

Integration

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Read first: [Differentiation](#), [Applications of differentiation](#)

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What you need to know

By the end of this topic you should be able to:

1. **Treat integration as differentiation backwards.** Integrating $f(x)$ means finding a function whose derivative is $f(x)$. Every indefinite integral needs an arbitrary constant, $+c$.
2. **Integrate sums of powers of x** , including $\frac{1}{x}$ (which gives $\ln x$) and $\frac{1}{ax+b}$.
3. **Integrate these standard forms**, with angles in radians: $(ax+b)^n$ for any rational n (including $n = -1$), $\sin(ax+b)$, $\cos(ax+b)$, $\sec^2(ax+b)$ and e^{ax+b} .
4. **Evaluate definite integrals and use them to find plane areas:** the area between a curve and the x -axis, between a line and a curve, between two curves, and areas made of two parts added together.

Motion problems (displacement, velocity and acceleration) use integration too; they are in the Kinematics note. Integration by parts and by substitution are **not** in the syllabus: when a question needs more than the standard forms, it gives you a derivative to use ("hence") or an identity or a split to rewrite with first.

Understand it

Integration undoes differentiation

Differentiating $x^3 + 5$ gives $3x^2$. So does differentiating $x^3 - 2$, or x^3 plus any constant, because constants differentiate to 0. Going backwards from $3x^2$, you can't tell which constant was there, so you write

$$\int 3x^2 dx = x^3 + c,$$

where c is the **arbitrary constant**. The $\int \dots dx$ means "integrate with respect to x ". You can always check an integral by differentiating your answer: you must get back what was inside.

Powers of x

To differentiate x^n you multiply by the power, then take one off the power. To integrate, do the opposite in the opposite order: **add one to the power, then divide by the new power**.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

For example $\int x^3 dx = \frac{x^4}{4} + c$, and $\int 3x^{-2} dx = \frac{3x^{-1}}{-1} + c = -\frac{3}{x} + c$. Integrate a sum term by term, and keep constant multiples in front.

Rewrite first so that every term is a single power: $\frac{2x-1}{\sqrt{x}} = 2x^{1/2} - x^{-1/2}$, which integrates to $\frac{4}{3}x^{3/2} - 2x^{1/2} + c$. Never integrate the top and bottom of a fraction separately.

The rule fails for $n = -1$ (you would divide by 0). Since $\frac{d}{dx} \ln x = \frac{1}{x}$,

$$\int \frac{1}{x} dx = \ln x + c \quad (x > 0).$$

The reverse chain rule: $ax + b$ inside

Differentiating $(2x + 3)^6$ gives $6(2x + 3)^5 \times 2$: the chain rule multiplies by 2, the derivative of the inside.

Integrating reverses this, so you **divide by** a , the coefficient of x inside the bracket:

- $\int (2x + 3)^4 dx = \frac{(2x + 3)^5}{5 \times 2} + c = \frac{(2x + 3)^5}{10} + c$
- $\int (1 - 2x)^{-3} dx = \frac{(1 - 2x)^{-2}}{(-2)(-2)} + c = \frac{1}{4(1 - 2x)^2} + c$ (here $a = -2$, so watch the signs)
- $\int \frac{1}{4x + 1} dx = \frac{1}{4} \ln(4x + 1) + c$
- $\int e^{5x-1} dx = \frac{1}{5} e^{5x-1} + c$
- $\int \sin 3x dx = -\frac{1}{3} \cos 3x + c$, $\int \cos 3x dx = \frac{1}{3} \sin 3x + c$, $\int \sec^2 4x dx = \frac{1}{4} \tan 4x + c$

This only works when the inside is **linear** ($ax + b$). It does not work for something like $(x^2 + 1)^3$: expand that instead, or look for a "hence".

The signs of the trigonometric ones come from the derivatives: $\frac{d}{dx} \cos x = -\sin x$, so the integral of $\sin x$ is $-\cos x$; and $\frac{d}{dx} \sin x = \cos x$, so the integral of $\cos x$ is $+\sin x$.

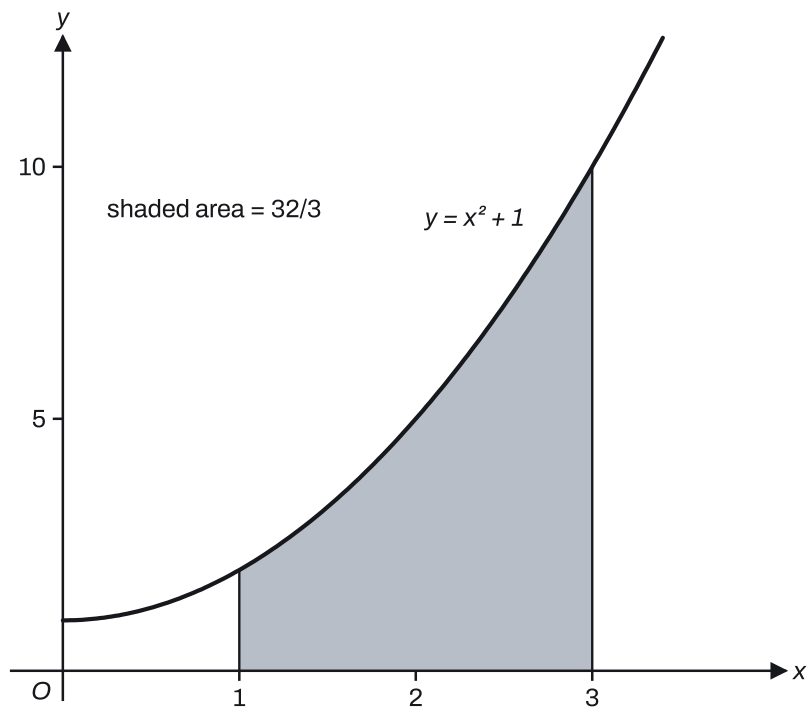
Definite integrals and area

A **definite integral** has limits:

$$\int_a^b f(x) dx = \left[F(x) \right]_a^b = F(b) - F(a),$$

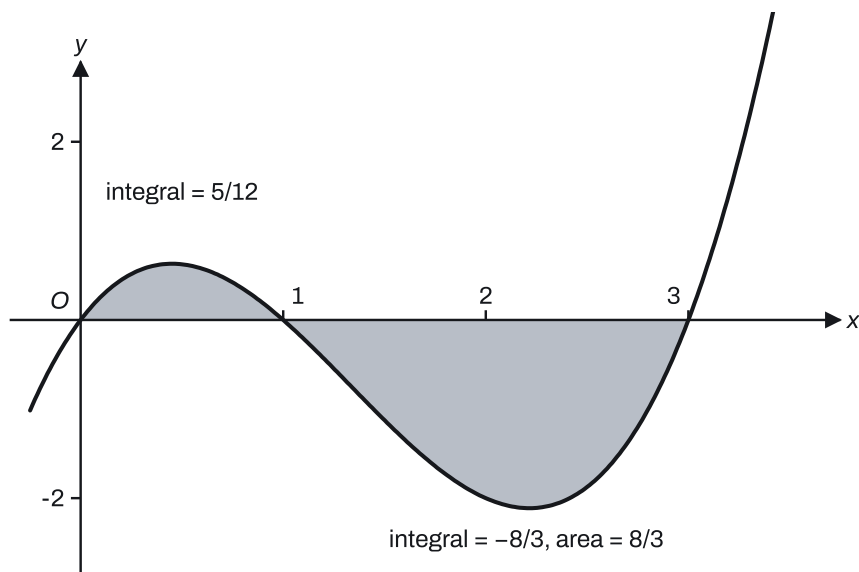
where F is any integral of f . The constant c would cancel, so leave it out. The answer is a number.

If the curve is above the x -axis between $x = a$ and $x = b$, this number is the **area** between the curve and the axis. The area is made of thin strips of height y and width δx ; adding them up and letting the width shrink to 0 is exactly what the integral does.



$$\int_1^3 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_1^3 = (9 + 3) - \left(\frac{1}{3} + 1 \right) = \frac{32}{3}.$$

Below the axis the integral is negative. For $y = x^2 - 4$ between $x = 0$ and $x = 2$ the curve is under the axis and $\int_0^2 (x^2 - 4) dx = -\frac{16}{3}$; the area is $\frac{16}{3}$. If a region is partly above and partly below, integrate each part separately and add their sizes. Integrating straight across would let the negative part cancel some of the positive part.

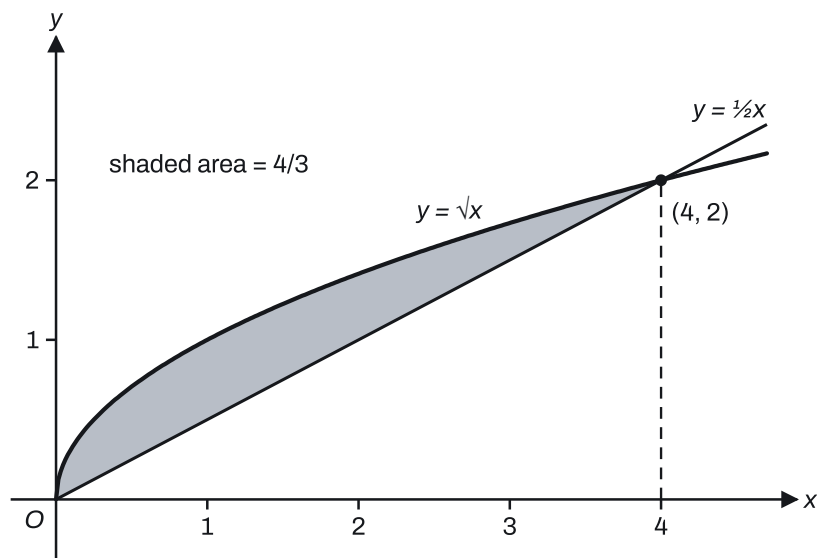


Area between two graphs

Where one graph is above another between $x = a$ and $x = b$,

$$\text{area} = \int_a^b (\text{upper} - \text{lower}) dx.$$

This works even if part of the region is below the x -axis, because the difference upper – lower stays positive. The limits are usually where the graphs meet, found by solving their equations together.



For $y = \sqrt{x}$ and $y = \frac{1}{2}x$: they meet where $\sqrt{x} = \frac{1}{2}x$, at $x = 0$ and $x = 4$. The curve is on top, so the area is

$$\int_0^4 \left(x^{1/2} - \frac{1}{2}x \right) dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{4}x^2 \right]_0^4 = \frac{16}{3} - 4 = \frac{4}{3}.$$

When the lower boundary is a straight line, you can also find the area under the curve ($\frac{16}{3}$) and take away the triangle under the line ($\frac{1}{2} \times 4 \times 2 = 4$). Triangles and trapezia need no integration: use their area formulas.

Key facts and formulas

The 0606 List of formulas has no calculus results, so every integral here must be learnt. "Given" means it is on the List of formulas in the exam.

Formula	Status
$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$	Learn it
$\int \frac{1}{x} dx = \ln x + c$	Learn it
$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c, n \neq -1$	Learn it
$\int \frac{1}{ax + b} dx = \frac{1}{a} \ln(ax + b) + c$	Learn it
$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$	Learn it

Formula	Status
$\int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + c$	Learn it
$\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + c$	Learn it
$\int \sec^2(ax + b) dx = \frac{1}{a} \tan(ax + b) + c$	Learn it
$\int_a^b f(x) dx = F(b) - F(a)$, where $F'(x) = f(x)$	Learn it
Area between graphs $= \int_a^b (\text{upper} - \text{lower}) dx$	Learn it
$\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\operatorname{cosec}^2 A = 1 + \cot^2 A$	Given
Triangle $\frac{1}{2} \times \text{base} \times \text{height}$; trapezium $\frac{1}{2}(a + b)h$	Learn it
Useful rewrites: $\sqrt{x} = x^{1/2}$, $\frac{k}{x^n} = kx^{-n}$, $\frac{1}{\cos^2 x} = \sec^2 x$, $\tan^2 x = \sec^2 x - 1$, $e^{2x} \times e^3 = e^{2x+3}$. Angles are always in radians.	

How to do it

In the Paper 1 and Paper 2 questions we have tagged for this topic (83 questions, 2021–2025), the types came up this often. 9 of the 83 questions mix two types, so the counts add up to more than 83.

Question type	Questions
Area of a region	27
"Hence" integration after differentiating	19
Finding a curve from its gradient	15
Standard integrals, indefinite and definite	14
Rewrite first, then integrate	12
Unknown limit or constant	5

1. Area of a region (27 questions)

- Find the limits.** They are where graphs meet each other or the x -axis. Solve the equations together; this is often a quadratic, or a quadratic in e^x or in x^2 . Sometimes one limit is a given line such as $x = 5$ or the x -coordinate of a named point.
- Plan the area** in one line before integrating: which integral, minus which triangle or trapezium. Decide which graph is on top.
- Integrate** (upper – lower), or each graph separately.
- Substitute the limits** and show the substitution: write $F(b) - F(a)$ with the numbers in.

5. **Add or subtract** the straight-edged pieces, and give the answer in the form asked (exact, or $p + \ln q$, and so on).

Variations you will meet:

- **A line from calculus.** The region is bounded by a tangent or a normal to the curve. Find the line first (Applications of differentiation), then where it meets the axis or the curve. Area under the curve plus or minus a triangle is usually quickest.
- **A chord AB between two points on the curve.** Find A and B from the curve's equation. The area between the chord and the curve is the trapezium under AB minus the area under the curve (or the other way round, if the curve is on top).
- **A horizontal line as the top edge**, for example $y = 9$ above $y = x^2$ from $x = 0$ to 3 : area = rectangle – area under the curve = $27 - 9 = 18$.
- **Two curves.** Subtract one from the other first, simplify to a single expression, then integrate it once between the curves' meeting points. If the meeting points are symmetric about the y -axis, you may integrate from 0 and double, but only if the region really is symmetric.
- **Part below the axis.** Split at the crossing point; take the size of each integral and add (Example 4).
- **A maximum or a stationary point** as a corner of the region: find it by differentiating first.
- **"Do not use a calculator"**: every step, including the limits, must be written out and the answer left exact.

2. "Hence" integration after differentiating (19 questions)

Part (a) asks you to differentiate something, usually a product or a quotient; part (b) says "hence" integrate. Since $\int \frac{dy}{dx} dx = y + c$, the derivative you found gives you an integral for free.

1. **Differentiate carefully** in part (a) (product rule, quotient rule, chain rule). Simplify.
2. **Write the result as an integral**: if $\frac{d}{dx}F(x) = g(x)$, then $\int g(x) dx = F(x) + c$.
3. **Match it to what you're asked.**
 - If the integrand is a **multiple** of your derivative, multiply: $\int k g(x) dx = kF(x) + c$ (Example 3).
 - If your derivative has **extra terms**, move them across and integrate those, which will be standard forms. For example $\frac{d}{dx}[(x+1)\sqrt{x+3}] = \sqrt{x+3} + \frac{x+1}{2\sqrt{x+3}}$. So $\int \left(\sqrt{x+3} + \frac{x+1}{2\sqrt{x+3}}\right) dx = (x+1)\sqrt{x+3} + c$, and moving the extra term across, $\int \frac{x+1}{2\sqrt{x+3}} dx = (x+1)\sqrt{x+3} - \int (x+3)^{1/2} dx = (x+1)\sqrt{x+3} - \frac{2}{3}(x+3)^{3/2} + c$, so $\int \frac{x+1}{\sqrt{x+3}} dx = 2(x+1)\sqrt{x+3} - \frac{4}{3}(x+3)^{3/2} + c$.
4. **Add $+ c$** , or put in limits if the integral is definite.

Variations you will meet:

- **Logarithms**: $\frac{d}{dx} \ln(x^2 + 7) = \frac{2x}{x^2+7}$, so $\int \frac{2x}{x^2+7} dx = \ln(x^2 + 7) + c$ and $\int \frac{x}{x^2+7} dx = \frac{1}{2} \ln(x^2 + 7) + c$.
- **tan of something**: $\frac{d}{dx} \tan(f(x)) = f'(x) \sec^2(f(x))$ gives the integral of that shape.
- **Check the multiple.** If the integrand is half, or three times, your derivative, the integral is half, or three times, your function. Differentiate your answer to be sure.
- **The same "hence" in a later part** of a longer question (after a tangent or normal): use the part-(a) derivative, not a new method. Integrating a product term by term is always wrong.

3. Finding a curve from its gradient (15 questions)

1. **Integrate** $\frac{dy}{dx}$, with $+ c$.
2. **Substitute the given point** (x, y) on the curve to find c .
3. **Write the equation** $y = \dots$ with the value of c in.

For example, $\frac{dy}{dx} = 6x^2 - 2$ and the curve passes through $(1, 5)$: $y = 2x^3 - 2x + c$, so $5 = 2 - 2 + c$, $c = 5$, and $y = 2x^3 - 2x + 5$.

Variations you will meet:

- **Second derivative given** ($\frac{d^2y}{dx^2}$ or $f''(x)$). Integrate twice with **two different constants**. The **gradient** information (such as "the gradient is 3 at $x = 1$ ", or "the tangent is parallel to...") fixes the first constant, in $\frac{dy}{dx}$. The **point** fixes the second, in y . Using the point in $\frac{dy}{dx}$ is the classic error (Example 2).
- **Then find the stationary point**: set the $\frac{dy}{dx}$ you were given (or found) equal to 0, then use your equation for y .
- **Two unknown constants** in $\frac{dy}{dx}$ itself (for example $\frac{dy}{dx} = kx + \dots$): use each piece of information once.
- **Show a value**, for example "show that $\frac{dy}{dx} = 52$ when $x = 11$ ": find the first constant, substitute, and write the result.

4. Standard integrals, indefinite and definite (14 questions)

1. **Rewrite** each term in one of the standard forms (powers, $ax + b$ inside a bracket, e^{ax+b} , trig of $ax + b$).
2. **Integrate term by term** using the table: new power, divide by it, divide by a .
3. **Indefinite**: add $+ c$. **Definite**: put square brackets round the result, substitute the top limit, subtract the bottom limit, and simplify.

Variations you will meet:

- **Exact answers** with e , $\ln$, π or surds: combine logs with the log laws ($\ln 9 - \ln 3 = \ln 3$, $2 \ln 3 = \ln 9$).
- **Trigonometric limits**: use exact values such as $\tan \frac{\pi}{4} = 1$, $\sin \frac{\pi}{6} = \frac{1}{2}$, $\cos \frac{\pi}{3} = \frac{1}{2}$.
- **"Show that" a definite integral equals a given value**: write the integrated expression, the substitution, and every step to the printed answer.
- **A domain note** such as "for $x < \frac{5}{2}$ " next to $\frac{1}{5-2x}$ just tells you the bracket is positive, so the log is defined:

$$\int \frac{1}{5-2x} dx = -\frac{1}{2} \ln(5 - 2x) + c.$$

5. Rewrite first, then integrate (12 questions)

The integrand isn't a standard form yet. Change it into a sum of standard forms, then integrate.

1. **Choose the rewrite**:
 - **Expand** a bracket squared, such as $(3 - \sqrt{x})^2 = 9 - 6x^{1/2} + x$.
 - **Divide** each term of the top by the bottom: $\frac{6x^2-5}{x} = 6x - \frac{5}{x}$.

- **Split into partial fractions** when an earlier part shows you the split. Then integrate each part:

$$\int \left(\frac{3}{x+1} - \frac{2}{(x+1)^2} \right) dx = 3 \ln(x+1) + \frac{2}{x+1} + c. \text{ Note that } \frac{1}{(x+1)^2} \text{ is a **power** (it does not give a log).}$$

- **Use an identity:** $\tan^2 x = \sec^2 x - 1$, $\sin^2 x + \cos^2 x = 1$, $\frac{1}{\cos^2 x} = \sec^2 x$.

2. **Integrate** each term as in type 4, and finish as asked.

"Show that" parts before the integral are there to give you the rewrite: use them.

6. Unknown limit or constant (5 questions)

1. **Integrate** and substitute the limits, keeping the letter: for example $\int_1^a 2x \, dx = a^2 - 1$.
2. **Set equal** to the given value and **solve**: $a^2 - 1 = 15$ gives $a = 4$ (the limit must be greater than 1). With logs or exponentials, combine into one log first, then take e of both sides (or $\ln$).
3. If the answer is to be "in terms of a " or "as a single logarithm", stop after simplifying with the log laws.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
 - **DM1** method mark that needs the M mark before it.
 - **A1** accuracy mark: for a correct answer; it needs the M mark before it.
 - **B1** mark for a correct result on its own, with no method needed.
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Example 1

(a) Find $\int \left(6\sqrt{3x-1} - \frac{2}{5-2x} \right) dx$ for $\frac{1}{3} < x < \frac{5}{2}$. **[4]**

(b) Find the exact value of $\int_0^{\pi/3} 4 \sin \left(2x + \frac{\pi}{6} \right) dx$. **[4]**

(a) $6(3x-1)^{1/2}$ integrates to $\frac{6(3x-1)^{3/2}}{\frac{3}{2} \times 3} = \frac{4}{3}(3x-1)^{3/2}$. **B1** for $k(3x-1)^{3/2}$, **B1** for $k = \frac{4}{3}$.

$-\frac{2}{5-2x}$ integrates to $-2 \times \frac{\ln(5-2x)}{-2} = \ln(5-2x)$. **B1** for $k \ln(5-2x)$, **B1** for $k = 1$ (with $+c$).

$$\frac{4}{3}(3x-1)^{3/2} + \ln(5-2x) + c$$

The coefficient of x inside $5-2x$ is -2 , and the two minus signs cancel.

(b) Here $a = 2$, and $\sin$ integrates to $-\cos$:

$$\int_0^{\pi/3} 4 \sin \left(2x + \frac{\pi}{6} \right) dx = \left[-2 \cos \left(2x + \frac{\pi}{6} \right) \right]_0^{\pi/3}$$

B1 for $k \cos \left(2x + \frac{\pi}{6} \right)$, **B1** for $k = -2$.

$$= -2 \cos \frac{5\pi}{6} - \left(-2 \cos \frac{\pi}{6} \right) = -2 \left(-\frac{\sqrt{3}}{2} \right) + 2 \left(\frac{\sqrt{3}}{2} \right) = 2\sqrt{3}.$$

M1 for substituting both limits, top minus bottom, **A1** for $2\sqrt{3}$.

Example 2

A curve is such that $\frac{d^2y}{dx^2} = 6x - \frac{4}{x^3}$ for $x > 0$. The normal to the curve at the point $P(1, 4)$ is parallel to the line $x + 3y = 6$. Find the equation of the curve. [7]

The line $x + 3y = 6$ has gradient $-\frac{1}{3}$, so the normal at P has gradient $-\frac{1}{3}$ and the tangent (the curve's gradient) at P is 3. **B1**

$$\frac{dy}{dx} = 3x^2 + \frac{-4x^{-2}}{-2} + c = 3x^2 + 2x^{-2} + c.$$

B1 for $3x^2$, **B1** for $2x^{-2}$.

Gradient 3 at $x = 1$: $3 + 2 + c = 3$, so $c = -2$. **M1** for using the gradient in $\frac{dy}{dx}$ (not in y).

$$y = x^3 + \frac{2x^{-1}}{-1} - 2x + d = x^3 - \frac{2}{x} - 2x + d.$$

M1 for integrating their $\frac{dy}{dx}$ again, with a new constant.

The curve passes through $(1, 4)$: $1 - 2 - 2 + d = 4$, so $d = 7$. **DM1**

$$y = x^3 - \frac{2}{x} - 2x + 7$$

A1

Example 3

(a) Given that $y = \frac{x}{\sqrt{x^2 + 5}}$, show that $\frac{dy}{dx} = \frac{5}{(x^2 + 5)^{3/2}}$. [3]

(b) Hence find the exact value of $\int_0^2 \frac{1}{(x^2 + 5)^{3/2}} dx$. [3]

(a) Quotient rule, with $u = x$ and $v = (x^2 + 5)^{1/2}$, so $v' = \frac{1}{2}(x^2 + 5)^{-1/2} \times 2x = x(x^2 + 5)^{-1/2}$. **B1**

$$\frac{dy}{dx} = \frac{(x^2 + 5)^{1/2} - x \times x(x^2 + 5)^{-1/2}}{x^2 + 5}.$$

M1 for the quotient rule structure.

Multiply the top and bottom by $(x^2 + 5)^{1/2}$:

$$\frac{dy}{dx} = \frac{(x^2 + 5) - x^2}{(x^2 + 5)^{3/2}} = \frac{5}{(x^2 + 5)^{3/2}}.$$

A1, the given answer with the simplification shown.

(b) From (a), $\int \frac{5}{(x^2 + 5)^{3/2}} dx = \frac{x}{\sqrt{x^2 + 5}} + c$, so the integral we want is $\frac{1}{5}$ of this. **M1**

$$\int_0^2 \frac{1}{(x^2 + 5)^{3/2}} dx = \frac{1}{5} \left[\frac{x}{\sqrt{x^2 + 5}} \right]_0^2 = \frac{1}{5} \left(\frac{2}{3} - 0 \right) = \frac{2}{15}.$$

M1 for the limits in the right order, **A1** for $\frac{2}{15}$.

You could never do this integral with the standard forms: the "hence" is the only way.

Example 4

The curve $y = x^3 - 4x^2 + 3x$ meets the x -axis at three points.

(a) Find the x -coordinates of these points. **[2]**

(b) Find the total area of the two regions enclosed by the curve and the x -axis. **[6]**

(a) $x(x^2 - 4x + 3) = x(x - 1)(x - 3) = 0$, so $x = 0, 1$ and 3 . **M1 A1**

(b) Between 0 and 1 the curve is above the axis (at $x = 0.5$, $y = 0.625$); between 1 and 3 it is below (at $x = 2$, $y = -2$). So integrate each part separately. **M1** for splitting at $x = 1$.

$$\int (x^3 - 4x^2 + 3x) dx = \frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}.$$

B1 for all three terms.

$$\int_0^1 = \left(\frac{1}{4} - \frac{4}{3} + \frac{3}{2} \right) - 0 = \frac{5}{12}. \quad \int_1^3 = \left(\frac{81}{4} - 36 + \frac{27}{2} \right) - \frac{5}{12} = -\frac{9}{4} - \frac{5}{12} = -\frac{8}{3}.$$

M1 for substituting the limits, **A1** for both values.

The second region's area is $\frac{8}{3}$ (the minus sign only says it is under the axis). Total area

$$\frac{5}{12} + \frac{8}{3} = \frac{37}{12} (= 3.08 \text{ to 3 s.f.}).$$

M1 A1

A single integral from 0 to 3 gives $-\frac{9}{4}$, which is **not** the area: the two parts partly cancel.

Common mistakes

- **Leaving out** $+ c$. Examiners report it every session on indefinite integrals, and it costs a mark each time. Don't add $+ c$ to a **definite** integral, though: a stray $+ c$ on a number is also penalised.

- **Forgetting to divide by a .** $\int e^{4x} dx = \frac{1}{4}e^{4x}$, not e^{4x} or $4e^{4x}$; $\int \cos 3x dx = \frac{1}{3} \sin 3x$. Differentiate your answer to check.
- **Getting the trig signs the wrong way round.** $\sin$ integrates to $-\cos$, and $\cos$ integrates to $+\sin$. Integrating $\sin$ to give $\cos$ (or $-\sin$) is a common slip.
- **Integrating a product or a fraction piece by piece.** $\int x \cos x dx$ is **not** $\frac{x^2}{2} \sin x$, and $\int \frac{x^2-1}{x} dx$ is not $\frac{x^3/3-x}{\ln x}$. Divide, expand, use the given partial fractions, or use the "hence".
- **Using $\ln$ for the wrong power.** Only $\frac{1}{ax+b}$ (power -1) gives a log. $\frac{1}{(ax+b)^2}$ is a power and gives $-\frac{1}{a(ax+b)}$.
- **Ignoring the "hence".** Starting again with an unfamiliar method (such as integration by parts, which is not in this syllabus) usually fails. Use the derivative you have just found.
- **Mixing up the two constants** with a second derivative. The gradient condition belongs in $\frac{dy}{dx}$, the point in y . Examiners often see the point's y -value put into $\frac{dy}{dx}$.
- **Answers with no working.** A calculator's value for a definite integral, with no integrated expression shown, earns little or nothing. Always write $[F(x)]_a^b$ and the substitution.
- **Forgetting part of the region.** Planning the area first avoids leaving out a triangle, subtracting the wrong piece, or using a limit that is outside the region (such as a crossing point beyond the given line $x = \dots$).
- **Adding a negative integral as if it were an area.** Split at the axis and add the sizes.
- **Radians.** Trig limits such as $\frac{\pi}{6}$ must be worked in radians; a calculator in degrees gives nonsense.

How to get the marks

- **Each correct term** of an integral usually earns its own mark, so write the integral out in full before substituting. A method mark for the form (such as $k \ln(5 - 2x)$ or $k(3x - 1)^{3/2}$) is given even when the constant k is wrong.
- **Definite integrals:** show the square brackets and the substitution of **both** limits, top minus bottom. With "exact", keep $\ln$, e , π and surds, and simplify as asked (such as $a + \ln b$).
- **"Show that":** the answer is printed, so every step must be visible, and the final line must match it exactly.
- **"Hence":** you must use the previous part. Write down the link, for example " $\int 5(x^2 + 5)^{-3/2} dx = \frac{x}{\sqrt{x^2+5}} + c$ ", before using it.
- **Areas:** write your plan (for example, "area under the curve — area of the trapezium"). Plan marks are given even if an earlier value is wrong. The final answer must be positive.
- **Finding a curve:** show the substitution that finds each constant, and give the final equation " $y = \dots$ " in full.
- **Accuracy:** exact answers where asked; otherwise 3 significant figures, with angles in radians.
- **"Do not use a calculator":** every value, including the limits and the log simplification, must come from written working.
- **Method marks follow through** from an earlier wrong answer, so keep going and use your values.

Quick check

1. Find $\int \frac{1 - \sin^2 x}{\cos^4 x} dx$.
2. Find $\int \left(2 + \frac{1}{x}\right)^2 dx$ for $x > 0$.
3. Given that $\int_0^k e^{2x} dx = 4$, find the exact value of the constant k .
4. Show that $\frac{d}{dx} (x\sqrt{2x+1}) = \frac{3x+1}{\sqrt{2x+1}}$, and hence find $\int_0^4 \frac{6x+2}{\sqrt{2x+1}} dx$.
5. A curve with $\frac{dy}{dx} = \frac{4}{2x-1}$, for $x > \frac{1}{2}$, passes through $(1, 3)$. Find its equation, and the value of y when $x = \frac{e^2 + 1}{2}$.

Answers

1. $1 - \sin^2 x = \cos^2 x$, so the integrand is $\frac{\cos^2 x}{\cos^4 x} = \frac{1}{\cos^2 x} = \sec^2 x$, and the integral is $\tan x + c$.
2. $(2 + x^{-1})^2 = 4 + 4x^{-1} + x^{-2}$, so the integral is $4x + 4 \ln x - \frac{1}{x} + c$.
3. $\left[\frac{1}{2}e^{2x}\right]_0^k = \frac{1}{2}e^{2k} - \frac{1}{2} = 4$, so $e^{2k} = 9$, $2k = \ln 9$ and $k = \ln 3$.
4. Product rule: $\sqrt{2x+1} + x \times \frac{1}{2}(2x+1)^{-1/2} \times 2 = \frac{(2x+1) + x}{\sqrt{2x+1}} = \frac{3x+1}{\sqrt{2x+1}}$. The integrand is 2 times this, so the integral is $2 \left[x\sqrt{2x+1} \right]_0^4 = 2(4 \times 3 - 0) = 24$.
5. $y = 4 \times \frac{1}{2} \ln(2x-1) + c = 2 \ln(2x-1) + c$. At $(1, 3)$: $3 = 2 \ln 1 + c$, so $c = 3$ and $y = 2 \ln(2x-1) + 3$. When $x = \frac{e^2+1}{2}$, $2x-1 = e^2$, so $y = 2 \times 2 + 3 = 7$.

Past questions to try

- 0606/11/M/J/25 Q9: area between a curve, its tangent and the x -axis.
- 0606/12/O/N/25 Q9: a curve from its second derivative, then a stationary point.
- 0606/12/F/M/25 Q12: differentiate a product, then "hence" integrate.
- 0606/13/M/J/25 Q8: area between a chord and a curve, with logarithms.