Cambridge IGCSE · Additional Mathematics 0606 · Notes

Straight-line graphs

Read online at **pastopic.com/cambridge/additional-mathematics-0606/notes/straight-line-graphs**

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Read first: **Simultaneous equations**, **Logarithmic and exponential functions**

What you need to know

By the end of this topic you should be able to:

1. **Use the equation of a straight line**: find its gradient, write its equation in any of the usual forms, and find where two lines (or a line and an axis) meet.
2. **Use the conditions for parallel and perpendicular lines**: parallel lines have equal gradients; perpendicular gradients multiply to -1.
3. **Solve problems with midpoints and lengths**, including finding the **perpendicular bisector** of a line segment and using it, for example to find a point on it or the area of a triangle.
4. **Change a relationship into straight-line form and back.** Rewrite a relationship such as $y = Ax^n$ or $y = Ab^x$ as $Y = mX + c$, so that plotting Y against X gives a straight line; then find the unknown constants from the gradient or intercept of that line. Going the other way, turn a straight line of, say, y^2 against x^3, or e^y against x, or y against $\ln x$, back into an equation for y in terms of x.

The equation of a circle belongs to the next topic, and gradients of curves belong to differentiation; this note only covers the straight-line work those questions use.

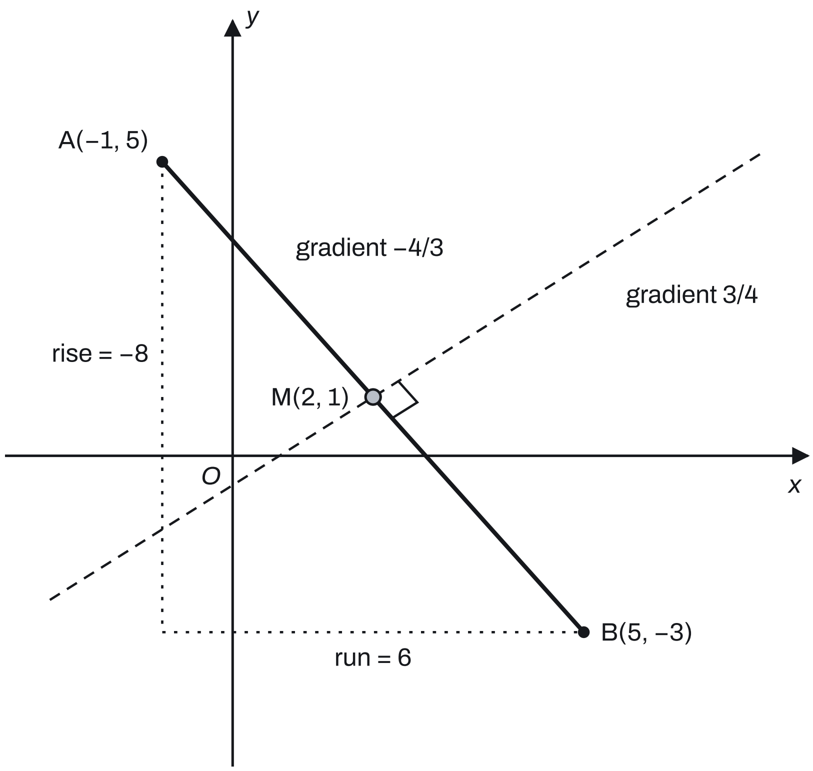
Understand it

Gradient

The gradient of a line says how far it goes up for every 1 across. Between two points (x_1, y_1) and (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}.$$

A positive gradient goes up from left to right, a negative one goes down. A horizontal line $y = k$ has gradient 0; a vertical line $x = k$ has no gradient (the run is 0).



For $A(-1, 5)$ and $B(5, -3)$: $m = \dfrac{-3-5}{5-(-1)} = \dfrac{-8}{6} = -\dfrac{4}{3}$.

The equation of a line

Three forms, all the same line written differently:

- $y = mx + c$: gradient m, crosses the y-axis at $(0, c)$.
- $y - y_1 = m(x - x_1)$: gradient m, through the point (x_1, y_1). This is the quickest form when you know a point and a gradient.
- $ax + by + c = 0$: questions often ask for this with integers. To read the gradient, rearrange to $y = \ldots$: the gradient is $-\frac{a}{b}$.

A point lies on a line exactly when its coordinates satisfy the equation. Two lines meet where both equations are true, so solve them as simultaneous equations. A line meets the y-axis where $x = 0$ and the x-axis where $y = 0$.

Parallel and perpendicular lines

- **Parallel** lines have the **same gradient**: $y = 2x + 1$ and $4x - 2y = 7$ (which is $y = 2x - 3.5$) are parallel.
- **Perpendicular** lines have gradients that **multiply to** -1: $m_1 m_2 = -1$, so $m_2 = -\dfrac{1}{m_1}$. Flip the fraction and change the sign. $x + 2y = 6$ has gradient $-\frac{1}{2}$, so it is perpendicular to $y = 2x + 1$.

Midpoint and length

The midpoint is the average of the ends, and the length comes from Pythagoras, using the run and the rise as the two shorter sides:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right), \qquad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

For $A(-1, 5)$ and $B(5, -3)$: $M = (2, 1)$ and $AB = \sqrt{6^2 + (-8)^2} = \sqrt{100} = 10$.

Points in between. To go a fraction of the way from A to B, add that fraction of the step from A to B. A third of the way: $(-1, 5) + \frac{1}{3}(6, -8) = \left(1, \frac{7}{3}\right)$. The midpoint is the special case of a half.

The perpendicular bisector

The **perpendicular bisector** of AB is the line that cuts AB in half at right angles. It passes through the **midpoint** and has the **perpendicular gradient**. Every point on it is the same distance from A as from B, and every point that is the same distance from both lies on it.

For $A(-1, 5)$ and $B(5, -3)$: midpoint $(2, 1)$, perpendicular gradient $-1 \div \left(-\frac{4}{3}\right) = \frac{3}{4}$, so

$$y - 1 = \frac{3}{4}(x - 2) \;\Rightarrow\; 4y - 4 = 3x - 6 \;\Rightarrow\; 3x - 4y - 2 = 0.$$

The same line comes from the "equal distances" idea: $(x + 1)^2 + (y - 5)^2 = (x - 5)^2 + (y + 3)^2$. The x^2 and y^2 cancel, leaving $12x - 16y - 8 = 0$, which is $3x - 4y - 2 = 0$ again.

Moving along the bisector. If M is the midpoint and C is on the bisector, the step from M to C runs along the bisector. A point twice as far from AB as C is $M \pm 2(C - M)$ (one on each side). The reflection of C in AB is $M - (C - M) = 2M - C$, because M is then the midpoint of C and its reflection.

Areas

- A triangle with one side along an axis (or two sides at right angles): use $\frac{1}{2} \times \text{base} \times \text{height}$, with lengths read from the coordinates.
- Any triangle or polygon with known corners: list the corners in order round the shape, repeat the first at the end, then

  $$\text{Area} = \frac{1}{2}\left|(x_1y_2 + x_2y_3 + x_3y_1) - (y_1x_2 + y_2x_3 + y_3x_1)\right|.$$

  For $(1, 1)$, $(7, 3)$, $(2, 6)$: $\frac{1}{2}\left|(3 + 42 + 2) - (7 + 6 + 6)\right| = \frac{1}{2} \times 28 = 14$.

Turning a curve into a straight line

Some relationships between x and y give curves, which are hard to read accurately. The trick is to rewrite the relationship as

$$Y = mX + c,$$

where Y and X are new quantities made from y and x (such as $\ln y$ and $\ln x$) and m and c are constants. Plotting Y against X then gives a straight line, and its gradient and intercept tell you the unknown constants. The vertical axis is always Y, and Y is the one you write first: "plot Y against X".

Powers and exponentials need logarithms (see the logarithms note). Take $\ln$ (or $\lg$) of both sides and use the laws:

- $y = Ax^n \ \Rightarrow\ \ln y = n \ln x + \ln A$. Plot $\ln y$ against $\ln x$: gradient n, intercept $\ln A$.
- $y = Ab^x \ \Rightarrow\ \ln y = x \ln b + \ln A$. Plot $\ln y$ against x: gradient $\ln b$, intercept $\ln A$.
- $y = Ae^{kx} \ \Rightarrow\ \ln y = kx + \ln A$. Plot $\ln y$ against x: gradient k, intercept $\ln A$.

Other forms just need rearranging so one side is "something $\times$ a constant, plus a constant":

- $y = \dfrac{a}{x} + bx$: multiply by x to get $xy = bx^2 + a$. Plot xy against x^2: gradient b, intercept a.
- $y^2 = Ax^3 + B$ is already in straight-line form: plot y^2 against x^3.

$\lg$ works exactly like $\ln$: with $\lg y$ the intercept is $\lg A$, so $A = 10^{\text{intercept}}$.

Turning a straight line back into an equation

Now you are told the line and have to find y. Write the line with the **real axis labels**, find m and c as for any line, then undo the transformation.

- $\ln y = 0.4x + 1.1$: $y = e^{0.4x+1.1} = e^{1.1} \times \left(e^{0.4}\right)^x = 3.00 \times 1.49^x$.
- $\lg y = 2 \lg x + 0.5$: $y = 10^{0.5} x^2 = 3.16x^2$.
- $e^y = 12 - 3x$: $y = \ln(12 - 3x)$. A logarithm needs a positive input, so this is only **valid** for $12 - 3x > 0$, that is $x < 4$.

Key facts and formulas

None of these is in the 0606 list of formulas, so every one must be learnt.

Formula	Status
Gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$	Learn it
$y = mx + c$; $y - y_1 = m(x - x_1)$; $ax + by + c = 0$ has gradient $-\frac{a}{b}$	Learn it
Parallel: $m_1 = m_2$	Learn it
Perpendicular: $m_1 m_2 = -1$, so $m_2 = -\dfrac{1}{m_1}$	Learn it
Midpoint $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$	Learn it
Length $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Learn it

Formula	Status
Triangle area $\frac12\left	(x_1y_2 + x_2y_3 + x_3y_1) - (y_1x_2 + y_2x_3 + y_3x_1)\right
$y = Ax^n \iff \ln y = n \ln x + \ln A$	Learn it
$y = Ab^x \iff \ln y = x \ln b + \ln A$; $y = Ae^{kx} \iff \ln y = kx + \ln A$	Learn it
Straight-line form $Y = mX + c$: $m = \dfrac{Y_2 - Y_1}{X_2 - X_1}$, c = value of Y when $X = 0$	Learn it

How to do it

In the Paper 1 and Paper 2 questions we have tagged for this topic (51 questions, 2021–2025), the types came up this often. Each question is counted once, under its main type.

Question type	Questions
A transformed straight line is given: find the relationship	19
Perpendicular bisector problems	16
Lines, midpoints, lengths and areas	9
Drawing a straight-line graph from data	7

1. A transformed straight line is given: find the relationship (19 questions)

You are told "when Y is plotted against X, a straight line is obtained", with two points on it (or one point and the gradient).

1. **Write the line with the real labels:** $Y = mX + c$, for example $\ln y = mx + c$ or $e^y = mx + c$. This line is often a mark on its own.
2. **Find the gradient** from the two points: $m = \dfrac{Y_2 - Y_1}{X_2 - X_1}$. The points are (X, Y) values, **not** (x, y) values: in $(2, 5)$ on a graph of $\ln y$ against x, $\ln y = 5$, not $y = 5$.
3. **Find** c by putting one point into $Y = mX + c$ (or solve two simultaneous equations).
4. **Rearrange for** y, undoing one layer at a time:
   - $\ln y = \ldots$ gives $y = e^{\cdots}$; $\lg y = \ldots$ gives $y = 10^{\cdots}$;
   - $e^y = \ldots$ gives $y = \ln(\ldots)$; $e^{ky} = \ldots$ gives $y = \frac{1}{k}\ln(\ldots)$;
   - $\ln(y + a) = \ldots$ gives $y = e^{\cdots} - a$; $\lg(py + q) = \ldots$ gives $y = \dfrac{10^{\cdots} - q}{p}$;
   - $\sqrt{y} = \ldots$ gives $y = (\ldots)^2$: square the **whole** right-hand side, never term by term.

Variations you will meet:

- **"Show that $y = Ab^x$" or "$y = Ax^n$", "where A and b are integers"**: after finding m and c (often logarithms such as $\ln 5$), use $e^c = A$ and $e^m = b$ and give the final line in exactly the form printed, with $\times$ between the constants.
- **The gradient is given instead of a second point**: skip step 2.
- **X is not just x**: $\ln x$, $\lg(x+1)$, x^2 or $\sqrt{x}$. Put the coordinate in as that quantity, and undo it at the end.
- **"Find the values of x for which y exists (is valid, can be defined)"**: whatever is inside $\ln$ or $\lg$ must be **positive**. For $y = \ln(15 - 5x)$, solve $15 - 5x > 0$, so $x < 3$. If X is x^2, the inequality is quadratic, for example $20 - 5x^2 > 0$ gives $-2 < x < 2$.
- **Use the equation**: find y for a given x (substitute, keeping brackets), or x for a given y (solve, often by taking logarithms or powers).
- **"Show that a graph of $\lg P$ against T is a straight line"**: take $\lg$ of the given relationship and write it as $\lg P = (\ldots)T + (\ldots)$, naming the gradient and intercept.

2. Perpendicular bisector problems (16 questions)

1. **Get the two end points.** If they are where a line meets a curve, substitute the line into the curve, solve the quadratic, and find both points (see the simultaneous equations note).
2. **Midpoint** of the two points.
3. **Gradient** of the segment, then the **perpendicular gradient** $-\frac{1}{m}$.
4. **Equation** through the **midpoint** with the perpendicular gradient: $y - y_M = m_\perp(x - x_M)$. Give the form asked for.

Variations you will meet:

- **A point on the bisector with one unknown coordinate**, such as $(a, -8)$ or $(2, k)$: substitute it into the bisector's equation and solve.
- **Where the bisector meets something**: an axis (put $x = 0$ or $y = 0$), or another line (solve together). Then find an area, often of a triangle with the origin (base $\times$ height $\div 2$) or of a triangle with three known corners (the area formula).
- **A point twice (or three times) as far from AB as C**, on the bisector: $D = M \pm 2(C - M)$, two answers.
- **The reflection of C in AB**: $2M - C$. Or, if C is not on the bisector, find the foot F of the perpendicular from C to AB and use $2F - C$.
- **"Such that AB bisects CD"**: the midpoint of CD is on AB; when C and D are both on the bisector, it is M, so $D = 2M - C$.
- **A diagonal of a square or rhombus**: the diagonals bisect each other at right angles, so one diagonal is the perpendicular bisector of the other.

A quick sketch showing A, B, M and the other points stops you mixing up which line is which.

3. Lines, midpoints, lengths and areas (9 questions)

1. **Gradient** from two points; **equation** from a point and a gradient.

2. **Parallel**: same gradient. **Perpendicular**: $-\frac{1}{m}$. Read the gradient of $ax + by = c$ by rearranging to $y = \ldots$.
3. **Intersection**: solve the two equations together.
4. **Length, midpoint, area** as needed.

Variations you will meet:

- **A line through the midpoint of AB, parallel (or perpendicular) to a given line**: midpoint first, then the given line's gradient.
- **B is the midpoint of AC, find C**: $C = 2B - A$ (add the step from A to B again).
- **A point dividing AB in a ratio**, such as $AC : CB = 1 : 2$: C is $\frac{1}{3}$ of the way from A, so $C = A + \frac{1}{3}(B - A)$. A line through C perpendicular to AB goes through C, **not** the midpoint.
- **A point on a line at a given distance**: write the point with one unknown (using the line's equation), set the length (or length squared) equal to the value, and solve the quadratic: two answers.
- **Distance from the origin** or between two points: the length formula; give exact surds if asked.
- **Three points and right angles**: two gradients that multiply to -1 show a right angle.

4. Drawing a straight-line graph from data (7 questions)

You get a table of x and y and a relationship such as $y = Ax^b$ or $y = Ab^x$.

1. **Take logarithms of the relationship** to find what to plot: for $y = Ax^b$, $\ln y = b \ln x + \ln A$; for $y = Ab^x$ or $y = Ae^{kx}$, $\ln y$ against x (or x^2 if the power is x^2).
2. **Make a new table** of X and Y values, to 2 decimal places.
3. **Plot** them accurately on the grid given and draw **one ruled straight line of best fit**.
4. **Gradient**: pick two points **on your line**, far apart (plotted points that lie on it are fine), and use $\frac{\text{rise}}{\text{run}}$.
5. **Intercept**: read where the line meets the vertical axis (only if that axis is at $X = 0$), or put a point into $Y = mX + c$.
6. **Match** to the log form: for $y = Ax^b$, gradient $= b$ and intercept $= \ln A$, so $A = e^{\text{intercept}}$. For $y = Ab^x$, $b = e^{\text{gradient}}$.
7. **"Use your graph to estimate"**: convert the given value ($\ln y$ for a given y, say), read the other coordinate from **your line**, then convert back.

Give answers to the accuracy asked (often 1 or 2 significant figures), and don't solve with the original data values when the question says "use your graph".

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

The line l_1 has equation $2x + 3y = 12$. The line l_2 passes through the point $(-1, -4)$ and is perpendicular to l_1.

(a) Find the equation of l_2, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. **[3]**

(b) Find the coordinates of P, the point where l_1 and l_2 meet. **[2]**

(c) Find the area of the triangle bounded by l_1, l_2 and the y-axis. **[3]**

(a) $3y = -2x + 12$, so l_1 has gradient $-\frac{2}{3}$. **B1**

Perpendicular gradient $= \frac{3}{2}$: $y + 4 = \frac{3}{2}(x + 1)$. **M1** for using $-\frac{1}{m}$ with the point.

$2y + 8 = 3x + 3$, so $3x - 2y - 5 = 0$. **A1**

(b) From (a), $y = \frac{3x-5}{2}$. Substituting into l_1: $2x + \frac{3(3x-5)}{2} = 12$, so $4x + 9x - 15 = 24$ and $13x = 39$. So $x = 3$ and $y = 2$. **M1 A1**: $P(3, 2)$.

(c) l_1 meets the y-axis at $(0, 4)$ and l_2 at $\left(0, -\frac{5}{2}\right)$. **B1** for both.

The base along the y-axis is $4 - \left(-\frac{5}{2}\right) = \frac{13}{2}$, and the height is the distance of P from the y-axis, 3. **M1**

Area $= \frac{1}{2} \times \frac{13}{2} \times 3 = \frac{39}{4} = 9.75$. **A1**

Example 2

The points $A(-4, 1)$ and $B(2, 7)$ are fixed. The point $Q(x, y)$ moves so that it is always the same distance from A as from B.

(a) Show that Q always lies on the line $x + y = 3$. **[3]**

(b) Find the coordinates of the point on this line that is closest to $C(6, 5)$, and find the shortest distance from C to the line, giving your answer as an exact surd. **[5]**

(a) $QA = QB$, so $QA^2 = QB^2$: **M1**

$$(x+4)^2 + (y-1)^2 = (x-2)^2 + (y-7)^2.$$

Expanding, $x^2 + 8x + 16 + y^2 - 2y + 1 = x^2 - 4x + 4 + y^2 - 14y + 49$. **M1** for expanding correctly.

$12x + 12y = 36$, so $x + y = 3$. **A1**

This is the perpendicular bisector of AB, as it should be: the midpoint $(-1, 4)$ satisfies $-1 + 4 = 3$, and AB has gradient 1 while the line has gradient -1.

(b) The shortest distance is along the perpendicular from C to the line. The line has gradient -1, so the perpendicular has gradient 1: $y - 5 = x - 6$, that is $y = x - 1$. **M1**

Solving with $x + y = 3$: $x + x - 1 = 3$, so $x = 2$, $y = 1$. **M1 A1**: the closest point is $(2, 1)$.

Distance $= \sqrt{(6-2)^2 + (5-1)^2} = \sqrt{32} = 4\sqrt{2}$. **M1 A1**

Example 3

Variables x and y are such that when xy is plotted against x^2, a straight line passing through the points $(1, 7)$ and $(4, 1)$ is obtained.

(a) Find y in terms of x. **[4]**

(b) Find the positive value of x for which $y = 3$. **[3]**

(a) $xy = mx^2 + c$. **B1** for the line with the right labels.

$m = \dfrac{1-7}{4-1} = -2$. **M1**

$7 = -2(1) + c$, so $c = 9$. **M1** for using a point to find c.

$xy = 9 - 2x^2$, so $y = \dfrac{9}{x} - 2x$. **A1**

(b) $\dfrac{9}{x} - 2x = 3$. Multiplying by x: $9 - 2x^2 = 3x$, so $2x^2 + 3x - 9 = 0$. **M1**

$(2x-3)(x+3) = 0$, so $x = \frac{3}{2}$ or $x = -3$. **M1** for solving the quadratic.

The positive value is $x = 1.5$. **A1**

Example 4

The table shows values of P measured at times t.

t	**0**	**2**	**4**	**6**	**10**
P	113	320	588	905	1660

It is thought that $P = A(t+2)^n$, where A and n are constants.

(a) Explain why the graph of $\ln P$ against $\ln(t+2)$ should be a straight line, and state what its gradient and intercept represent. **[2]**

(b) Draw the graph of $\ln P$ against $\ln(t+2)$. **[3]**

(c) Use your graph to estimate n and A, each to 2 significant figures. **[4]**

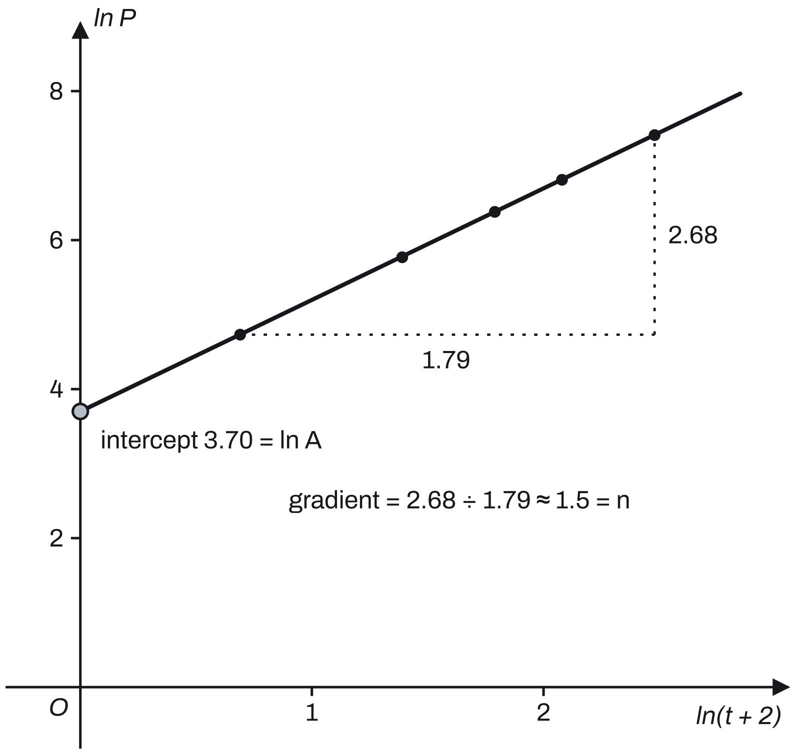
(d) Use your graph to estimate the value of t when $P = 1000$. **[2]**

(a) Taking $\ln$ of both sides: $\ln P = \ln A + \ln(t+2)^n = n\ln(t+2) + \ln A$. **M1** This has the form $Y = mX + c$ with $Y = \ln P$ and $X = \ln(t+2)$, so the graph is a straight line with gradient n and intercept $\ln A$. **A1**

(b) The new table:

$\ln(t+2)$	**0.69**	**1.39**	**1.79**	**2.08**	**2.48**
$\ln P$	4.73	5.77	6.38	6.81	7.41

B1 for the table, **B1** for plotting the points, **B1** for one ruled straight line through them.



(c) Using the points $(0.69, 4.73)$ and $(2.48, 7.41)$ on the line:

$$n = \text{gradient} = \frac{7.41 - 4.73}{2.48 - 0.69} = \frac{2.68}{1.79} = 1.50 = 1.5 \text{ (2 s.f.)}.$$

M1 A1

$\ln A = 4.73 - 1.497 \times 0.69 = 3.70$ (this is also where the line meets the vertical axis), so $A = e^{3.70} = 40$ (2 s.f.). **M1 A1**

(d) $\ln 1000 = 6.91$. On the line, $\ln P = 6.91$ when $\ln(t+2) = \dfrac{6.91 - 3.70}{1.5} = 2.14$. **M1**

$t + 2 = e^{2.14} = 8.5$, so $t = 6.5$. **A1** A reading between 6.4 and 6.7 from a good graph is fine.

Common mistakes

- **Using an end point instead of the midpoint.** A perpendicular bisector goes through the **midpoint**. Examiners see lines through A with the right gradient every year, and they score little.
- **Using the gradient of AB for the bisector**, or finding the equation of AB when the question wants the perpendicular. Finding the equation of AB itself is often unnecessary: you only need its gradient.
- **Assuming lines are perpendicular because they look it.** The bisector of AB is at right angles to AB, not to some other line in the question. Work out every gradient; a quick sketch helps.
- **Gradient upside down or sign slips.** The gradient is $\frac{\text{change in } y}{\text{change in } x}$, not the other way up. Subtract in the same order top and bottom, and add (don't subtract) in the midpoint formula.
- **Using the midpoint when the point is somewhere else.** "$AC : CB = 1 : 2$" means C is a third of the way along, and "perpendicular through C" goes through that C.
- **Treating graph coordinates as x and y.** On a graph of $\ln y$ against $\ln x$, the point $(4, 7)$ means $\ln x = 4$ and $\ln y = 7$. The same goes for y^2, e^y and $\log(x+1)$ axes.
- **Stopping at the line.** "Find y in terms of x" is not finished at $\ln y = 3x - 4$: rearrange to $y = e^{3x-4}$.
- **Undoing term by term.** $\sqrt{y} = 3x - 2$ gives $y = (3x-2)^2$, not $9x^2 + 4$. Keep brackets round everything you square, cube or take the log of.
- **Mixing $\ln$ and $\lg$.** Use the one on the axis: $\lg y = c$ means $y = 10^c$, not e^c.
- **Writing $+$ instead of $\times$** in $y = Ab^x$, or leaving the answer in a different form from the one printed.
- **Not knowing what the gradient and intercept stand for.** Write the log form first ($\ln y = b \ln x + \ln A$) so it is clear the gradient is b and the intercept is $\ln A$, not A.
- **Ignoring "use your graph".** Solving simultaneous equations from the original data does not earn the marks, and in the last part read the answer from your line rather than from possibly wrong values of A and b.
- **Poor plotting.** One wrongly plotted point costs a mark; the line must be single, ruled and straight.

How to get the marks

- **Write the general line first**, with the real labels: $\ln y = mx + c$ or $e^y = mx + c$. It usually earns a mark by itself, and it stops you mixing up x and $\ln x$.
- **Show the gradient calculation** with the numbers in, then how you found c. Method marks follow through, so a wrong gradient used correctly still earns later marks.

- **Perpendicular bisector**: the marks are for the midpoint, the perpendicular gradient and a line through that midpoint. Show all three.
- **"In the form $ax + by + c = 0$ where a, b, c are integers"**: clear every fraction. Any correct equation usually earns most of the marks, but the last one needs the stated form.
- **"Show that"** means the answer is printed: every step must be visible and your last line must match it exactly, including "$\times$".
- **"Exact"** means fractions, surds or logarithms such as $\ln 15.5$; otherwise give 3 significant figures unless told otherwise. Data questions often ask for 1 or 2 significant figures: round to exactly that.
- **"Draw"** means an accurate plot with a ruled line on the grid given; **"sketch"** means the right shape and key points only.
- **"Use your graph"** means the answer must come from the straight line you drew. State the log equation, the gradient and the intercept you used.
- **Validity questions** want an inequality in x, from "the inside of the log is positive".

Quick check

1. Show that the lines $3x - 2y = 4$ and $4x + 6y = 1$ are perpendicular.
2. The triangle PQR has vertices $P(-2, 1)$, $Q(4, 3)$ and $R(-1, 8)$. Show that the triangle is isosceles, and find its area using the midpoint of PQ.
3. For each relationship, say what to plot to get a straight line, and give its gradient and intercept. (a) $T = 8 \times 2^{t/5}$, using $\lg$. (b) $y = a\sqrt{x} + \dfrac{b}{\sqrt{x}}$.
4. When $\dfrac{1}{y}$ is plotted against $\dfrac{1}{x}$, a straight line passing through $\left(\frac{1}{2}, 3\right)$ and $(2, 9)$ is obtained. Find y in terms of x, and the value of x when $y = \frac{1}{2}$.
5. When $\ln\left(\dfrac{y}{x}\right)$ is plotted against x, a straight line passing through $(1, 0.7)$ and $(3, -0.3)$ is obtained. Find y in terms of x, and the positive value of x for which $y = x$.

Answers

1. $3x - 2y = 4$ is $y = \frac{3}{2}x - 2$, gradient $\frac{3}{2}$. $4x + 6y = 1$ is $y = -\frac{2}{3}x + \frac{1}{6}$, gradient $-\frac{2}{3}$. Since $\frac{3}{2} \times \left(-\frac{2}{3}\right) = -1$, the lines are perpendicular.
2. $PR = \sqrt{1^2 + 7^2} = \sqrt{50}$ and $QR = \sqrt{5^2 + 5^2} = \sqrt{50}$, so $PR = QR$ and the triangle is isosceles ($PQ = \sqrt{40}$ is different). The midpoint of PQ is $M(1, 2)$. Since R is the same distance from P and Q, it lies on the perpendicular bisector of PQ, so RM is the height: $RM = \sqrt{2^2 + 6^2} = \sqrt{40}$. Area $= \frac{1}{2} \times \sqrt{40} \times \sqrt{40} = 20$.
3. (a) $\lg T = \dfrac{\lg 2}{5}\, t + \lg 8$: plot $\lg T$ against t; gradient $\frac{\lg 2}{5} = 0.0602$, intercept $\lg 8 = 0.903$. (b) Multiply by $\sqrt{x}$: $y\sqrt{x} = ax + b$. Plot $y\sqrt{x}$ against x; gradient a, intercept b.
4. $\dfrac{1}{y} = m\dfrac{1}{x} + c$ with $m = \dfrac{9-3}{2-\frac{1}{2}} = 4$ and $c = 3 - 4 \times \frac{1}{2} = 1$. So $\dfrac{1}{y} = \dfrac{4}{x} + 1 = \dfrac{x+4}{x}$ and $y = \dfrac{x}{x+4}$. When $y = \frac{1}{2}$, $\frac{1}{y} = 2$: $\frac{4}{x} + 1 = 2$, so $x = 4$.
5. $\ln \dfrac{y}{x} = mx + c$ with $m = \dfrac{-0.3 - 0.7}{3 - 1} = -0.5$ and $c = 0.7 + 0.5 = 1.2$. So $\dfrac{y}{x} = e^{1.2 - 0.5x}$ and $y = xe^{1.2-0.5x}$. $y = x$ when $e^{1.2-0.5x} = 1$, that is $1.2 - 0.5x = 0$, so $x = 2.4$.

Past questions to try

- **0606/13/O/N/24 Q6**: drawing $\ln y$ against x^2 from data and using the graph.
- **0606/12/M/J/22 Q3**: a line of $\lg(2y+1)$ against x, then using the equation.
- **0606/13/O/N/22 Q13**: a point on a perpendicular bisector, then points three times as far from the segment.
- **0606/22/O/N/22 Q11**: a point dividing a line in a ratio, a perpendicular, and points at a given distance.