hence"), or try values that divide the constant term until $p(a) = 0$. In a non-calculator question, show each trial you used, with its value.
2. **Find the quadratic factor** by observation, comparing coefficients or long division. Write the result as a product: $(x - a)(\ldots x^2 + \ldots x + \ldots)$.
3. **Factorise the quadratic** if you can; otherwise use the quadratic formula.
4. **Write the answer in the form asked**: "as a product of linear factors" means all three brackets, such as $(x - 2)(x + 1)(3x - 4)$, not just the quadratic. For "solve", set each factor to 0 and list every root.

Variations you will meet:

- **"Show that $x = \frac{2}{3}$ is a root"**: work out $p\left(\frac{2}{3}\right)$ with the substitution written out and end with "$= 0$". A factorised form found by inspection does not show it unless you multiply it out.
- **A factor $x + a$ with an unknown integer a**: the root $-a$ must divide the constant term, so test those values.
- **"With integer coefficients"** means clear fractions: $\left(x - \frac{4}{3}\right)$ becomes $(3x - 4)$, and the 3 is taken out of the quadratic factor.
- **Factors given already**: if two factors of a cubic are known, the third comes from the leading coefficient and the constant. For $3x^3 + \ldots + 8 = (x - 2)(x + 4)(\ldots)$, the third factor is $3x - 1$ because $x \times x \times 3x$ gives $3x^3$ and $(-2)(4)(-1) = 8$.

3. Where a curve meets a line or another curve (8 questions)

1. **Set the two equations equal** and move everything to one side, giving a cubic equal to 0.
2. **Find a root.** Often it is given: the point you were told about (for example, where a normal was drawn) is always one of the meeting points, so its x-coordinate is a root.
3. **Divide out that factor** to get the quadratic factor, and solve it by factorising or the formula.

4. **Give what is asked**: x-coordinates only, or full points (substitute into the simpler equation). "Exact" means surds, such as $x = 1 \pm \sqrt{3}$.

Variations you will meet:

- **"The normal cuts the curve again"**: first find the normal (differentiation), then use the point of contact as the known root.
- **"Show that the line meets the curve only once"**: the quadratic factor has a negative discriminant (type 4).
- **An unknown k in the line**: the given meeting point satisfies both equations, which gives k first.
- **"Show that B is the mid-point of AC"**: find all three points, then average the coordinates of A and C.

4. Show there is only one real root, or a repeated root (8 questions)

1. **Factorise** $p(x)$ as $(\text{linear})(\text{quadratic})$.
2. **Work out the discriminant** $b^2 - 4ac$ of the quadratic factor, with the numbers shown.
3. **Conclude** with a sentence: "$b^2 - 4ac = -8 < 0$, so the quadratic factor has no real roots, and $p(x) = 0$ has only one real root, $x = -\frac{1}{2}$."

For a **repeated root**, the quadratic factor is a perfect square (discriminant 0), or it shares a root with the linear factor. For example, $(2x+3)(x^2-4x+4) = (2x+3)(x-2)^2$ gives the repeated root $x = 2$. Don't mix up the two: a repeated **factor** gives a repeated **root**.

5. Find a remainder or a quotient (6 questions)

- **Just the remainder**: use the remainder theorem, $p(a)$ or $p\left(\frac{b}{a}\right)$. No division needed.
- **"Write $p(x)$ in the form $(2x-5)q(x)+r$"**: divide (long division or comparing coefficients) to get the quotient $q(x)$, and check the remainder with $r = p\left(\frac{5}{2}\right)$.
- **"Hence write $p(x) - r$ as a product of linear factors"**: $p(x) - r = (2x-5)q(x)$, so factorise $q(x)$. "Hence solve $p(x) = r$" then needs no more work: set each factor to 0.

6. A cubic equation in disguise (4 questions)

Some equations are the cubic you just factorised with x replaced by something else, such as e^{2y}, $\sin\theta$ or $\operatorname{cosec} 2\theta$.

1. **Spot the replacement**: in $4e^{3y} - 12e^{2y} + 5e^y + 6 = 0$, $e^{3y} = (e^y)^3$ and $e^{2y} = (e^y)^2$, so it is $4x^3 - 12x^2 + 5x + 6 = 0$ (Example 2 below) with $x = e^y$.
2. **Use the roots you already have** for x.
3. **Reject impossible values**: e^{2y} is always positive; $\sin\theta$ and $\cos\theta$ lie between -1 and 1; $\operatorname{cosec}\theta$ is never between -1 and 1. Say which you rejected.
4. **Solve each remaining equation** for the angle or variable, in the range given. Here $e^y = 2, -\frac{1}{2}$ or $\frac{3}{2}$; reject $-\frac{1}{2}$, so $y = \ln 2$ or $y = \ln\frac{3}{2}$.

7. Build a polynomial from its roots (1 question)

If $p(x) = 0$ has roots α, β and γ and $p(x)$ starts kx^3, then $p(x) = k(x - \alpha)(x - \beta)(x - \gamma)$. A fractional root gives a whole-number factor: roots $\frac{1}{2}$, -1 and 3 with $k = 2$ give $(2x - 1)(x + 1)(x - 3) = 2x^3 - 5x^2 - 4x + 3$. If a root is unknown, compare the constant terms (or another coefficient) to find it, and use any conditions given, such as "n is a negative integer", to choose between answers.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

The polynomial p is given by $p(x) = 2x^3 - 3x^2 + 4x + 3$.

(a) Find the quotient and the remainder when $p(x)$ is divided by $x - 2$. **[3]**

(b) Show that $2x + 1$ is a factor of $p(x)$. **[1]**

(c) Hence show that the equation $p(x) = 0$ has only one real root, and state it. **[4]**

(a) Long division: $2x^3 \div x = 2x^2$; $2x^2(x - 2) = 2x^3 - 4x^2$; subtract to leave $x^2 + 4x$. Then $x^2 \div x = x$; $x(x - 2) = x^2 - 2x$; subtract to leave $6x + 3$. Then $6x \div x = 6$; $6(x - 2) = 6x - 12$; subtract to leave 15. **M1** for a full division attempt.

Quotient $2x^2 + x + 6$, remainder 15. **A1 A1**

(Check: $p(2) = 16 - 12 + 8 + 3 = 15$.)

(b) $p\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{8}\right) - 3\left(\frac{1}{4}\right) + 4\left(-\frac{1}{2}\right) + 3 = -\frac{1}{4} - \frac{3}{4} - 2 + 3 = 0$, so $2x + 1$ is a factor. **B1**

(c) $p(x) = (2x + 1)(x^2 + Bx + 3)$. Comparing x^2 terms: $2B + 1 = -3$, so $B = -2$. **M1** for a quadratic factor with two terms right.

$p(x) = (2x + 1)(x^2 - 2x + 3)$. **A1**

For $x^2 - 2x + 3$: $b^2 - 4ac = 4 - 12 = -8 < 0$, so it has no real roots. **M1**

So the only real root of $p(x) = 0$ is $x = -\frac{1}{2}$. **A1**

Example 2

The polynomial p is given by $p(x) = 4x^3 + ax^2 + bx + 6$, where a and b are integers. It is given that $x - 2$ is a factor of $p(x)$, and that when $p(x)$ is divided by $2x - 1$ the remainder is 6.

(a) Find the values of a and b. **[4]**

(b) Using your values of a and b, solve the equation $p(x) = 0$. **[3]**

(c) Hence solve $4\sin^3\theta + a\sin^2\theta + b\sin\theta + 6 = 0$ for $0^\circ \leqslant \theta \leqslant 360^\circ$. **[3]**

(a) $p(2) = 32 + 4a + 2b + 6 = 0$, so $2a + b = -19$. **B1**

$p\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{a}{4} + \frac{b}{2} + 6 = 6$. Multiply by 4: $2 + a + 2b = 0$, so $a + 2b = -2$. **B1**

From the first, $b = -19 - 2a$. Then $a - 38 - 4a = -2$, so $-3a = 36$. **M1** for solving.

$a = -12$, $b = 5$. **A1**

(b) $p(x) = 4x^3 - 12x^2 + 5x + 6 = (x - 2)(4x^2 + Bx - 3)$. Comparing x^2 terms: $B - 8 = -12$, so $B = -4$. **M1**

$p(x) = (x - 2)(4x^2 - 4x - 3) = (x - 2)(2x + 1)(2x - 3)$. **A1**

$x = 2,\ -\frac{1}{2},\ \frac{3}{2}$. **A1**

(c) This is $p(x) = 0$ with $x = \sin\theta$, so $\sin\theta = 2, -\frac{1}{2}$ or $\frac{3}{2}$. $\sin\theta$ cannot be more than 1, so only $\sin\theta = -\frac{1}{2}$ is possible. **B1**

The basic angle is 30°, and $\sin\theta$ is negative in the third and fourth quadrants: $\theta = 180^\circ + 30^\circ = 210^\circ$ and $\theta = 360^\circ - 30^\circ = 330^\circ$. **B1 B1**

Example 3

DO NOT USE A CALCULATOR IN THIS QUESTION.

Find the exact coordinates of the points where the line $y = 2x - 2$ meets the curve $y = x^3 - 3x^2 + 4$. **[6]**

$x^3 - 3x^2 + 4 = 2x - 2 \Rightarrow x^3 - 3x^2 - 2x + 6 = 0$. **B1**

Try factors of 6: $x = 1$ gives $1 - 3 - 2 + 6 = 2$; $x = 3$ gives $27 - 27 - 6 + 6 = 0$. So $x - 3$ is a factor. **M1** for a trial shown.

$x^3 - 3x^2 - 2x + 6 = (x - 3)(x^2 - 2)$. **M1** for the quadratic factor (by grouping: $x^2(x - 3) - 2(x - 3)$). **A1**

$x^2 - 2 = 0$ gives $x = \pm\sqrt{2}$. **A1** for all three x-values: 3, $\sqrt{2}$, $-\sqrt{2}$.

Using $y = 2x - 2$: the points are $(3, 4)$, $(\sqrt{2},\ 2\sqrt{2} - 2)$ and $(-\sqrt{2},\ -2\sqrt{2} - 2)$. **A1**

Example 4

The polynomial p is given by $p(x) = x^3 + ax^2 + bx + c$, where a, b and c are constants.

- When $p(x)$ is divided by x the remainder is -12.
- $x - 3$ is a factor of $p(x)$.
- $p'(-2) = 0$.

(a) Find the values of a, b and c. **[5]**

(b) Show that the equation $p(x) = 0$ has a repeated root, and find it. **[3]**

(a) Dividing by x: $p(0) = c = -12$. **B1**

$p(3) = 27 + 9a + 3b - 12 = 0$, so $3a + b = -5$. **B1**

$p'(x) = 3x^2 + 2ax + b$, so $p'(-2) = 12 - 4a + b = 0$, giving $-4a + b = -12$. **M1** for differentiating and substituting. **A1**

Subtracting: $7a = 7$, so $a = 1$ and $b = -8$. **A1**

(b) $p(x) = x^3 + x^2 - 8x - 12 = (x - 3)(x^2 + 4x + 4)$. **M1 A1**

$x^2 + 4x + 4 = (x + 2)^2$, so $p(x) = (x - 3)(x + 2)^2$ and $x = -2$ is a repeated root (the other root is $x = 3$). **A1**

Common mistakes

- **Getting the sign of the substituted value wrong.** Dividing by $x + 2$ means $p(-2)$, not $p(2)$. Say "$x + 2 = 0$ when $x = -2$" to yourself first.
- **Setting a remainder equation equal to 0.** "Remainder 5 when divided by $x - 3$" means $p(3) = 5$. Only a factor gives 0.
- **Slips with negative numbers and fractions**, such as $(-2)^3 = -8$ or $2\left(-\frac{1}{2}\right)^3 = -\frac{1}{4}$. Examiners report these every year. Write the substitution out in full before simplifying.
- **Writing "$p(3) = 0$" without showing the working.** For "show that ... is a factor (or root)", show the substituted expression and write "$= 0$".
- **Stopping at the quadratic factor.** "As a product of linear factors" needs all three brackets written together, not just the quadratic, and not a list of roots.
- **Misusing synthetic division with $2x - 1$ or $2x + 1$.** It gives a quotient twice as big: for $4x^3 - 7x + 3$ it leads to $(2x - 1)(4x^2 + 2x - 6)$, which is twice the right answer $(2x - 1)(2x^2 + x - 3)$. Use long division or compare coefficients, and multiply back to check.
- **Using $x - \frac{1}{2}$ instead of $2x - 1$ as a factor.** Factors should have whole-number coefficients.
- **Guessing factors with a calculator in a non-calculator question.** You must show how you found the first root (a trial with its value) and how you solved the quadratic factor.

- **An incomplete "only one real root" argument.** A negative discriminant alone is not the answer: say that the quadratic factor has no real roots, so $p(x) = 0$ has only the one root, and name it.
- **Confusing repeated factors with repeated roots.** $(3x-1)^2$ is a repeated factor, which gives one repeated root, $x = \frac{1}{3}$.
- **Not rejecting impossible values** in disguised equations, such as $\sin\theta = \frac{3}{2}$ or $e^{2y} = -1$.

How to get the marks

- **Each condition is worth a mark.** Write each equation clearly, such as $p(2) = 32 + 4a + 2b + 6 = 0$, even before simplifying. Method marks are given for substituting correctly even with one slip.
- **Solving the simultaneous equations** earns a method mark if you get at least one unknown from your own equations, so keep going after an error.
- **"Show that"** gives the answer. Show every step, and your last line must match the printed result (for example "$= 0$" or "$7a - 3b = 39$").
- **"Hence"** means use the previous part: the factor you just found, or the roots you just found. Starting again another way may earn nothing.
- **Quadratic factor marks**: a quadratic factor with two of its three terms right earns the method mark, so write it down even if unsure.
- **"Only one real root"** needs the correct quadratic factor, a discriminant worked out with its value, and a conclusion that mentions both the quadratic and the cubic.
- **"Exact"** means surds or fractions, such as $x = \pm\sqrt{2}$ or $y = \ln\frac{3}{2}$, not decimals.
- **"Write down"** (for example the remainder when divided by x) needs little or no working: here it is the constant term.
- **"Integers"** is a hint: a fractional answer for an integer coefficient means an error somewhere.

Quick check

1. Find the remainder when $x^3 + 4x^2 - 3x + 5$ is divided by $2x + 1$.
2. Given that $x - 4$ is a factor of $2x^3 - 9x^2 + kx + 12$, find k and write the cubic as a product of linear factors.
3. Without a calculator, solve $x^3 - 7x - 6 = 0$.
4. Show that $x^3 + 2x^2 + 3x + 6 = 0$ has only one real root.
5. A cubic polynomial with leading coefficient 3 has roots $-\frac{1}{3}$, 2 and 4. Write it in the form $ax^3 + bx^2 + cx + d$.

Answers

1. $2x+1=0$ when $x=-\frac{1}{2}$. $p\left(-\frac{1}{2}\right)=-\frac{1}{8}+1+\frac{3}{2}+5=\frac{59}{8}$.
2. $p(4)=128-144+4k+12=4k-4=0$, so $k=1$. Then $2x^3-9x^2+x+12=(x-4)(2x^2-x-3)=(x-4)(2x-3)(x+1)$.
3. $p(1)=-12$, $p(-1)=-1+7-6=0$, so $x+1$ is a factor. $x^3-7x-6=(x+1)(x^2-x-6)=(x+1)(x-3)(x+2)$, so $x=-2,-1,3$.
4. By grouping, $x^2(x+2)+3(x+2)=(x+2)(x^2+3)$. $x^2+3=0$ has no real roots (discriminant $0-12=-12<0$, or $x^2\geqslant 0$), so the only real root is $x=-2$.
5. $3\left(x+\frac{1}{3}\right)(x-2)(x-4)=(3x+1)(x^2-6x+8)=3x^3-17x^2+18x+8$.

Past questions to try

- **0606/13/O/N/24 Q5**: three unknowns from a factor, a remainder and a remainder of $p'(x)$.
- **0606/23/M/J/21 Q6**: show a root, then show the equation has a repeated root.
- **0606/22/M/J/23 Q3**: a factor, then where a line meets a curve, then a mid-point.
- **0606/12/F/M/24 Q5**: a factor of both $p(x)$ and $p'(x)$, then a cubic in $\operatorname{cosec} 2\theta$.