

Permutations and combinations

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What you need to know

By the end of this topic you should be able to:

1. **Tell a permutation from a combination.** A permutation is an arrangement, where the order matters; a combination is a selection, where it doesn't. Decide which one a question needs before you calculate.
2. **Use factorials and the two formulas.** Know what $n!$ means, that $0! = 1$, and how to work with nP_r (the number of ways to arrange r of n different items) and nC_r (the number of ways to choose r of them), both with numbers and with algebra such as ${}^{n+1}C_3$.
3. **Solve counting problems** about arranging and selecting, set either in an everyday situation (people in a line, teams, digits, passwords) or as an equation in n to solve or an identity to prove.

Every item in these problems is different. Three kinds of problem are **not** in the syllabus: repeated items (such as the letters of BANANA), items arranged in a circle, and counting problems that need both a permutation and a combination (such as choosing a group and then arranging it). Each counting problem you meet needs one or the other.

Understand it

Arranging or choosing?

Eight runners reach a final. Handing out gold, silver and bronze is an **arrangement**: Ali gold and Ben silver is different from Ben gold and Ali silver. Picking which 3 runners go through to the next round is a **selection**: the same 3 people in any order are the same group.

- Medals: $8 \times 7 \times 6 = 336$ ways (8 choices for gold, then 7 for silver, then 6 for bronze).
- Going through: 56 ways.

Each group of 3 can be put in order in $3 \times 2 \times 1 = 6$ ways, so there are 6 times as many arrangements as selections: $336 = 6 \times 56$. Ask yourself: **if I swap two of the chosen items, is it a new outcome?** Yes means permutation; no means combination.

The product rule and the addition rule

If one job can be done in a ways and, after that, a second job in b ways, the two together can be done in $a \times b$ ways. A café with 3 starters and 4 mains offers $3 \times 4 = 12$ two-course meals. Almost every answer in this topic is built from this rule.

If instead you do **one job or the other** (a starter **or** a main, not both), you **add**: $3 + 4 = 7$ ways. In counting, "and then" means multiply and "or" (separate cases that can't overlap) means add.

Factorials

$n!$ ("n factorial") is $n \times (n - 1) \times \cdots \times 2 \times 1$. It counts the ways to arrange n different items in a row: 5 different books on a shelf can be arranged in $5! = 120$ ways.

Two facts make algebra with factorials work:

- $n! = n \times (n - 1)!$, so for example $(n + 1)! = (n + 1)n(n - 1)!$ and $\frac{(n + 1)!}{(n - 1)!} = (n + 1)n$.
- $0! = 1$. This is defined, not calculated, so that the formulas below still work when $r = n$ or $r = 0$.

The two formulas

Arranging r of n different items in a row: n choices for the first place, $n - 1$ for the second, down to $n - r + 1$ for the last. That product is

$${}_nP_r = n(n - 1)(n - 2) \cdots (n - r + 1) = \frac{n!}{(n - r)!}.$$

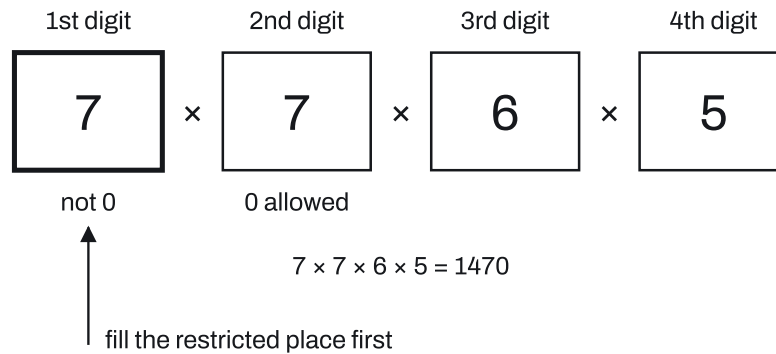
Choosing r of them: every group of r can be ordered in $r!$ ways, so there are $r!$ times fewer selections than arrangements:

$${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n - r)!}.$$

For example ${}_6P_2 = 6 \times 5 = 30$ and ${}_6C_2 = \frac{30}{2} = 15$. The exam paper also writes ${}_nC_r$ as $\binom{n}{r}$. Choosing the r items to take is the same as choosing the $n - r$ to leave behind, so ${}_nC_r = {}_nC_{n-r}$: ${}^{10}C_7 = {}^{10}C_3 = 120$. Your calculator has nPr and nCr keys for numbers; with a letter n you must use the factorial forms.

Deal with the restricted places first

When some places have rules, fill those places first, then the free places. For 4-digit numbers made from the digits 0 to 7, each used at most once, the first digit cannot be 0:



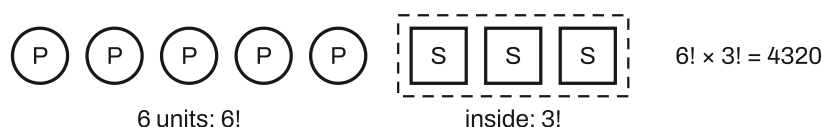
If you filled the boxes left to right without thinking, you might write $8 \times 7 \times 6 \times 5$, which wrongly includes numbers starting with 0. Another correct way is "all orders minus the bad ones": ${}^8P_4 - {}^7P_3 = 1680 - 210 = 1470$.

Items that must stay together, or apart

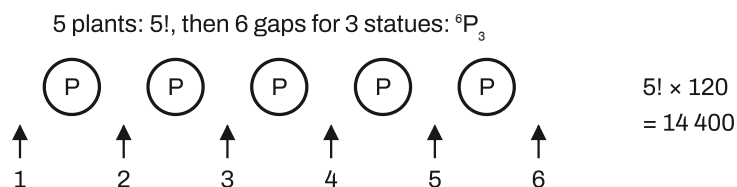
Five different plants and three different statues are placed in a row along a path.

- **Together.** Tie the 3 statues into one block. Now there are $5 + 1 = 6$ units to arrange, in $6!$ ways, and the statues can be ordered inside the block in $3!$ ways: $6! \times 3! = 4320$.
- **No two next to each other.** Place the 5 plants first, in $5!$ ways. That leaves 6 gaps: one before the first plant, one after the last, and 4 between. Put the 3 statues into 3 different gaps, in ${}^6P_3 = 120$ ways, and no two can touch: $5! \times 120 = 14\,400$.

Statues together: one block



Statues apart: use the gaps



"Not all together" is a different condition: it only rules out the three standing in one block, so its count is everything minus "together". For two items, "not together" and "not next to each other" mean the same thing: total minus together.

Counting what you don't want

"At least one" usually has many cases but only one opposite case: **none**. Choosing 4 roses from 6 red and 3 white with at least one white:

$${}^9C_4 - {}^6C_4 = 126 - 15 = 111.$$

Adding the cases (1 white, 2 white, 3 white) gives the same answer, $60 + 45 + 6 = 111$, with more work.

Symmetry

Of all the ways to line up 6 people, Asha is to the left of Bilal in exactly half (swap the two of them in any line and you get a line where it is the other way round). So there are $\frac{6!}{2} = 360$ such lines, without listing a single one.

Key facts and formulas

"Given" means it is in the list of formulas printed in the exam paper (as part of the binomial theorem). "Learn it" means you must remember it.

Formula	Status
${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	Given
$n! = n(n-1)(n-2) \cdots 2 \times 1$, and $0! = 1$	Learn it
$n! = n \times (n-1)!$	Learn it
${}^nP_r = \frac{n!}{(n-r)!} = n(n-1) \cdots (n-r+1)$	Learn it
${}^nP_r = r! \times {}^nC_r$	Learn it
${}^nC_r = {}^nC_{n-r}$	Learn it
n different items in a row: $n!$ ways	Learn it
"And then": multiply; separate cases ("or"): add	Learn it
Wanted = total – unwanted	Learn it
k items together: treat them as one unit, then multiply by $k!$	Learn it

Useful expansions: ${}^nC_2 = \frac{n(n-1)}{2}$, ${}^nC_3 = \frac{n(n-1)(n-2)}{6}$, ${}^nP_2 = n(n-1)$, ${}^nP_3 = n(n-1)(n-2)$.

How to do it

In the questions we have tagged for this topic (43 questions, 2021–2025), the types came up this often. 27 of the 43 questions mix two or three types in their parts, so the counts add up to more than 43.

Question type	Questions
Selections with conditions	21
Equations and proofs with nP_r and nC_r	17

Question type	Questions
Forming numbers from digits	16
Arranging people or objects in a line	9
Codes and passwords from mixed characters	8
Splitting people into groups	2

1. Selections with conditions (21 questions)

Teams, committees, groups of letters or photographs: the order inside the group doesn't matter, so use nC_r .

1. **Split the pool into categories** (for example doctors, nurses, dentists) and write how many of each there are.
2. **List the cases** that meet the condition, as numbers from each category adding up to the group size. Write them down; examiners reward clearly stated cases.
3. **For each case, multiply** one nC_r per category.
4. **Add** the cases.

Variations you will meet:

- **No restriction:** just ${}^{\text{total}}C_{\text{size}}$.
- **"At least one ...":** total minus the "none" case. For "at least 2", subtract "none" and "exactly 1". Before counting, check what is automatic: choosing 6 from 10 people of whom only 4 are not teachers forces at least 2 teachers.
- **"At least one of each kind"** with three kinds: add the cases directly (usually quicker and safer). If you use the complement, a group missing **two** kinds is subtracted twice, so add it back.
- **A named person must be in:** put them in, then choose the rest from everyone else. **Must be out:** remove them from the pool.
- **Two named people, both in / exactly one / at most one:** both in is ${}^{n-2}C_{r-2}$; exactly one is $2 \times {}^{n-2}C_{r-1}$; at most one is total minus both in.
- **A family or group that must not be separated:** two cases, all of them in (choose the rest from the others) or none of them in. Add.
- **"The same number of A as B":** list 1 and 1, 2 and 2, ..., making up the rest from the other categories, and stop when a category runs out.
- **Letters of a word** with vowels and consonants: a selection of letters is a combination; treat vowels and consonants as the two categories.

2. Equations and proofs with nP_r and nC_r (17 questions)

1. **Write each term with factorials**, carefully replacing n by what is on top, for example ${}^{n+1}C_4 = \frac{(n+1)!}{4!(n-3)!}$.

2. **Cancel the factorials** so no "!" is left with a letter in it. Use $m! = m \times (m - 1)!$ to make the larger factorial match the smaller one, then cancel. Simplify the numbers ($\frac{5!}{3!} = 20$) separately.
3. **Rearrange** to a polynomial equation, usually linear or quadratic, and solve it.
4. **Choose the right root:** n must be a positive integer, and large enough for every term to make sense (you need $n \geq r$ in nC_r). Reject the others and say so.

For example, ${}^nP_2 + {}^nC_2 = 45$ gives $n(n - 1) + \frac{n(n - 1)}{2} = 45$, so $\frac{3n(n - 1)}{2} = 45$, $n(n - 1) = 30$, $n^2 - n - 30 = 0$, $(n - 6)(n + 5) = 0$, and $n = 6$ (reject -5).

Variations you will meet:

- **In words:** "the number of combinations of n objects taken 3 at a time" is nC_3 ; "the number of permutations of $2n$ items taken 3 at a time" is ${}^{2n}P_3$. Translate first, then solve.
- **"Show that n satisfies ...", then "hence":** reach the printed equation with every step visible, then solve it.
- **Find n , then a further value:** once you have n , work out what is asked, such as another nC_r , on your calculator.
- **Prove an identity**, for example that one combination expression equals another: start from one side (usually the more complicated one), write everything with factorials, take out the common factorial, simplify the bracket and finish at the other side. Don't use the result you are proving.
- **"Use an algebraic method"** means trial and improvement on the calculator earns little or nothing, even with the right answer.

3. Forming numbers from digits (16 questions)

1. **Draw a box for each place** and fill the restricted places first: usually the first digit (not 0; or a range such as "greater than 50 000") and the last digit (even, odd, 0 or 5 for a multiple of 5).
2. **Fill the remaining boxes** with the digits left, counting down one each time.
3. **Multiply along the boxes.**

If the first and last places both have rules and their allowed digits overlap, split into cases. For example, 4-digit numbers from the digits 1 to 8 that begin with a prime digit and end with an odd digit: the first digit is 2, 3, 5 or 7 and the last is 1, 3, 5 or 7, and 3, 5, 7 belong to both lists.

- First digit 2: last digit any of the 4 odd digits, middle two from the 6 left: $1 \times 4 \times 6 \times 5 = 120$.
- First digit 3, 5 or 7: last digit one of the 3 odd digits left: $3 \times 3 \times 6 \times 5 = 270$.
- Total 390.

Split by the first digit **or** by the last digit, but don't mix the two splits.

Variations you will meet:

- **0 is available:** it can't go first, but it can go anywhere else, which changes the count for the second box. When the last digit must be even, split into "ends in 0" and "ends in another even digit", because when 0 is used up at the end, the first place has one more choice.

- **Divisible by 5:** ends in 0 or ends in 5, two cases. **Divisible by 10:** ends in 0.
- **Greater than a value like 750 000:** split by the first digit (8 or 9 is enough on its own; 7 needs the second digit checked).
- **Codes that may start with 0** are simpler: no special first place.
- **A digit condition like "prime" or "a multiple of 3":** list which digits qualify, then treat them like "odd" or "even".

4. Arranging people or objects in a line (9 questions)

1. **No restriction:** $n!$.
2. **Fixed places** (a woman at each end, a named person in the middle): fill those places first, then arrange the rest.
3. **Together:** make each "together" set one block, arrange the blocks and the loose items, then multiply by the arrangements inside each block. Three kinds all in blocks: $3!$ for the order of the blocks, times each block's inside order.
4. **Not next to each other:** for two items, total minus together. For several items that must all be separated, place the others first and put them in the gaps.

Variations you will meet:

- **Alternating** (no two of one kind next to each other when there is exactly one more of that kind): the pattern is fixed, so multiply the arrangements of each kind.
- **Exactly k people between two named people:** count the positions of the pair (for n places there are $n - k - 1$ positions for the left one), times 2 for which of them is on the left, times the arrangements of the others.
- **Some letters together in a fixed order:** the block counts as one unit and has only **1** inside order.
- **Letters of a word with all letters different:** the same as arranging people.

5. Codes and passwords from mixed characters (8 questions)

Characters (letters, numbers, symbols) used at most once, so the order matters: permutations.

1. **No restriction:** $^{\text{characters}}P_{\text{length}}$.
2. **Places with a type** ("starts with two letters", "ends with a symbol"): fill those places from their own type, counting down, then fill the rest from **all** the characters left.
3. **"At least one symbol":** total minus the passwords with no symbols.

For example, 3-character codes from 5 letters and 3 symbols with at least one symbol: ${}^8P_3 - {}^5P_3 = 336 - 60 = 276$.

Variations you will meet:

- **"Starts with a symbol then a number, or a number then a symbol":** work out one order and multiply by 2.

- **Exactly so many of each type**: choose which places get which type, then fill them. For 4 letters and 1 number in a 5-character code, the number can go in any of 5 places.
- **"Includes all the symbols but doesn't start or end with one"**: put the symbols in the allowed middle places first (a permutation of places), then fill the others.

6. Splitting people into groups (2 questions)

1. **Choose each group in turn** from the people left, starting with any group that has a named person in it. For 10 people split into a car (4 seats) and a minibus (6 seats), with Dev in the car: choose Dev's 3 companions, ${}^9C_3 = 84$, and the other 6 take the minibus in ${}^6C_6 = 1$ way, so 84 splits.
2. **Equal-sized groups with no names** (not "group A, group B"): choosing the same groups in a different order is not new, so divide by the number of orders of those groups. Nine people into three groups of 3:
$$\frac{{}^9C_3 \times {}^6C_3}{{}^3P_3} = \frac{1680}{6} = 280.$$
3. **Named people with a condition** (two people must be apart): start with the named people. Put the first one in a group and choose the rest of that group, then do the same for the second one from the people left; each named person labels their group, so no division is needed.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
 - **A1** accuracy mark: for a correct answer; it needs the M mark before it.
 - **B1** mark for a correct result on its own, with no method needed.
-

Example 1

Seven runners, including Kofi and Lena, line up in a row for a photograph.

- (a) Find the number of arrangements in which Kofi stands in the middle place. **[1]**
 - (b) Find the number of arrangements in which Kofi stands somewhere to the left of Lena. **[2]**
 - (c) The three tallest runners (not Kofi or Lena) must not all stand together. Find the number of arrangements. **[3]**
- (a)** Kofi's place is fixed; the other 6 runners fill the other 6 places in $6! = 720$ ways. **B1**
- (b)** There are $7! = 5040$ arrangements in all. Swapping Kofi and Lena turns each arrangement with Kofi on the left into one with Kofi on the right, so exactly half have Kofi on the left. **M1**

$$\frac{5040}{2} = 2520.$$

A1

(c) Count the arrangements where the three tallest **are** together, then subtract. With the three as one block there are 5 units, arranged in $5!$ ways, and $3!$ orders inside the block: $5! \times 3! = 720$. **M1**

$$7! - 5! \times 3! = 5040 - 720 = 4320.$$

M1 for subtracting from $7!$, **A1**. (Placing them in gaps would answer a different question, "no two of them next to each other".)

Example 2

Four-digit numbers are formed from the digits 0, 1, 2, 3, 4, 5, 6, 7. Each digit is used at most once, and a number cannot start with 0.

(a) Find how many numbers can be formed. **[2]**

(b) Find how many of these numbers contain the digit 3. **[2]**

(c) Find how many of these numbers begin with an even digit and end with a digit greater than 3. **[3]**

(d) Find how many of these numbers have their digits in increasing order from left to right. **[2]**

(a) First digit: 7 choices (not 0). Then 7, 6 and 5 for the other places. **M1**

$$7 \times 7 \times 6 \times 5 = 1470.$$

A1

(b) Count the numbers **without** a 3: the digits are 0, 1, 2, 4, 5, 6, 7, so $6 \times 6 \times 5 \times 4 = 720$. **M1**

$$1470 - 720 = 750.$$

A1

(c) The first digit is 2, 4 or 6 (not 0); the last is 4, 5, 6 or 7. The lists share 4 and 6, so split by the first digit.

- First digit 2: 4 choices for the last digit.
- First digit 4 or 6: 3 choices for the last digit each (the first digit is used up).

M1 for a correct split into cases. That gives $4 + 3 + 3 = 10$ ways to fill the two end places. The middle two places use 2 of the 6 digits left: $6 \times 5 = 30$ ways. **M1**

$$10 \times 30 = 300.$$

A1

(d) Once 4 digits are chosen, there is only one way to put them in increasing order, so each **selection** gives exactly one number. The digit 0 could only come first in an increasing number, which isn't allowed, so choose from 1 to 7: **M1**

$${}^7C_4 = 35.$$

A1. This part is a combination even though the question is about numbers: the order is decided for you.

Example 3

A radio presenter chooses 6 songs for a show from a list of 5 ballads, 4 anthems and 3 folk songs, all different.

(a) Find the number of possible choices. **[1]**

(b) Find the number of choices that contain more ballads than anthems and folk songs put together. **[3]**

(c) Find the number of choices that contain songs of exactly two of the three kinds. **[3]**

(a) ${}^{12}C_6 = 924$. **B1**

(b) With b ballads out of 6 songs, "more ballads than the rest" means $b > 6 - b$, so $b = 4$ or $b = 5$. The other songs come from the $4 + 3 = 7$ non-ballads. **M1** for these two cases.

- 4 ballads: ${}^5C_4 \times {}^7C_2 = 5 \times 21 = 105$.
- 5 ballads: ${}^5C_5 \times {}^7C_1 = 7$.

M1 for a correct product for either case.

$$105 + 7 = 112.$$

A1

(c) Take each pair of kinds and choose all 6 songs from those two kinds together:

- ballads and anthems: ${}^9C_6 = 84$;
- ballads and folk: ${}^8C_6 = 28$;
- anthems and folk: ${}^7C_6 = 7$.

M1 for a combination for each pair. Each of these totals would also include choices that use only **one** kind, which must be removed. But no kind has 6 songs, so there are none of those here. **M1** for checking this.

$$84 + 28 + 7 = 119.$$

A1

Example 4

It is given that $4 \times {}^{n+2}C_4 = 35 \times {}^nP_2$, where $n \geq 2$.

(a) Show that $n^2 + 3n - 208 = 0$. **[4]**

(b) Hence find the value of n . **[2]**

(a) Write each side with factorials: **B1** for each.

$$4 \times \frac{(n+2)!}{4!(n-2)!} = 35 \times \frac{n!}{(n-2)!}.$$

Multiply both sides by $(n-2)!$ and use $(n+2)! = (n+2)(n+1) \times n!$: **B1** for cancelling the factorials correctly.

$$\frac{4(n+2)(n+1)n!}{24} = 35 \times n! \Rightarrow \frac{(n+2)(n+1)}{6} = 35.$$

So $(n+2)(n+1) = 210$, $n^2 + 3n + 2 = 210$, and

$$n^2 + 3n - 208 = 0.$$

B1, the given equation with every step shown.

(b) $(n-13)(n+16) = 0$, so $n = 13$ or $n = -16$. **M1**

n must be a positive integer, so $n = 13$ only. **A1**

Check: $4 \times {}^{15}C_4 = 4 \times 1365 = 5460$ and $35 \times {}^{13}P_2 = 35 \times 156 = 5460$.

Common mistakes

- **Combinations when the order matters, or permutations when it doesn't.** Passwords, numbers and lines are arrangements; teams and committees are selections. Examiners often see nC_r used for a number or password question, and sometimes even when the question itself uses nP_r notation.
- **Adding instead of multiplying.** "The plants in their places **and** the statues in the gaps" is $5! \times {}^6P_3$, not $5! + {}^6P_3$. Stating the separate counts without forming a product earns little.
- **Forgetting that 0 can't come first.** Fill the first place before the others, and remember 0 is still available for the other places.
- **Mixing two case splits.** In a number question, split by the first digit or by the last digit. Using one split for some cases and the other for the rest double-counts or misses numbers.
- **Missing cases.** When you add cases, list them all first (for example 2, 3, 4 and 5 teachers). Writing the list is also good practice because it shows the examiner your method.
- **Over-subtracting with the complement.** For "at least one of each of three kinds" A, B and C, subtracting "no A", "no B" and "no C" removes a choice made of only one kind twice. Add the cases directly, or add those back.
- **Slips with algebraic factorials.** ${}^{n+1}C_5$ has $(n+1-5)! = (n-4)!$ at the bottom, not $(n-5)!$. Write the factorial form out in full first, then cancel one step at a time.
- **Keeping a negative or impossible n .** A quadratic usually has one root that is negative or too small. Reject it and give only the valid value.
- **Guessing n on a calculator.** Trial and improvement is not an algebraic method and earns at most the final mark.
- **Using a given result to prove itself.** In a "show that" identity, start from one side and work to the other; don't multiply across the equation you were told to prove.

How to get the marks

- **Show the product before the answer.** Write $3 \times {}^8P_5$ or ${}^6C_2 \times {}^9C_4$, then its value. A correct product earns the method mark even if you slip on the arithmetic; a bare wrong number earns nothing.
- **Give exact integers.** Counting answers are whole numbers: never round them or give them in standard form.
- **Name your cases** in words, for example "starts with 5 or 7" or "2 women, 3 men". Mark schemes award marks for each correct case, and a labelled case is easy to credit.
- **"Find the number of ways"** for a single part worth 1 mark usually needs one formula; a part worth 3 or more almost always needs cases or a complement.
- **"Show that"**: the answer is printed, so every step must be visible, from the factorial form through the cancelling to the final equation, and your last line must match the printed one exactly.
- **"Hence"**: use the equation or result you have just shown, rather than starting again.
- **"Use an algebraic method"**: write the factorial forms and solve the equation you get; a trial on the calculator is not accepted.
- **Finish an equation question with one value**: state which root you reject and why, for example " $n = -16$ is not possible".

Quick check

1. Work out 7P_3 and 7C_3 . Which one counts the ways to give first, second and third prizes to 3 of 7 finalists, and which counts the ways to choose 3 of the 7 for a photograph?
2. Eight different postcards, including a beach card and a bridge card, are shared out so that Ana gets 4, Ben gets 3 and Chen gets 1. Find the number of ways this can be done if the beach card and the bridge card go to different people.
3. A 4-character door code is made from the letters P, Q, R, S, T and the digits 1 to 6, each used at most once. Find how many codes contain exactly two digits, with the two digits next to each other.
4. Given that ${}^{n+1}P_2 = 5 \times {}^{n-1}C_2$, where $n \geq 3$, find the value of n . Hence find the value of ${}^{n+3}P_3$.
5. A reading group chooses 5 books from 7 novels and 4 poetry books, all different. One particular poetry book must be chosen, and there must be more novels than poetry books. Find the number of choices.

Answers

- ${}^7P_3 = 7 \times 6 \times 5 = 210$ counts the prizes (order matters). ${}^7C_3 = \frac{210}{3!} = 35$ counts the photograph groups (order doesn't matter).
- With no condition: Ana's 4, then Ben's 3 from the 4 left, then Chen's 1: ${}^8C_4 \times {}^4C_3 \times {}^1C_1 = 70 \times 4 \times 1 = 280$. Subtract the shares where the two cards go to the same person. Both to Ana: she gets 2 more from the other 6, ${}^6C_2 = 15$, then Ben's 3 from the 4 left, ${}^4C_3 = 4$: 60. Both to Ben: Ana's 4 from the other 6, ${}^6C_4 = 15$, then Ben's third card from the 2 left: 2, so 30. Both to Chen is impossible (he gets 1). Answer $280 - 60 - 30 = 190$.
- The two digits fill places 1–2, 2–3 or 3–4: 3 ways. The digits in those places: $6 \times 5 = 30$. The letters in the other two places: $5 \times 4 = 20$. Total $3 \times 30 \times 20 = 1800$.
- ${}^{n+1}P_2 = \frac{(n+1)!}{(n-1)!} = (n+1)n$ and ${}^{n-1}C_2 = \frac{(n-1)(n-2)}{2}$. So $2n(n+1) = 5(n-1)(n-2)$, $2n^2 + 2n = 5n^2 - 15n + 10$, $3n^2 - 17n + 10 = 0$, $(3n-2)(n-5) = 0$. $n = \frac{2}{3}$ is not a whole number, so $n = 5$. (Check: ${}^6P_2 = 30 = 5 \times {}^4C_2 = 5 \times 6$.) Then ${}^8P_3 = 8 \times 7 \times 6 = 336$.
- The named book is in, so there is at least 1 poetry book; more novels than poetry books means 1 or 2 poetry books. Only the named one: ${}^7C_4 = 35$. The named one and 1 of the other 3: ${}^3C_1 \times {}^7C_3 = 3 \times 35 = 105$. Total $35 + 105 = 140$.

Past questions to try

- 0606/22/M/J/25 Q5:** numbers with 0 among the digits, even and multiple-of-5 cases, then a combinations equation.
- 0606/21/M/J/25 Q3:** kinds standing together, then at least one of each kind.
- 0606/22/O/N/24 Q7:** named people in or out of a group, and people a fixed distance apart.
- 0606/11/M/J/25 Q12:** proving an identity with combinations.