Cambridge IGCSE · Additional Mathematics 0606 · Notes

Functions

Read online at **pastopic.com/cambridge/additional-mathematics-0606/notes/functions**

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What you need to know

By the end of this topic you should be able to:

1. **Use the words** function, domain, range (also called the image set), one-one function, many-one function, inverse function and composite function, and explain in words why a given rule is (or is not) a function.
2. **Find the domain and range** of a function, including inverse functions and composite functions. Restrict a domain so that f^{-1} or gf exists, knowing that the domain of gf sits inside the domain of f and the range of gf sits inside the range of g.
3. **Read and write function notation**: $f(x) = 2e^x$, $f : x \mapsto \lg x$ for $x > 0$, $f^{-1}(x)$, $fg(x)$ meaning $f(g(x))$, and $f^2(x)$ meaning $f(f(x))$. (f^2 is never used for a trigonometric function, where $\sin^2 x$ means something else.)
4. **Draw** $y = |f(x)|$ **from** $y = f(x)$, where f is linear, quadratic, cubic, or a trigonometric function of the form $a \sin bx + c$, $a \cos bx + c$ or $a \tan bx + c$.
5. **Explain in words why a function has no inverse.**
6. **Find the inverse of a one-one function**, written with the correct notation, for example $f(x) = e^{2x}$ gives $f^{-1}(x) = \frac{1}{2} \ln x$.
7. **Form and use composite functions**, knowing that the order matters: fg is usually not the same as gf.
8. **Sketch a function and its inverse** on the same axes, as reflections of each other in the line $y = x$.

Understand it

What a function is

A **function** is a rule that takes each input and gives **exactly one** output. The set of inputs you are allowed to use is the **domain**; the set of outputs you actually get is the **range**. A function is not complete without its domain: $f(x) = x^2$ for $x \in \mathbb{R}$ and $f(x) = x^2$ for $x \geqslant 0$ are different functions.

To check a graph, imagine vertical lines. If **every vertical line** through the domain meets the graph **exactly once**, the graph is a function. A circle fails: the line $x = 0$ meets it twice, so one input would have two outputs. That sentence is the kind of explanation examiners want.

Write the domain in terms of x and the range in terms of the function: domain $x \geqslant 0$, range $f(x) \geqslant 3$ (or $f \geqslant 3$), never "range $x \geqslant 3$".

One-one and many-one

- A **one-one** function gives different outputs for different inputs. On a graph, every **horizontal** line meets it at most once.
- A **many-one** function has at least two inputs with the same output. x^2 for $x \in \mathbb{R}$ is many-one, because $f(2) = f(-2) = 4$.

Only one-one functions have inverses, because the inverse has to send each output back to a single input. x^2 for $x \in \mathbb{R}$ would have to send 4 back to both 2 and -2, which is not a function. So to explain why there is no inverse, say "f is many-one", and back it up with two inputs that give the same output.

Finding the range

Sketch the graph over the given domain and read off the lowest and highest y-values. The shapes you need most:

- **Quadratic**: complete the square. $(x-3)^2+2$ has its lowest point at $(3,2)$, so over all real x the range is $f(x) \geqslant 2$. If the domain stops short of the vertex, the range starts at the end point instead.
- **Exponential**: $e^x > 0$ for every x, so $e^x + 2 > 2$ and $5 - 3e^x < 5$. e^{-x} is positive too, and $e^{-x} \leqslant 1$ once $x \geqslant 0$.
- **Logarithm**: $\ln(\ldots)$ only works when the bracket is positive. Over its largest domain, $\ln(ax+b)$ takes every real value, so its range is $f(x) \in \mathbb{R}$.
- **Reciprocal**: $\dfrac{1}{x-1}$ can never equal 0, and has a vertical asymptote at $x = 1$.

The **largest possible domain** of a formula is every x where it makes sense: no division by 0, no square root of a negative number, no log of zero or a negative number. For example, $\ln(4x-3)$ needs $4x - 3 > 0$, so $x > \frac{3}{4}$.

Inverse functions

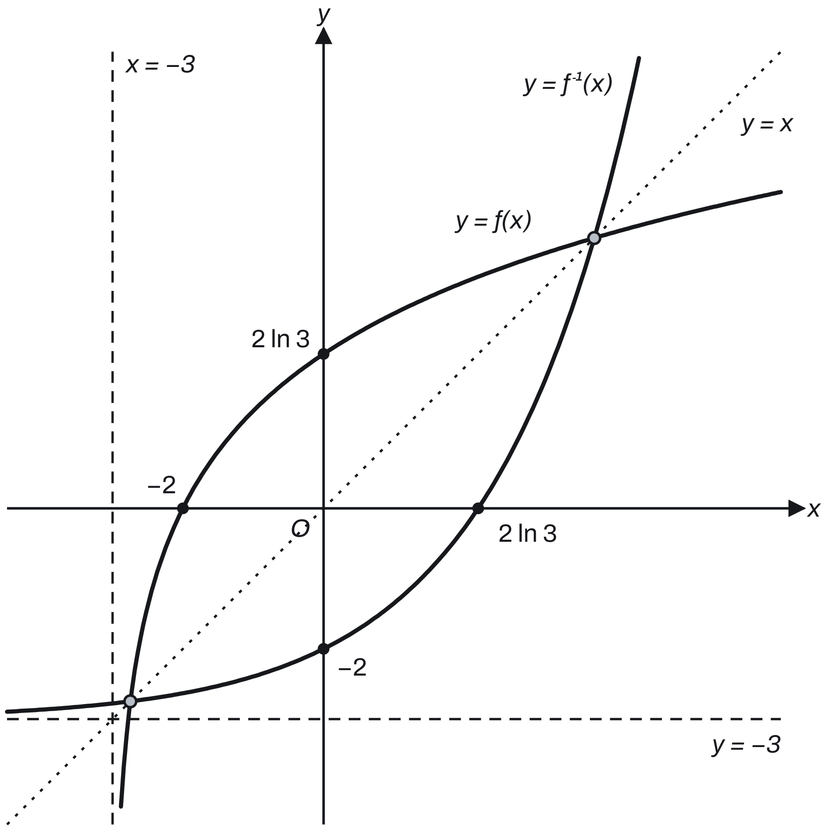
The **inverse** f^{-1} undoes f: if $f(2) = 7$ then $f^{-1}(7) = 2$. Its inputs are f's outputs, so the two swap over:

$$\text{domain of } f^{-1} = \text{range of } f, \qquad \text{range of } f^{-1} = \text{domain of } f.$$

To find the rule, write $y = f(x)$, make x the subject, then swap the letters x and y. With $f(x) = 2\ln(x+3)$:

$$y = 2\ln(x+3) \;\Rightarrow\; \frac{y}{2} = \ln(x+3) \;\Rightarrow\; x = e^{y/2} - 3, \quad \text{so } f^{-1}(x) = e^{x/2} - 3.$$

Swapping x and y swaps every point (a,b) to (b,a), and that is a **reflection in the line** $y = x$. So the graph of f^{-1} is the graph of f reflected in $y = x$: intercepts swap axes, and a vertical asymptote $x = -3$ becomes a horizontal asymptote $y = -3$.



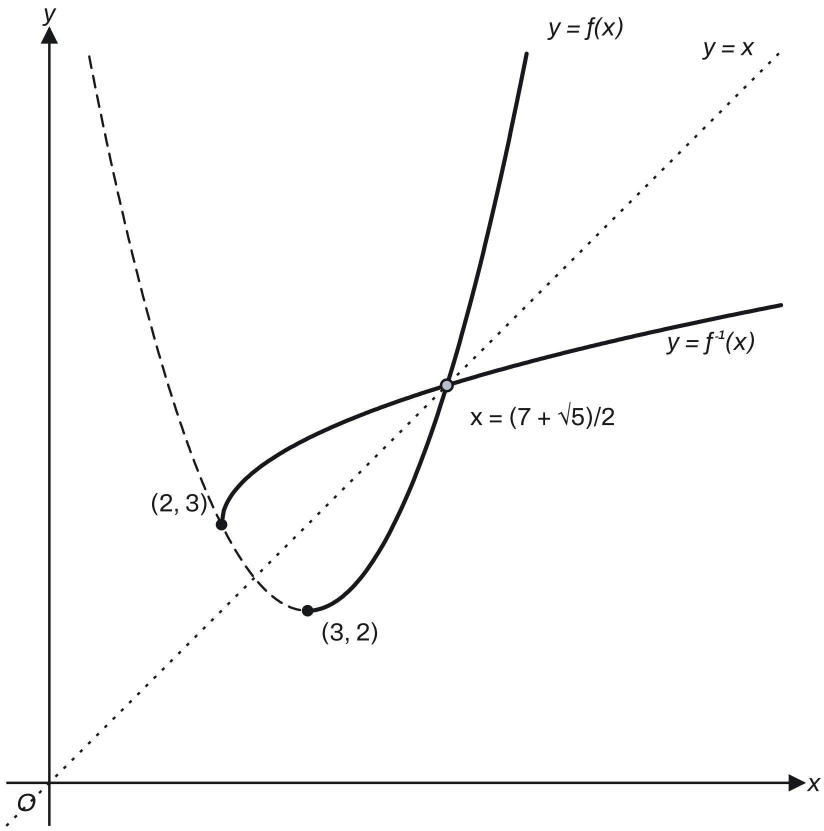
Where f and f^{-1} meet, they meet on $y = x$ (for an increasing function). So to solve $f(x) = f^{-1}(x)$ you can solve the easier equation $f(x) = x$, then keep only the roots inside the domain.

Because the graphs are reflections, a tangent to f with gradient m at (a, b) reflects to a tangent to f^{-1} with gradient $\frac{1}{m}$ at (b, a).

A function that is its own inverse is **self-inverse**: $ff(x) = x$, and its graph is symmetrical about $y = x$. Examples are $f(x) = \frac{1}{x}$ and $f(x) = 6 - x$.

Restricting the domain

A many-one function can be made one-one by cutting its domain down. For a quadratic, keep only one side of the vertex. $f(x) = x^2 - 6x + 11 = (x - 3)^2 + 2$ is many-one over $\mathbb{R}$, but for $x \geqslant 3$ it is one-one, so it has an inverse. The **least** value of k in "$x \geqslant k$" is the x-coordinate of the vertex, $k = 3$.



When you make x the subject of a quadratic you get a $\pm$ square root. Choose the sign from the **range of the inverse** (the domain of f): here $f^{-1}(x) \geqslant 3$, so $f^{-1}(x) = 3 + \sqrt{x-2}$, with the plus sign. If the domain were $x \leqslant 3$ you would need $3 - \sqrt{x-2}$.

Composite functions

$fg(x)$ means "do g first, then f": $fg(x) = f(g(x))$. The function nearest to x acts first. With $f(x) = 2x + 3$ and $g(x) = x^2 - 1$:

$$fg(x) = 2(x^2 - 1) + 3 = 2x^2 + 1, \qquad gf(x) = (2x+3)^2 - 1 = 4x^2 + 12x + 8.$$

These are different, so the order matters. $f^2(x)$ means $ff(x)$, and $f^{-1}f(x) = x$.

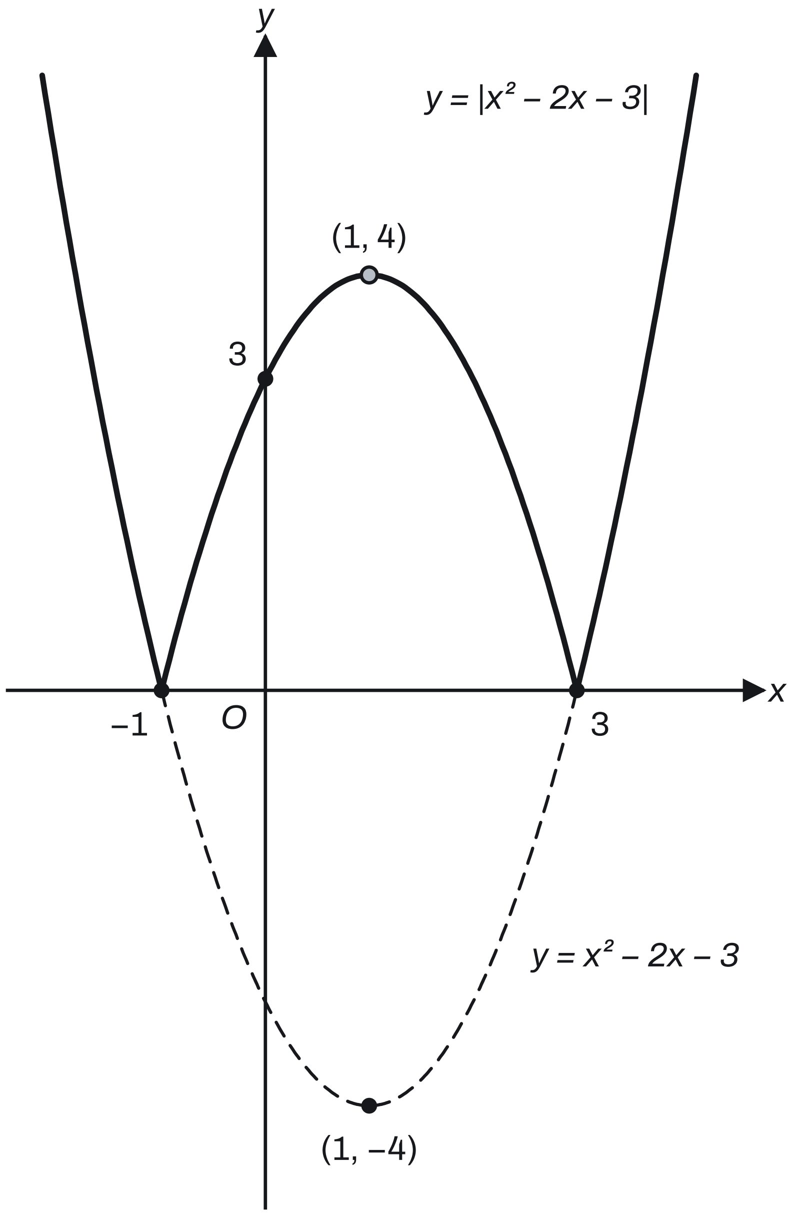
gf only **exists** if every output of f is an allowed input of g: **the range of f must lie inside the domain of g**. For example, if $g(x) = \ln(x-4)$ for $x > 4$, then f's outputs must all be greater than 4. The domain of gf is the domain of f (or part of it), and the range of gf is part of the range of g. To find the range of gf, feed the range of f into g.

The modulus graph $y = |f(x)|$

$|a|$ is the size of a without its sign: $|5| = 5$ and $|-5| = 5$. So $|f(x)|$ is never negative. To draw $y = |f(x)|$:

1. draw $y = f(x)$;
2. keep every part on or above the x-axis;
3. reflect every part below the x-axis in the x-axis.

The graph then touches the x-axis at the roots of $f(x) = 0$, where it usually has a sharp corner, and the y-intercept is $|f(0)|$.



For a straight line such as $y = |3x - 2|$ you get a V: it touches the x-axis at $x = \frac{2}{3}$, crosses the y-axis at 2, and both arms are straight (use a ruler). For a trigonometric graph such as $y = |3 \sin x - 1|$, every dip below the axis is flipped up, so the graph runs between 0 and the largest value of $|3 \sin x - 1|$, which is 4.

Key facts and formulas

None of these is in the List of formulas on page 2 of the exam paper, so you must learn them all.

Formula	Status
$fg(x) = f(g(x))$: do g first; $f^2(x) = f(f(x))$	Learn it

Formula	Status
gf exists only if the range of f lies inside the domain of g	Learn it
Domain of gf $\subseteq$ domain of f; range of gf $\subseteq$ range of g	Learn it
f^{-1} exists only if f is one-one	Learn it
Domain of f^{-1} = range of f; range of f^{-1} = domain of f	Learn it
$ff^{-1}(x) = f^{-1}f(x) = x$	Learn it
The graph of $y = f^{-1}(x)$ is the reflection of $y = f(x)$ in $y = x$	Learn it
Self-inverse: $ff(x) = x$, i.e. $f^{-1} = f$	Learn it
$\lvert a \rvert = a$ if $a \geqslant 0$ and $\lvert a \rvert = -a$ if $a < 0$	Learn it
$y = e^x$ and $y = \ln x$ are inverses: $e^{\ln x} = x$ for $x > 0, \ln(e^x) = x$	Learn it
$e^x > 0$ for all x; $\ln(ax+b)$ needs $ax + b > 0$	Learn it
$a(x-p)^2 + q$ has vertex (p, q)	Learn it

How to do it

In the Paper 1 and Paper 2 questions we have tagged for this topic (36 questions, 2021–2025), the types came up this often. Almost every question is several parts on one pair of functions, so 33 of the 36 mix types and the counts add up to far more than 36.

Question type	Questions
Domain and range	29
Composite functions	25
Finding an inverse function	24
Graphs of a function and its inverse	15
Explaining in words (function, one-one, inverse exists, self-inverse)	5
The graph of $y = \lvert f(x) \rvert$	1

1. Domain and range (29 questions)

1. **Sketch the graph** over the given domain, even roughly in the margin.
2. **Find the end values**: substitute the ends of the domain, and find any vertex (complete the square) or asymptote inside it.
3. **Write the range** in terms of f, with the right inequality: $\geqslant$ where the value is reached, $>$ where it is only approached.

Variations you will meet:

- **"Write down the least possible value of a"** for $\ln(5x-2)$, $x > a$: solve $5x - 2 = 0$, so $a = \frac{2}{5}$. Give $a = \frac{2}{5}$, not "$a > \frac{2}{5}$".
- **"The least value of k such that f^{-1} exists"** for a quadratic on $x \geqslant k$: k is the x-coordinate of the vertex. For $x \leqslant p$, the **greatest** p is the vertex's x-coordinate.
- **"Does f^{-1} exist?"** for a quadratic on a given interval: if the vertex is inside the domain, the function is many-one, so no.
- **Domain and range of f^{-1}**: swap those of f. Examiners often see the range of f^{-1} left out.
- **Domain of a composite**, "the least value of a so that gf exists": the range of f must sit inside the domain of g. For $f(x) = 4e^x + a$ with $x \geqslant 0$, the range is $f(x) \geqslant 4 + a$; if g needs $x \geqslant 6$, then $4 + a \geqslant 6$, so the least a is 2.
- **Largest domain** of a formula such as $\dfrac{1}{2x-5}$: every x except $\frac{5}{2}$.
- **Range of gf**: work out the range of f first, then put its end values into g. For $f(x) = \sin x$ with $45^\circ \leqslant x \leqslant 135^\circ$, the range of f is $\frac{\sqrt{2}}{2} \leqslant f(x) \leqslant 1$.

2. Composite functions (25 questions)

1. **Form the composite** by putting the inner function in place of x in the outer one: for $fg(x)$, write $f(\ )$ and put $g(x)$ in the bracket. Simplify.
2. **To solve** $fg(x) = k$, set your expression equal to k and solve. Or work from the outside in: solve $f(u) = k$ to get u, then solve $g(x) = u$.
3. **Check every answer** against the domain and reject any that fall outside it.

For example, with $f(x) = \ln(x+1)$ for $x > -1$ and $g(x) = x^2 + 3$: $fg(x) = \ln(x^2+4) = 3$ gives $x^2 + 4 = e^3$, so $x = \pm\sqrt{e^3 - 4}$.

Variations you will meet:

- **A number**, such as $fg(0)$: find $g(0)$, then put that into f.
- **"Explain why fg does not exist"** (or why f^2 can be formed): compare the range of g with the domain of f in one sentence, for example "the range of g is $g(x) > 0$, which is not in the domain of f, $x < 0$".
- $gg(x)$ **or** $f^2(x)$: put the function into itself. Solving $gg(x) = k$ is often easier from the outside in: solve $g(u) = k$, then $g(x) = u$.
- **Composites with an inverse**, such as $f^{-1}g(x)$: find f^{-1} first, then compose.
- **Given f and fg, find g**: $g(x) = f^{-1}\big(fg(x)\big)$. Choose the sign of any square root so that g's outputs lie in the domain of f.
- **Match expressions to notation** such as f^2, fg, g^{-1}: work out each composite and compare.

3. Finding an inverse function (24 questions)

1. Write $y = f(x)$.
2. **Make x the subject**, undoing the steps of f in reverse order. Use $\ln$ to undo e, e to undo $\ln$, and completing the square (then $\pm\sqrt{\ }$) for a quadratic.

3. **Swap x and y** (before or after rearranging) and write $f^{-1}(x) = \ldots$
4. **State the domain and range** of f^{-1} if asked: the range and domain of f.

For example, $f(x) = 4 + e^{-2x}$: $e^{-2x} = y - 4$, so $-2x = \ln(y-4)$ and $f^{-1}(x) = -\frac{1}{2}\ln(x-4)$, for $x > 4$.

Variations you will meet:

- **A quadratic**: complete the square first. $f(x) = 7 - (x-1)^2$ for $x \geqslant 1$ gives $(x-1)^2 = 7 - y$, $x = 1 \pm \sqrt{7-y}$; the range of f^{-1} is $\geqslant 1$, so take $+$: $f^{-1}(x) = 1 + \sqrt{7-x}$ for $x \leqslant 7$.
- **"Show that $f^{-1}(x) = \ldots$"** where the rearrangement is a quadratic in y, such as $2y^2 - 3xy - x = 0$: use the quadratic formula with x as a constant, then **justify** the sign ("the range of f^{-1} is the domain of f, $x < 0$, so take the negative square root").
- **A fraction**, such as $y = \dfrac{2x+1}{x-2}$: multiply up, collect the x terms on one side and factorise: $y(x-2) = 2x + 1 \Rightarrow x(y-2) = 2y + 1 \Rightarrow x = \dfrac{2y+1}{y-2}$. (This one is self-inverse.)
- **Solve $f^{-1}(x) = k$**: this is the same as $x = f(k)$, which needs no inverse.
- **Solve $f(x) = f^{-1}(x)$**: for an increasing f, solve $f(x) = x$ and keep the roots in the domain.

4. Graphs of a function and its inverse (15 questions)

1. **Sketch** $y = f(x)$ over its domain only, with the right shape: where it crosses each axis and any asymptote.
2. **Reflect it in** $y = x$ to get $y = f^{-1}(x)$; don't draw it from its formula. Draw $y = x$ lightly to help.
3. **Label** both curves and state all intercepts (the intercepts of f^{-1} are those of f with the coordinates swapped) and **both asymptotes** as equations.

Variations you will meet:

- **Exponential and log pairs**: $y = ae^{kx} + b$ has asymptote $y = b$; its inverse has asymptote $x = b$. $y = \ln(ax + b)$ has asymptote $x = -\frac{b}{a}$; its inverse has asymptote $y = -\frac{b}{a}$.
- **"The equation $f(x) = f^{-1}(x)$ has two roots"**: make your curves cross twice, on $y = x$.
- **Half a parabola**: draw only the side of the vertex in the domain, with the end point, and its reflection.
- **Given a graph**, sketch its inverse on the same diagram: reflect key points (intercepts, ends, points on $y = x$ stay put).
- **Gradient of the inverse**: at the reflected point (b, a) the tangent to f^{-1} has gradient $\frac{1}{m}$, and the two tangents meet on $y = x$.

5. Explaining in words (5 questions)

Give a reason, not just "yes" or "no":

- **Why a graph is (not) a function**: "a vertical line meets the graph more than once, so one input has two outputs" (or "every vertical line meets it once").
- **One-one or many-one**: use a horizontal line; a graph symmetrical about $y = x$ is **its own inverse**.

- **Why f^{-1} exists**: "f is one-one". Why it doesn't: "f is many-one", for example "$f(1) = f(-1)$", or "the domain includes the turning point".
- **Self-inverse** from $ff(x)$: if $ff(x)$ simplifies to x, then $f^{-1} = f$, and the graph is symmetrical about $y = x$.
- **Whether g^2 exists**: is the range of g inside the domain of g?

6. The graph of $y = |f(x)|$ (1 question)

1. Find $f(x)$ if it is given in another form (for example $f^2(x)$ from $f(x)$).
2. Sketch $y = f(x)$, then reflect the parts below the x-axis in the x-axis.
3. State the x-intercepts (roots of $f(x) = 0$) and the y-intercept ($|f(0)|$).

Variation: a line $y = ax + b$ lies on or above $y = |f(x)|$ only between two given x-values. Then the line meets the V exactly at those two x-values: work out the height of $y = |f(x)|$ at each, and find the line through the two points. For example, if $y = ax + b$ is on or above $y = |3x - 2|$ exactly when $0 \leqslant x \leqslant 2$, the line passes through $(0, 2)$ and $(2, 4)$, so $a = 1$ and $b = 2$.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

The functions f and g are defined for all real x by $f(x) = 2x + 3$ and $g(x) = x^2 - 1$.

(a) Find expressions for $fg(x)$ and $gf(x)$. **[3]**

(b) Solve the equation $fg(x) = 19$. **[2]**

(c) Find an expression for $f^{-1}(x)$. **[2]**

(d) Explain why g^{-1} does not exist. **[1]**

(a) $fg(x) = f(x^2 - 1) = 2(x^2 - 1) + 3 = 2x^2 + 1$. **B1**

$gf(x) = g(2x + 3) = (2x + 3)^2 - 1$ **M1** for the correct order, $= 4x^2 + 12x + 8$. **A1**

(b) $2x^2 + 1 = 19 \Rightarrow x^2 = 9$ **M1**, so $x = 3$ or $x = -3$. **A1** (both needed)

(c) $y = 2x + 3 \Rightarrow x = \dfrac{y-3}{2}$, then swap the letters: **M1**

$$f^{-1}(x) = \frac{x-3}{2}.$$

A1

(d) g is many-one: for example $g(2) = g(-2) = 3$, so an inverse could not send 3 back to a single value. **B1**

Example 2

The function f is defined by $f(x) = x^2 - 6x + 11$ for $x \geqslant k$, where k is a constant.

(a) Write $x^2 - 6x + 11$ in the form $(x-a)^2 + b$. **[2]**

(b) Write down the least value of k for which f^{-1} exists. **[1]**

For the rest of the question, use this value of k.

(c) Write down the range of f. **[1]**

(d) Find $f^{-1}(x)$, stating its domain and range. **[4]**

(e) Solve the equation $f(x) = f^{-1}(x)$, giving your answer in exact form. **[3]**

(a) $x^2 - 6x + 11 = (x-3)^2 - 9 + 11 = (x-3)^2 + 2$. **B1 B1** $(a = 3, b = 2)$

(b) The vertex is at $x = 3$, so $k = 3$. **B1**

(c) $f(x) \geqslant 2$. **B1**

(d) $y = (x-3)^2 + 2 \Rightarrow (x-3)^2 = y - 2 \Rightarrow x = 3 \pm \sqrt{y-2}$, then swap the letters. **M1**

The range of f^{-1} is the domain of f, $x \geqslant 3$, so take the $+$ sign:

$$f^{-1}(x) = 3 + \sqrt{x-2}.$$

A1

Domain: $x \geqslant 2$. Range: $f^{-1}(x) \geqslant 3$. **B1 B1**

(e) The graphs meet on the line $y = x$ (the second diagram), so solve $f(x) = x$: **M1**

$$x^2 - 6x + 11 = x \;\Rightarrow\; x^2 - 7x + 11 = 0 \;\Rightarrow\; x = \frac{7 \pm \sqrt{5}}{2}.$$

A1

$\frac{7-\sqrt{5}}{2} \approx 2.38$ is less than 3, outside the domain, so reject it: $x = \dfrac{7+\sqrt{5}}{2}$. **A1**

Example 3

The function f is defined by $f(x) = 2\ln(x+3)$ for $x > a$, where a is as small as possible.

(a) Write down the value of a and the range of f. **[2]**

(b) Find $f^{-1}(x)$, stating its domain and range. **[4]**

(c) On one set of axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$. State the exact intercepts with the axes and the equations of the asymptotes. **[4]**

(a) $\ln$ needs $x + 3 > 0$, so $a = -3$. **B1** The range is $f(x) \in \mathbb{R}$ (every real number). **B1**

(b) $y = 2\ln(x+3) \Rightarrow \dfrac{y}{2} = \ln(x+3) \Rightarrow x + 3 = e^{y/2}$ **M1**, so

$$f^{-1}(x) = e^{x/2} - 3.$$

A1

Domain: $x \in \mathbb{R}$. Range: $f^{-1}(x) > -3$. **B1 B1**

(c) The sketch is the first diagram in "Understand it". **B1** for $y = f(x)$: a log shape rising to the right from the asymptote $x = -3$. **B1** for its intercepts: $x = -2$ (where $x + 3 = 1$) and $y = 2\ln 3$. **B1** for $y = f^{-1}(x)$ as the reflection in $y = x$, meeting $y = f(x)$ twice on that line. **B1** for its intercepts $y = -2$ and $x = 2\ln 3$, and the asymptote $y = -3$.

Example 4

The functions f and g are defined by

$f(x) = 3e^x + a$ for $x \geqslant 0$, where a is an integer,

$g(x) = \ln(x-4)$ for $x > 4$.

(a) Find the least value of a for which gf exists. **[2]**

(b) Using $a = 2$, solve the equation $gf(x) = \ln 7$, giving your answer in exact form. **[3]**

(c) Using $a = 2$, state the range of gf. **[1]**

(a) $e^x \geqslant 1$ for $x \geqslant 0$, so the range of f is $f(x) \geqslant 3 + a$. **B1**

This must lie inside the domain of g, $x > 4$: $3 + a > 4$, so $a > 1$, and the least integer is $a = 2$. **B1**

(b) $gf(x) = \ln(3e^x + 2 - 4) = \ln(3e^x - 2)$. **B1** for the correct order.

$\ln(3e^x - 2) = \ln 7 \Rightarrow 3e^x - 2 = 7 \Rightarrow e^x = 3$ **M1**, so $x = \ln 3$. **A1**

(c) The range of f is $f(x) \geqslant 5$, and $g(5) = \ln 1 = 0$, with g increasing, so $gf(x) \geqslant 0$. **B1**

Common mistakes

- **Wrong order in a composite.** $fg(x)$ means g first. Examiner reports keep mentioning gf worked out when fg was asked, and "$fg(x) = f(x) \times g(x)$". Write $f(g(x))$ as a first line.
- **Wrong notation.** The domain must be in terms of x and the range in terms of f (or f^{-1}, gf). "$f^{-1}(x) = \ldots$" is needed for the inverse; "$y = \ldots$" loses the mark in several mark schemes.
- **Leaving out the range of f^{-1}**, or getting domain and range the wrong way round. Swap them from f.
- **Writing an inequality where a value is asked.** "The least possible value of a" is $a = 2$, not $a \geqslant 2$.
- **Keeping $\pm$ in the inverse of a quadratic**, or picking the wrong sign. The range of f^{-1} (the domain of f) decides the sign.
- **Not rejecting roots.** After solving $f(x) = x$ or $fg(x) = k$, check each answer against the domain; a root outside it must be crossed out, with a reason.
- **Drawing f^{-1} from its formula, or reflecting in the wrong line.** Reflect $y = f(x)$ in $y = x$, not in an axis. Sketch only over the domain and show asymptotes as dashed lines with their equations.
- **Confusing $f^{-1}(x)$ with $\frac{1}{f(x)}$.** They are completely different.
- **Log slips when finding an inverse**, such as $e^{-y} = x - 5$ becoming $-y = -\ln(x-5)$. Take $\ln$ of the whole side: $-y = \ln(x-5)$.
- **Freehand modulus graphs of straight lines.** $y = |3x - 2|$ is two straight arms: use a ruler and make the corner sharp on the x-axis.

How to get the marks

- **"Write down"** or **"state"** means no working is needed, but the notation must be right: "range $f(x) \geqslant 2$", "domain $x \geqslant 2$".
- **Inverse functions** earn an **M1** for a complete method (make x the subject and swap the letters at some point) even with one slip, then **A1** for the correct $f^{-1}(x) = \ldots$ Brackets must be right: $\ln(x-2)$, not $\ln x - 2$.
- **Composite equations**: the first method mark is for the correct order, so show the composite, for example $\ln\big(3(x^2+2) - 1\big) = 5$, before solving.
- **"Exact"** means keep e, $\ln$ and surds: $x = \pm\sqrt{\frac{e^5 - 5}{3}}$, not ± 6.93. Otherwise give 3 significant figures.
- **"Explain"** needs one clear sentence with the reason: "range of g is not a subset of the domain of g", "f is one-one", "a vertical line meets the graph twice".
- **"Hence"** means use the part before: the completed square gives the vertex, which gives the least k and the range.
- **Sketches** are marked point by point: the right shape and quadrants, the reflection in $y = x$, every intercept marked or stated, and each asymptote given as an equation. Label the curves $y = f(x)$ and $y = f^{-1}(x)$.
- **"Show that $f^{-1}(x) = \ldots$"**: the answer is given, so show every step and justify the choice of sign; the last mark is often for that sentence.

Quick check

1. $f(x) = 5 - 2x$ and $g(x) = x^2 + 3$ for all real x. Find $fg(x)$, and solve $gf(x) = 19$.
2. $h(x) = \dfrac{x+1}{x-1}$ for $x \neq 1$. Find $hh(x)$ and say what it tells you about h^{-1}.
3. $f(x) = e^{3x} + 1$ for $x \in \mathbb{R}$. State the range of f, and find $f^{-1}(x)$ with its domain.
4. $p(x) = x^2 + 4x$ for $x \geqslant -2$. Find the range of p and an expression for $p^{-1}(x)$.
5. Sketch $y = |2x - 5|$, stating its intercepts, and solve $|2x - 5| = 3$.

Answers

1. $fg(x) = 5 - 2(x^2 + 3) = -2x^2 - 1$. $gf(x) = (5 - 2x)^2 + 3 = 19$, so $(5 - 2x)^2 = 16$ and $5 - 2x = \pm 4$: $x = \frac{1}{2}$ or $x = \frac{9}{2}$.
2. $hh(x) = \dfrac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} = \dfrac{2x}{2} = x$. So h is self-inverse: $h^{-1}(x) = \dfrac{x+1}{x-1}$, and its graph is symmetrical about $y = x$.
3. $e^{3x} > 0$, so $f(x) > 1$. $y - 1 = e^{3x} \Rightarrow x = \frac{1}{3}\ln(y-1)$, so $f^{-1}(x) = \frac{1}{3}\ln(x-1)$, domain $x > 1$.
4. $p(x) = (x+2)^2 - 4$, so $p(x) \geqslant -4$. $(x+2)^2 = y + 4 \Rightarrow x = -2 \pm \sqrt{y+4}$; the range of p^{-1} is $\geqslant -2$, so $p^{-1}(x) = -2 + \sqrt{x+4}$, for $x \geqslant -4$.
5. A V shape with its corner on the x-axis at $(2.5, 0)$ and y-intercept 5. $2x - 5 = 3$ gives $x = 4$; $2x - 5 = -3$ gives $x = 1$.

Past questions to try

- **0606/12/M/J/25 Q9**: least value for a log domain, the inverse, and both sketches.
- **0606/22/F/M/25 Q7**: the least a so that gf exists, then a composite equation.
- **0606/23/O/N/24 Q2**: completing the square, restricting the domain, and the inverse with its domain and range.
- **0606/21/O/N/24 Q11**: explaining why a graph is a function, and a self-inverse function.