Cambridge IGCSE · Additional Mathematics 0606 · Notes

Quadratic functions

Read online at **pastopic.com/cambridge/additional-mathematics-0606/notes/quadratic-functions**

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Read first: **Functions**

What you need to know

By the end of this topic you should be able to:

1. **Find the maximum or minimum value of a quadratic** $f(x) = ax^2 + bx + c$, either by completing the square or by differentiating, and say at which x it happens.
2. **Use that maximum or minimum point** to sketch $y = f(x)$, and to write down the range of f for a given domain, in correct notation such as $f(x) \geqslant -5$.
3. **Use the discriminant** $b^2 - 4ac$ to decide whether $f(x) = 0$ has two real roots, two equal roots or no real roots, and use the same test to decide whether a given line crosses a given curve, is a tangent to it, or misses it.
4. **Solve quadratic equations for real roots** by factorising, by the quadratic formula or by completing the square, including equations with surds in their coefficients and equations that turn into quadratics.
5. **Solve quadratic inequalities**, from a graph or with algebra, and write the solution set in the correct form, such as $-2 < x < 5$, or $x < -1$ or $x > 3$.

Understand it

The shape of a quadratic graph

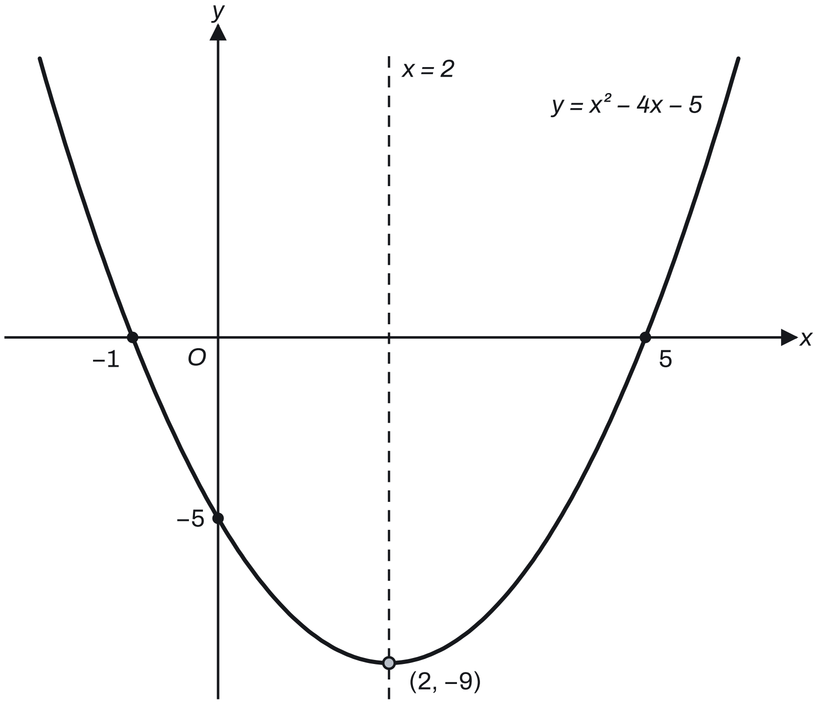
The graph of $y = ax^2 + bx + c$ is a **parabola**. If $a > 0$ it is U-shaped and has a lowest point (a **minimum**). If $a < 0$ it is ∩-shaped and has a highest point (a **maximum**). This turning point is called the **vertex** or **stationary point**. The parabola is symmetrical about the vertical line through the vertex.

Completing the square

Any quadratic can be rewritten as $a(x + p)^2 + q$. For $x^2 - 4x - 5$: halve the x-coefficient to get -2, so start with $(x - 2)^2 = x^2 - 4x + 4$. That has 4 too many, so

$$x^2 - 4x - 5 = (x - 2)^2 - 4 - 5 = (x - 2)^2 - 9.$$

Why this helps: $(x-2)^2$ is a square, so it is never negative, and it is 0 only when $x=2$. So the smallest value of $(x-2)^2-9$ is -9, at $x=2$. The **minimum point is** $(2,-9)$. In general, $a(x+p)^2+q$ has its vertex at $(-p,\ q)$: watch the sign of p.



When $a \neq 1$, take a out of the x^2 and x terms first, complete the square inside the bracket, then multiply back:

$$2x^2+12x+7=2\left(x^2+6x\right)+7=2\left[(x+3)^2-9\right]+7=2(x+3)^2-11.$$

Minimum -11 at $x=-3$.

When $a<0$ the square is subtracted, so the vertex is a maximum. For example, $5+4x-x^2=9-(x-2)^2$: the largest value is 9, when $x=2$, because you can only take away from 9.

Finding the vertex by differentiation

The vertex is where the gradient is 0. For $y=ax^2+bx+c$, $\frac{dy}{dx}=2ax+b=0$ gives $x=-\frac{b}{2a}$. For $y=x^2-4x-5$: $2x-4=0$, so $x=2$ and $y=-9$, as before. Use this only when the question lets you: after "Hence" following a completed square, the marks are for reading the vertex from that square.

The vertex is also halfway between the two roots, so if you know the roots, average them: -1 and 5 give $x=2$.

Sketching the graph

A sketch needs the right **shape** (U or ∩), the **vertex** in the right place, and every **intercept** labelled:

- the y**-intercept** is c (put $x=0$);
- the x**-intercepts** are the roots of $ax^2+bx+c=0$ (if there are any).

For $y = x^2 - 4x - 5 = (x - 5)(x + 1)$: U-shaped, vertex $(2, -9)$, crosses the y-axis at -5 and the x-axis at -1 and 5.

The range of a quadratic function

The **range** is the set of values $f(x)$ can take.

- **For all real** x, the range stops at the vertex. $f(x) = (x - 2)^2 - 9$ has range $f(x) \geqslant -9$; $g(x) = 9 - (x - 2)^2$ has range $g(x) \leqslant 9$. Use $\geqslant$ or $\leqslant$, since the vertex value is reached.
- **For a restricted domain**, sketch it and check the ends. For $f(x) = x^2 - 4x - 5$ with $0 \leqslant x \leqslant 3$, the vertex $x = 2$ lies inside the domain, so the least value is -9; the ends give $f(0) = -5$ and $f(3) = -8$, so the greatest is -5 and the range is $-9 \leqslant f(x) \leqslant -5$. With $3 \leqslant x \leqslant 6$ the vertex is outside, so just use the ends: $-8 \leqslant f(x) \leqslant 7$.

A quadratic on all real x is not one-one (it turns back at the vertex), so it has no inverse. Restricting the domain to one side of the vertex, such as $x \geqslant 2$, makes it one-one; see the Functions note.

Solving quadratic equations

Rearrange to $ax^2 + bx + c = 0$ first. Then:

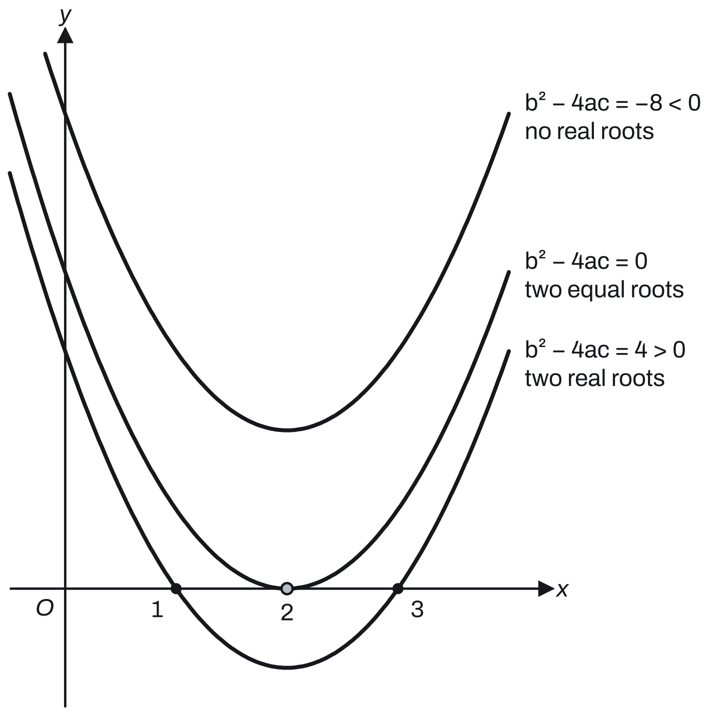
- **factorise** if you can see the factors: $x^2 - 4x - 5 = (x - 5)(x + 1) = 0$ gives $x = 5$ or $x = -1$;
- use the **quadratic formula** (it is on the List of formulas): $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$;
- or **complete the square**: $(x - 2)^2 - 9 = 0$ gives $x - 2 = \pm 3$, so $x = 5$ or -1. Remember the $\pm$.

The discriminant

In the formula, everything depends on the part under the square root, $b^2 - 4ac$, called the **discriminant**:

- $b^2 - 4ac > 0$: two different real roots (the graph crosses the x-axis twice);
- $b^2 - 4ac = 0$: two **equal** roots, one repeated root (the graph touches the x-axis at its vertex);
- $b^2 - 4ac < 0$: no real roots, because you can't square-root a negative number (the graph misses the x-axis).

"**Real roots**" covers the first two cases together, so it means $b^2 - 4ac \geqslant 0$.

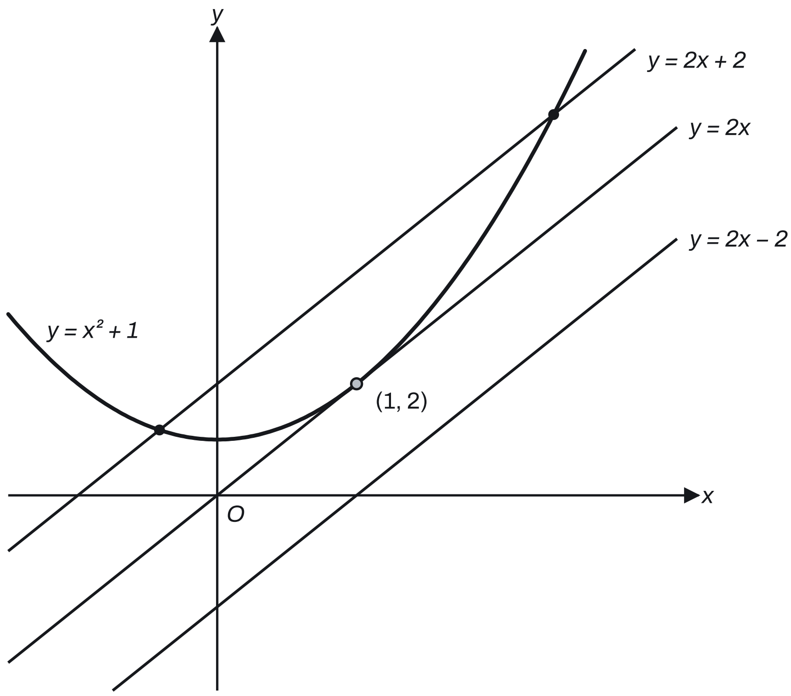


For example, $2x^2 + 3x + 5 = 0$ has $b^2 - 4ac = 9 - 40 = -31 < 0$, so it has no real roots.

A line and a curve

To find where the line $y = mx + k$ meets the curve $y = ax^2 + bx + c$, set the two expressions equal. That gives a quadratic, and each root is the x-coordinate of a meeting point. So the discriminant of **that** quadratic tells you how they meet:

- > 0: the line **crosses** the curve at two points;
- $= 0$: the line **touches** the curve at one point, so it is a **tangent**;
- < 0: the line **does not meet** the curve.



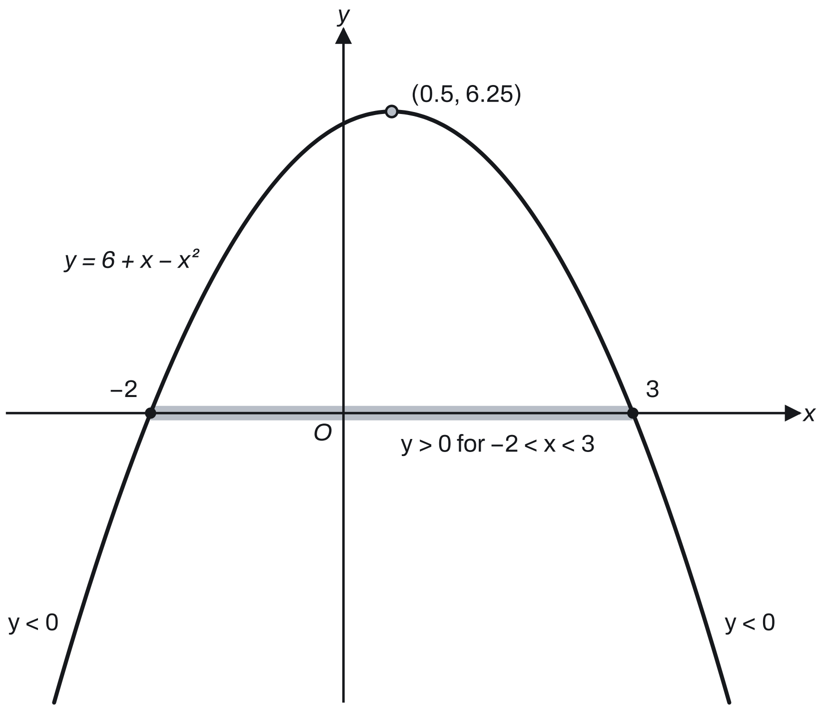
With $y = x^2 + 1$: the line $y = 2x$ gives $x^2 - 2x + 1 = 0$, discriminant $4 - 4 = 0$, so it is a tangent (at $x = 1$, the point $(1, 2)$). The line $y = 2x + 2$ gives $x^2 - 2x - 1 = 0$, discriminant $8 > 0$: two points. The line $y = 2x - 2$ gives $x^2 - 2x + 3 = 0$, discriminant $-8 < 0$: no meeting point.

The same works for a line and a circle: substitute the line into the circle's equation and look at the discriminant.

Quadratic inequalities

To solve $6 + x - x^2 > 0$, don't try to divide or square-root. Instead:

1. find the **critical values**, where it equals 0: $6 + x - x^2 = -(x + 2)(x - 3) = 0$ gives $x = -2$ and $x = 3$;
2. **sketch** the graph: $a = -1 < 0$, so it is ∩-shaped through -2 and 3;
3. **read off** where the graph is above the axis (> 0) or below it (< 0).



So $6+x-x^2>0$ for $-2<x<3$ (one piece, between the roots), and $6+x-x^2<0$ for $x<-2$ or $x>3$ (two pieces, outside the roots). For a U-shaped graph it is the other way round: "<0" is between the roots, ">0" is outside. Use $\leqslant$ or $\geqslant$ when the question has them.

Key facts and formulas

"Given" means it is in the List of formulas on page 2 of the exam paper. "Learn it" means you must remember it.

Formula	Status
Quadratic formula: $ax^2+bx+c=0 \Rightarrow x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	Given
$x^2+bx+c=\left(x+\frac{b}{2}\right)^2-\frac{b^2}{4}+c$	Learn it
$a(x+p)^2+q$ has vertex $(-p,q)$: a minimum if $a>0$, a maximum if $a<0$	Learn it
Vertex at $x=-\dfrac{b}{2a}$, from $\dfrac{dy}{dx}=2ax+b=0$	Learn it
Range for all real x: $f(x)\geqslant q$ if $a>0$; $f(x)\leqslant q$ if $a<0$	Learn it
$b^2-4ac>0$: two distinct real roots; $=0$: equal roots; <0: no real roots; $\geqslant 0$: real roots	Learn it
Line and curve: substitute, then discriminant >0 meets twice, $=0$ tangent, <0 does not meet	Learn it
$ax^2+bx+c>0$ for all x exactly when $a>0$ and $b^2-4ac<0$	Learn it
$(x-\alpha)(x-\beta)<0 \iff \alpha<x<\beta$; $(x-\alpha)(x-\beta)>0 \iff x<\alpha$ or $x>\beta$ (for $\alpha<\beta$)	Learn it

How to do it

In the Paper 1 and Paper 2 questions we have tagged for this topic (52 questions, 2021–2025), the types came up this often. 15 of the 52 questions mix types, so the counts add up to more than 52.

Question type	Questions
Completing the square: vertex, maximum or minimum, range	18
Discriminant and the nature of roots	13
Quadratic inequalities	12
Sketching and reading quadratic graphs	10
A line and a curve: tangent, meets, does not meet	9
Solving quadratic equations exactly	6

1. Completing the square: vertex, maximum or minimum, range (18 questions)

1. **Take out** a from the x^2 and x terms if $a \neq 1$.
2. **Halve the** x**-coefficient** inside the bracket, square the bracket, and subtract the extra constant.
3. **Multiply back** by a and collect the constants. Check by expanding.
4. **Read off** the vertex $(-p, q)$, whether it is a maximum or minimum, and the range.

For example, $3x^2 + 12x + 7 = 3\left[(x+2)^2 - 4\right] + 7 = 3(x+2)^2 - 5$, minimum point $(-2, -5)$. And $1 - 8x - 2x^2 = -2\left(x^2 + 4x\right) + 1 = -2\left[(x+2)^2 - 4\right] + 1 = 9 - 2(x+2)^2$, maximum value 9 at $x = -2$.

Variations you will meet:

- **The form is given**, such as $a(x+b)^2 + c$ or $a - b(x+c)^2$. Match it exactly and state a, b, c; "integers" or "rational numbers" tells you what to expect. You can also expand the form and compare coefficients.
- **"Hence write down the stationary point"**: read it from your square. No calculus: after "Hence" it earns nothing here.
- **"Find the range of** f**"** for all real x: $f(x) \geqslant q$ or $f(x) \leqslant q$. For a restricted domain, check the ends and whether the vertex is inside.
- **"State the least value of** p **so that** f **(with domain** $x \geqslant p$**) has an inverse"**: p is the x-coordinate of the vertex. The question may then ask for f^{-1} (see the Functions note).
- **"The minimum value of** y **is 3; find** c**"**: complete the square with c left in, then set the constant part equal to 3.
- **"Use your answer to solve…"**: set the completed square to 0 and square-root with $\pm$, keeping surds exact until the end. Sometimes the unknown is a power, such as $3u^2 + \ldots$ with $u = y^3$; solve for u first, then for y.

2. Discriminant and the nature of roots (13 questions)

1. **Rearrange** to $ax^2 + bx + c = 0$ and write down a, b and c (they may contain k).
2. **Choose the condition** from the wording:
   - two distinct (different) real roots: $b^2 - 4ac > 0$;
   - equal roots (repeated root): $b^2 - 4ac = 0$;
   - real roots: $b^2 - 4ac \geqslant 0$;
   - no real roots: $b^2 - 4ac < 0$;
   - curve completely above the x-axis, or y always positive: $b^2 - 4ac < 0$ (with $a > 0$).
3. **Simplify** $b^2 - 4ac$ to a quadratic in k (or a linear one) and find its **critical values**.
4. **Solve the inequality in** k with a sketch, as in type 3, and give the final answer as a set of values.

For example, $x^2 + kx + 9 = 0$ has equal roots when $k^2 - 36 = 0$, so $k = 6$ or $k = -6$. And $x^2 + kx + 4 > 0$ for all x when $k^2 - 16 < 0$, that is $-4 < k < 4$.

Variations you will meet:

- **"Show that the equation has no real roots"** (often "use algebra"): rearrange, find $b^2 - 4ac$, show it is negative, and **write the conclusion**: "$b^2 - 4ac < 0$, so there are no real roots". Completing the square also works: $2(x-1)^2 + 3$ is always at least 3, so it is never 0.
- **"Determine whether the equation has two distinct real roots, two equal roots or no real roots"**: tidy it into $ax^2 + bx + c = 0$ first (clear any fractions), then find the discriminant and state which case it is.
- **k in the x^2 term**, such as $kx^2 + 4x + (k-3) = 0$: work the same way. If the answer includes $k = 0$, the equation is no longer a quadratic there; check what happens (see Example 3).
- **"Show that it has distinct real roots for every k"**: show the discriminant is always positive, for example that it simplifies to a positive number, or to a completed square plus a positive number.

3. Quadratic inequalities (12 questions)

1. **Rearrange** so one side is 0: expand brackets and collect everything on one side. Never divide by something that could be negative.
2. **Find the critical values** by factorising or with the formula.
3. **Sketch** the parabola through them (U if the x^2 coefficient is positive, ∩ if negative).
4. **Read off** the answer: between the roots is **one** inequality, $\alpha < x < \beta$; outside is **two**, $x < \alpha$ or $x > \beta$. Keep $\leqslant/\geqslant$ if the question has them.

It is often safest to multiply through by -1 to make the x^2 coefficient positive, **reversing the sign**: $-2x^2 + 2x + 12 \geqslant 0$ becomes $x^2 - x - 6 \leqslant 0$, that is $(x-3)(x+2) \leqslant 0$, so $-2 \leqslant x \leqslant 3$.

Variations you will meet:

- **Already factorised**, such as $(x+3)(2x-7) \leqslant 0$: the critical values are -3 and $\frac{7}{2}$; it is U-shaped, so $-3 \leqslant x \leqslant \frac{7}{2}$.

- **Using a graph you have drawn**: find where the graph meets the horizontal line $y = k$ (by solving $f(x) = k$), then read off which parts of the graph are above or below the line.
- **Hidden quadratic inequalities**: a modulus inequality squared on both sides, or a fraction multiplied by a squared (always positive) denominator, leads to a quadratic inequality; finish in the same way.
- **"Hence find the area"** between the curve and the x-axis: the critical values are the limits of integration (see Integration).

4. Sketching and reading quadratic graphs (10 questions)

1. **Shape**: U if $a > 0$, ∩ if $a < 0$.
2. **Vertex**: from the completed square (or $x = -\frac{b}{2a}$).
3. **Intercepts**: y-intercept c; x-intercepts from solving $f(x) = 0$.
4. **Draw** a smooth symmetrical curve through them and **label every intercept** with its value. The vertex must be in the right quadrant.

Variations you will meet:

- **Number of roots of $f(x) = k$**: draw the horizontal line $y = k$ and count crossings. For $y = x^2 - 4x - 5$ (vertex $(2, -9)$), $f(x) = k$ has two roots for $k > -9$ and one (repeated) root for $k = -9$.
- **The modulus graph $y = |f(x)|$**: reflect the part below the x-axis upwards, so the vertex $(2, -9)$ becomes a top point $(2, 9)$ with sharp corners on the x-axis at -1 and 5. Then $|f(x)| = k$ has **4** roots for $0 < k < 9$, **3** for $k = 9$, and **2** for $k > 9$ or $k = 0$.
- **Finding the quadratic from its graph**: roots 1 and 4 mean $y = a(x-1)(x-4)$; the y-intercept 8 gives $4a = 8$, so $a = 2$ and $y = 2x^2 - 10x + 8$. If a graph could be a U or a ∩ through the same points, both are possible answers.
- **Sketching f and f^{-1} together**: sketch the allowed part of the parabola and reflect it in $y = x$; the intercepts swap over.

5. A line and a curve: tangent, meets, does not meet (9 questions)

1. **Eliminate y**: set the line equal to the curve (or substitute the line into a circle's equation).
2. **Rearrange** to $Ax^2 + Bx + C = 0$, where A, B, C may contain k.
3. **Use the discriminant**: $= 0$ for a tangent, > 0 to meet at two distinct points, $\geqslant 0$ to meet at all, < 0 to not meet ("does not touch or cut").
4. **Solve for k**, giving exact values (surds) when asked.
5. If asked for the **point of contact**, put your k back into the quadratic (it has a repeated root $x = -\frac{B}{2A}$) and find y from the line.

Variations you will meet:

- **"Find the non-zero value of k"**: the quadratic in k has two roots, one of them 0; give the other.

- **A line and a circle** (from the Coordinate geometry of the circle topic): substitute the line into the circle's equation, expand, and use the discriminant in exactly the same way. If the line is $x+3y=k$, it is easier to substitute $x=k-3y$ and get a quadratic in y.
- **Equating gradients** also works for a tangent, but examiners found the discriminant method more reliable.

6. Solving quadratic equations exactly (6 questions)

1. **Surd coefficients, no calculator**: use the quadratic formula with exact values. Simplify the discriminant (it usually comes out as a perfect square or a simple surd), then **rationalise** each root to get the form $a+b\sqrt{n}$, showing every step.
2. **Hidden quadratics**: spot a power that is the square of another, put u equal to the smaller one, solve for u, then go back to x.

For example, $x^2-2\sqrt{5}\,x+1=0$: $b^2-4ac=20-4=16$, so $x=\dfrac{2\sqrt{5}\pm 4}{2}=\sqrt{5}\pm 2$. And $x^{2/3}+2x^{1/3}-8=0$ with $u=x^{1/3}$ is $u^2+2u-8=(u+4)(u-2)=0$, so $u=2$ or $u=-4$, giving $x=8$ or $x=-64$.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

The function f is defined by $f(x)=2x^2-12x+13$.

(a) Write $f(x)$ in the form $a(x+b)^2+c$, where a, b and c are integers. **[3]**

(b) Hence write down the coordinates of the stationary point on the curve $y=f(x)$, and state whether it is a maximum or a minimum. **[2]**

(c) Sketch $y=f(x)$, stating the exact coordinates of the points where it meets the axes. **[3]**

(d) Find the range of f when the domain is $0\leqslant x\leqslant 5$. **[2]**

(a) $2x^2-12x+13=2\left(x^2-6x\right)+13=2\left[(x-3)^2-9\right]+13=2(x-3)^2-5$.

B1 for $a=2$, **B1** for $(x-3)^2$, **B1** for $c=-5$.

(b) $(3, -5)$, a minimum because $a = 2 > 0$. **B1** for each coordinate (read from part (a), no calculus).

(c) y-intercept: $f(0) = 13$, so $(0, 13)$. **B1**

x-intercepts: $2(x-3)^2 = 5$, so $x - 3 = \pm\sqrt{\frac{5}{2}} = \pm\frac{\sqrt{10}}{2}$ and $x = 3 \pm \frac{\sqrt{10}}{2}$ (about 1.42 and 4.58). **B1**

A U-shaped curve with its minimum at $(3, -5)$ in the fourth quadrant, crossing the positive x-axis twice. **B1** for the shape with the vertex in the right place.

(d) The vertex $x = 3$ is inside $0 \leqslant x \leqslant 5$, so the least value is -5. At the ends, $f(0) = 13$ and $f(5) = 2(2)^2 - 5 = 3$, so the greatest is 13. **M1** for checking the vertex and both ends.

$-5 \leqslant f(x) \leqslant 13$. **A1**

Example 2

Solve the inequality $(2x-1)(x+3) > 3x + 1$. **[4]**

Expand and collect on one side: $2x^2 + 5x - 3 - 3x - 1 > 0$, so $2x^2 + 2x - 4 > 0$. **M1** for a 3-term quadratic compared with 0.

$2(x+2)(x-1) > 0$: critical values $x = -2$ and $x = 1$. **M1** for factorising, **A1** for both critical values.

The graph is U-shaped, so it is above the axis outside the roots: $x < -2$ or $x > 1$. **A1**

Example 3

Find the set of values of k for which the equation $kx^2 + 4x + (k-3) = 0$ has two distinct real roots. **[5]**

$a = k$, $b = 4$, $c = k - 3$. Two distinct real roots need $b^2 - 4ac > 0$:

$$16 - 4k(k-3) > 0 \;\Rightarrow\; 16 - 4k^2 + 12k > 0 \;\Rightarrow\; k^2 - 3k - 4 < 0.$$

M1 for $b^2 - 4ac$ with the right a, b, c; **A1** for a correct 3-term quadratic in k (note the sign flip on dividing by -4).

$(k-4)(k+1) < 0$: critical values 4 and -1. **M1 A1**

Between the roots: $-1 < k < 4$. But when $k = 0$ the equation is $4x - 3 = 0$, which has only one root, so leave it out:

$$-1 < k < 0 \quad \text{or} \quad 0 < k < 4.$$

A1

Example 4

The line $y = mx - 1$ is a tangent to the curve $y = x^2 + 3x + 3$. Find the possible values of m and, for each one, the coordinates of the point where the line touches the curve. **[6]**

Equate: $x^2 + 3x + 3 = mx - 1$, so $x^2 + (3 - m)x + 4 = 0$. **M1** for equating and rearranging to a 3-term quadratic.

For a tangent, $b^2 - 4ac = 0$: $(3 - m)^2 - 16 = 0$. **M1 A1**

$3 - m = \pm 4$, so $m = -1$ or $m = 7$. **A1**

$m = 7$: $x^2 - 4x + 4 = (x - 2)^2 = 0$, so $x = 2$ and $y = 7(2) - 1 = 13$. The point is $(2, 13)$.

$m = -1$: $x^2 + 4x + 4 = (x + 2)^2 = 0$, so $x = -2$ and $y = -(-2) - 1 = 1$. The point is $(-2, 1)$. **M1** for putting m back and solving, **A1** for both points.

Check on the curve: $2^2 + 6 + 3 = 13$ and $4 - 6 + 3 = 1$.

Common mistakes

- **Using calculus after "Hence".** When the question says "Hence write down the stationary point" after completing the square, examiners give no credit for differentiating. Read the vertex straight from $a(x + p)^2 + q$.
- **Getting the vertex sign wrong.** $(x - 3)^2 - 5$ has its vertex at $x = 3$, not $x = -3$.
- **Completing the square with $a \neq 1$ without taking a out**, or forgetting to multiply the subtracted constant by a. Expand your answer to check it.
- **Not giving the form asked for.** Solving the equation or using calculus to find the vertex, but never writing $a(x + b)^2 + c$, loses the marks for that part.
- **Choosing the wrong discriminant condition.** "Equal roots" means $b^2 - 4ac = 0$, not $\geqslant 0$. "Real roots" includes equal roots, so it is $\geqslant 0$; answers that treat it as "> 0", or pick the region between the critical values when it should be outside, are common.
- **Stopping at the critical values.** Finding $k = 2$ and $k = -2.5$ is not the answer to "find the set of values of k". Finish with the inequality, using a sketch to choose between or outside.
- **Writing the final inequality badly.** "$x \leqslant 1.5, x \leqslant 7$" or "$-2 \geqslant x \geqslant 3$" are wrong. Between the roots is one statement, $1.5 \leqslant x \leqslant 7$; outside is two, joined by "or".
- **Slips with the constant term.** When moving terms across, collect every constant, including those containing k, before writing c. Many errors come from one lost term.
- **No conclusion in a "show that" about roots.** A negative discriminant is not enough on its own: write "so the equation has no real roots". Saying there is a "negative root" is wrong: a negative number can be a perfectly good root.
- **Missing working in no-calculator questions.** With surds, show the discriminant simplified and each rationalising step, or the method marks can't be given.
- **Sketches without intercepts, or with the wrong shape.** Mark every intercept with its value and put the vertex in the right quadrant; with a modulus graph, the reflected part must have sharp corners on the x-axis, not a smooth minimum.
- **Mixing up domain and range.** The range of $f(x) = (x + 2)^2 + 1$ is $f(x) \geqslant 1$ (with $\geqslant$, not $>$), not a statement about x.

How to get the marks

- **Discriminant questions** are marked in a set pattern: a method mark for writing $b^2 - 4ac$ with your a, b and c (with any sign, $>$, $=$ or $<$), an accuracy mark for simplifying it to a correct quadratic in k, a method mark for solving that quadratic, then the critical values and the final answer. Write each stage down.
- **Line and curve**: show the line and curve set equal and rearranged to $\ldots = 0$ before using the discriminant.
- **"Find the set of values"** or **"range of values"** wants an inequality, not two numbers. The final answer is what is marked, so write it clearly on its own line.
- **"Hence"** means use the previous part; **"write down"** means little or no working is expected, so the answer should come straight from what you have.
- **"Show that"** or **"Use algebra to show"**: every step visible, and a clear final statement.
- **"Exact"** or **"simplest surd form"**: keep surds, for example $k > 2 + 2\sqrt{3}$, not $k > 5.46$. Otherwise give non-exact answers to 3 significant figures.
- **"DO NOT USE A CALCULATOR"**: show the quadratic formula with numbers in, the simplified discriminant, and each rationalising step.
- **Sketches**: the marks are for the correct shape, the vertex in the right place and each intercept labelled. Draw it large and smooth.
- **Follow-through**: a wrong completed square used correctly for the vertex or range usually still earns the later marks, so keep going.

Quick check

1. Write $5 + 6x - x^2$ in the form $a - (x + b)^2$. Hence state the maximum value of $5 + 6x - x^2$, the value of x where it occurs, and the range of $f(x) = 5 + 6x - x^2$ for all real x.
2. Find the values of k for which $x^2 + (k + 1)x + 4 = 0$ has equal roots.
3. Use algebra to show that the line $y = 3x - 5$ does not meet the curve $y = x^2 - x + 1$.
4. Without a calculator, solve $\sqrt{2}\,x^2 + 3x - 2\sqrt{2} = 0$, giving your answers in the form $a + b\sqrt{2}$.
5. Solve $2x - 7\sqrt{x} + 3 = 0$.

Answers

1. $5+6x-x^2=5-\left[(x-3)^2-9\right]=14-(x-3)^2$, so $a=14, b=-3$. The maximum value is 14, when $x=3$, and the range is $f(x)\leqslant 14$.
2. $b^2-4ac=(k+1)^2-16=0$, so $k+1=\pm 4$ and $k=3$ or $k=-5$.
3. $x^2-x+1=3x-5$ gives $x^2-4x+6=0$. $b^2-4ac=16-24=-8<0$, so there are no real roots and the line does not meet the curve.
4. $a=\sqrt{2}, b=3, c=-2\sqrt{2}$, so $b^2-4ac=9+4(\sqrt{2})(2\sqrt{2})=9+16=25$ and $x=\dfrac{-3\pm 5}{2\sqrt{2}}$. With $+$: $x=\dfrac{2}{2\sqrt{2}}=\dfrac{1}{\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$. With $-$: $x=\dfrac{-8}{2\sqrt{2}}=\dfrac{-4}{\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}=-2\sqrt{2}$. So $x=0+\frac{1}{2}\sqrt{2}$ or $x=0-2\sqrt{2}$.
5. Let $u=\sqrt{x}$: $2u^2-7u+3=(2u-1)(u-3)=0$, so $u=\frac{1}{2}$ or $u=3$. Then $x=\frac{1}{4}$ or $x=9$.

Past questions to try

- **0606/11/M/J/23 Q1**: completing the square, the stationary point, a sketch and roots of a modulus equation.
- **0606/23/O/N/23 Q2**: the discriminant for a curve completely above the x-axis.
- **0606/22/O/N/23 Q2**: a line that is a tangent to a curve.
- **0606/21/O/N/23 Q8**: solving a quadratic with surd coefficients without a calculator.