

Hooke's law A2

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Read first: [Circular motion](#)

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What you need to know

By the end of this topic you should be able to:

1. **Use Hooke's law.** Model the pull in a stretched elastic string, or the pull or push in a spring, as proportional to its extension or compression: $T = \frac{\lambda x}{l}$. Know what the **modulus of elasticity** λ means.
2. **Use the elastic potential energy formula** $E = \frac{\lambda x^2}{2l}$ for the energy stored in a stretched string or a stretched or compressed spring. You do not have to prove it.
3. **Solve problems with elastic strings and springs**, using forces (equilibrium and Newton's second law) and work and energy. The particle may move horizontally, vertically or on an inclined plane, be attached to one or more strings or springs, or move in a horizontal circle on an elastic string (an elastic "conical pendulum").

Motion in a straight line under the variable force of a string, solved with a differential equation, belongs to the topic Linear motion under a variable force. Horizontal circles themselves are in Circular motion.

Understand it

Stretching a string: Hooke's law

Hang a light elastic string of natural (unstretched) length l from a fixed point. Pull the other end so the string's length becomes $l + x$. The amount x it has been stretched is the **extension**, and the string pulls back with a **tension** T . Hooke's law says the tension is proportional to the extension:

$$T = \frac{\lambda x}{l}.$$

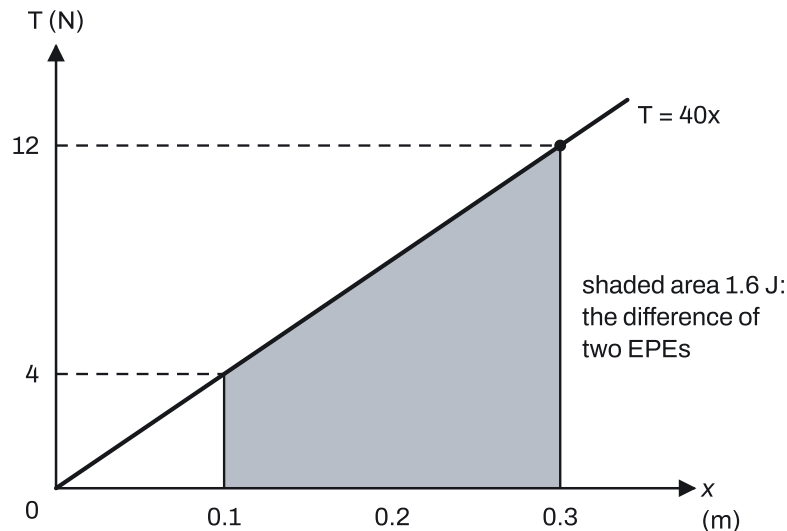
The constant λ is the **modulus of elasticity**. It is a force, measured in newtons (or given as, say, $4mg$), and it describes the material: put $x = l$ and $T = \lambda$, so λ is the tension needed to double the string's length. Dividing by l makes sense: a long string of the same material stretches more under the same pull than a short one.

A **string** can only pull. Once its length is back to l it goes **slack**: the tension is 0 and it stores no energy, however short the distance between its ends. A **spring** can also be squashed. If its length is $l - x$, the **compression** is x and it pushes outwards with a **thrust** $\frac{\lambda x}{l}$, given by the same formula.

For example, a spring of natural length 0.5 m and modulus 20 N stretched by 0.3 m has tension $\frac{20 \times 0.3}{0.5} = 12$ N.

Energy stored: elastic potential energy

As you stretch the string, the force you need grows in proportion to x , so the work you do is the area under the graph of T against x : a triangle with base x and height $\frac{\lambda x}{l}$.



$$E = \frac{1}{2} \times x \times \frac{\lambda x}{l} = \frac{\lambda x^2}{2l}.$$

That work is stored in the string as **elastic potential energy** (EPE), and the string gives it back when it shortens. For the spring above at $x = 0.3$: $E = \frac{20 \times 0.3^2}{2 \times 0.5} = 1.8$ J.

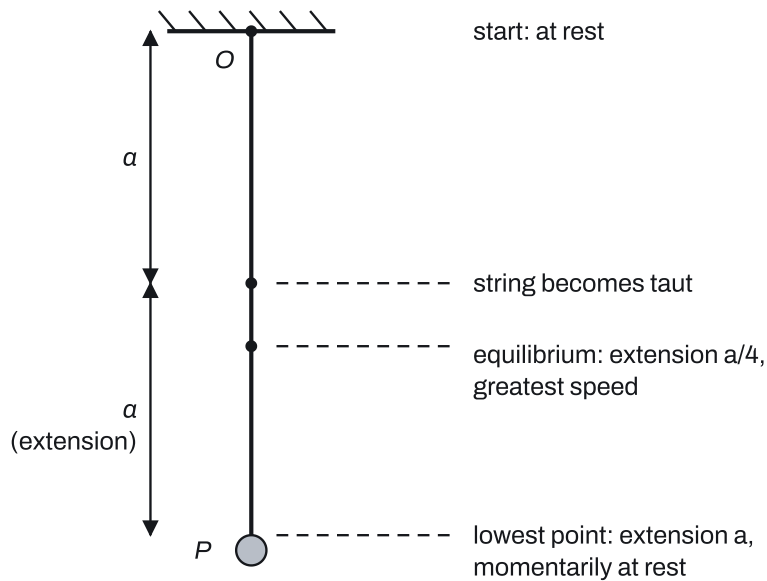
The shaded strip in the graph shows an important point. Stretching from $x = 0.1$ to $x = 0.3$ takes $1.8 - 0.2 = 1.6$ J, the **difference of the two EPEs**. It is not $\frac{20 \times 0.2^2}{2 \times 0.5} = 0.8$ J, which is the energy for stretching an unstretched string by 0.2 m.

Energy is conserved

On smooth surfaces, with light strings, no energy is lost: it only moves between kinetic, gravitational and elastic forms, so

$$\text{KE} + \text{GPE} + \text{EPE} = \text{constant}.$$

When there is friction, the energy lost equals the work done against it: (energy at the start) – (energy at the end) = friction force $\times$ distance. Energy is the main tool in this topic, because the tension changes as the particle moves, so the acceleration is **not** constant and the suvat equations do not apply.



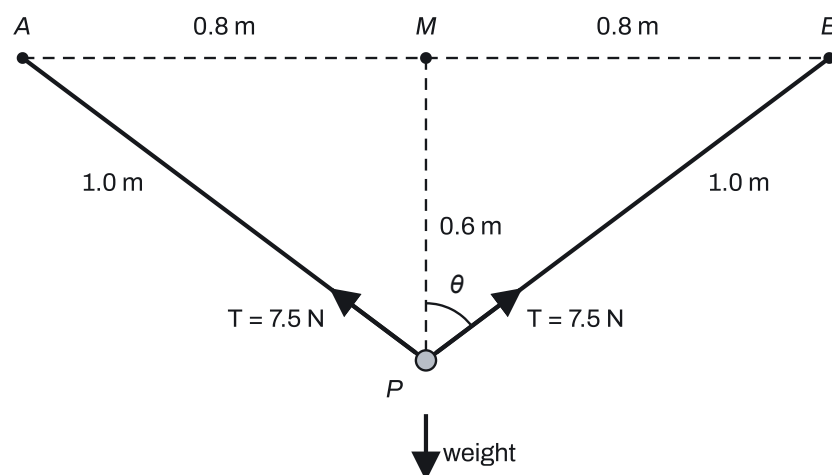
The diagram follows a particle dropped from the fixed point (Example 2 works it through). For the first a the string is slack, so only KE and GPE change. After that the string stretches, and the EPE grows until the particle stops. The speed is greatest where the tension exactly balances the weight: above that point the particle speeds up, below it it slows down.

Equilibrium

At rest, the forces balance, so Hooke's law and resolving give the extension. A particle of mass m hanging on one string has $\frac{\lambda e}{l} = mg$, so the **equilibrium extension** is $e = \frac{mgl}{\lambda}$.

Two pieces of one string

If a particle is tied to the **midpoint** of a string of natural length l , each half is a string of natural length $\frac{l}{2}$ with the **same** modulus λ (the material hasn't changed). Work with each half separately.



In the diagram each half has natural length 0.8 m and stretched length $\sqrt{0.8^2 + 0.6^2} = 1.0$ m, so the extension is 0.2 m and the tension is $\frac{30 \times 0.2}{0.8} = 7.5$ N. The vertical parts of the two tensions hold up the weight: $2 \times 7.5 \times 0.6 = 9$ N, so P has mass 0.9 kg.

Measuring heights

GPE only needs a **change** in height, so choose any level as zero, often the lowest point in the motion or the fixed point. On a plane inclined at α , a particle that moves d along a line of greatest slope changes height by $d \sin \alpha$.

Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. "Learn it" means you must remember it. Take $g = 10 \text{ m s}^{-2}$ unless told otherwise.

Formula	Status
Hooke's law: $T = \frac{\lambda x}{l}$ (tension, or thrust in a compressed spring)	Given
Elastic potential energy: $E = \frac{\lambda x^2}{2l}$	Given
Work done stretching from x_1 to x_2 : $\frac{\lambda(x_2^2 - x_1^2)}{2l}$	Learn it
Equilibrium extension of a hanging particle: $\frac{\lambda e}{l} = mg$	Learn it
KE = $\frac{1}{2}mv^2$, GPE = mgh	Learn it
Energy: KE + GPE + EPE is constant (no friction); otherwise the loss equals the work done against friction	Learn it
Work done against friction = $F \times d$, with $F = \mu R$ while sliding	Learn it
Newton's second law: resultant force = ma	Learn it
Circular motion: acceleration $r\omega^2 = \frac{v^2}{r}$ towards the centre	Given
Half of a string of natural length l has natural length $\frac{l}{2}$ and the same λ	Learn it

In every formula x is the **extension** (or compression), never the stretched length, and l is the **natural** length.

How to do it

In the Paper 3 questions we have tagged for this topic (41 questions, 2021–2025), the types came up this often. 20 of the 41 questions mix more than one type, so the counts add up to more than 41.

Question type	Questions
Vertical motion: energy with a hanging or standing string or spring	17
Equilibrium: Hooke's law with resolving	14
Elastic string in a horizontal circle	9
Smooth horizontal surface: energy	6
Acceleration at an instant	6
Inclined plane	5
Work done against friction	3
Greatest speed	2

1. Vertical motion: energy with a hanging or standing string or spring (17 questions)

- 1. Draw the two positions** (start and end) and mark every length: the natural length, the extension at each position, and the height change between them.
- 2. Find the starting extension.** If the particle starts in equilibrium, or is pulled down from equilibrium, first find the equilibrium extension with $\frac{\lambda e}{l} = mg$.
- 3. Write one energy equation** with every term: KE at each position, GPE change, and an EPE term for each position where the string is taut:

$$\frac{1}{2}mv_1^2 + \frac{\lambda x_1^2}{2l} = \frac{1}{2}mv_2^2 + \frac{\lambda x_2^2}{2l} \pm mgh.$$

Put the GPE gain on the side of the end position (or the loss on the side of the start).
- 4. Solve.** With an unknown extension this is usually a quadratic in x and a (every term has the same power, so divide through by mg and it becomes "homogeneous"). Keep the root that fits the situation and say why you reject the other.

For example, a particle of mass 0.5 kg hangs on a string of natural length 0.8 m and modulus 20 N. It is pulled down until the extension is 0.5 m and released. When the extension is 0.1 m it has risen 0.4 m, so

$$\frac{20 \times 0.5^2}{1.6} = \frac{1}{2}(0.5)v^2 + \frac{20 \times 0.1^2}{1.6} + 0.5 \times 10 \times 0.4,$$

that is $3.125 = 0.25v^2 + 0.125 + 2$, so $v^2 = 4$ and $v = 2 \text{ m s}^{-1}$.

Variations you will meet:

- **The string goes slack.** Above the natural-length point the string does nothing, so from there on only KE and GPE change. "Just reaches O " means the speed is 0 at O ; a particle dropped from O falls freely for the first l .
- **A spring standing on the floor,** compressed and released. Use the **compression** in E while it is shorter than l and the extension once it is longer; the particle can go past the natural length.
- **Mass added or removed.** A particle hangs in equilibrium, then extra mass is attached (or part of it is detached) and it is released. The **starting extension is the old equilibrium extension**, but the KE and GPE use the **new** mass.
- **Two strings, one above and one below** the particle (for instance A above, B below): each has its own natural length, modulus and extension, and one may be slack. In equilibrium, upward tension = downward tension + weight.
- **Unknown modulus.** Two speeds at two positions give two energy equations; eliminate the speed to find λ (or k in kmg).
- **A rod and a spring.** The particle turns on a light rod while a spring pulls it. The energy equation is the same; get the heights and the spring's lengths from the geometry.

2. Equilibrium: Hooke's law with resolving (14 questions)

1. **Draw the forces:** tension along each string, weight, any applied force, reaction and friction.
2. **Write each tension** with Hooke's law, in terms of the unknown length or extension.
3. **Resolve** (vertically and horizontally, or along and across a plane) and solve.

For example, a particle of weight 12 N hangs from a ceiling on a string of natural length 1 m and modulus 26 N, and a horizontal force of 5 N holds it with the string at angle θ to the ceiling. The tension is $\sqrt{12^2 + 5^2} = 13$ N and $\sin \theta = \frac{12}{13}$. Hooke's law gives $13 = 26x$, so $x = 0.5$ m, the string is 1.5 m long, and the particle is $1.5 \times \frac{12}{13} = 1.38$ m below the ceiling.

Variations you will meet:

- **A particle on the midpoint of a string** between two fixed points: each half has natural length $\frac{l}{2}$ and modulus λ . Use Pythagoras for each half's length and resolve vertically, as in the third diagram.
- **Two strings in a vertical line** above and below the particle: write each tension with the same unknown (say the length BP), using the fact that the two lengths add up to the distance between the fixed points.
- **"Show that"** an expression for a length or for k : write Hooke's law, set the forces equal, and rearrange one step at a time to the printed answer.
- **Friction in equilibrium.** After the particle stops on a rough plane, the friction needed is whatever balances the other forces along the plane (it need not be μR). Work out the tension at that extension, then resolve along the plane.
- **Equilibrium as a first step.** Many energy questions start with the particle "hanging in equilibrium": that gives the starting extension.

3. Elastic string in a horizontal circle (9 questions)

1. **Radius and length.** The stretched length is $l + x$. On a horizontal table with the string fixed at the centre, the radius is $l + x$, **not** l . For a conical pendulum, radius = $(l + x) \sin \theta$.
2. **Hooke's law** for the tension: $T = \frac{\lambda x}{l}$.
3. **Newton's second law towards the centre:** horizontal part of T (plus or minus friction) = $mr\omega^2$ or $\frac{mv^2}{r}$.
For a conical pendulum also resolve vertically: $T \cos \theta = mg$.
4. **Equate and solve** for x ; with v given this is often a quadratic.

For example, a particle of mass 0.3 kg on a smooth table is joined to O by a string of natural length 0.5 m and modulus 12 N, and moves round O with $\omega = 4 \text{ rad s}^{-1}$. Then $24x = 0.3 \times (0.5 + x) \times 16$, so $19.2x = 2.4$ and $x = 0.125 \text{ m}$.

Variations you will meet:

- **Conical pendulum with ω given.** Since the radius is $(l + x) \sin \theta$, the equation $T \sin \theta = m(l + x) \sin \theta \omega^2$ loses $\sin \theta$: $T = m(l + x)\omega^2$. Equate with Hooke's law to get x , then $\cos \theta = \frac{mg}{T}$ (Example 4).
- **String through a hole** in a smooth table, with a particle hanging below: the tension is the same all along the string, so it equals the hanging weight; the total stretched length is the part on the table plus the part below.
- **Rough turntable** with the particle about to slip outwards: friction μR acts towards the centre, and the string's vertical part changes R .
- **Two different particles** on the same string: write the circle equation for each and eliminate the unknown speed or radius.

4. Smooth horizontal surface: energy (6 questions)

1. No height changes, so the energy equation has only KE and EPE: $\frac{1}{2}mv_1^2 + \frac{\lambda x_1^2}{2l} = \frac{1}{2}mv_2^2 + \frac{\lambda x_2^2}{2l}$.
2. Turn each given **length** into an **extension** first.
3. Solve for the unknown (λ , a speed, or a length).

For example, a particle of mass 0.5 kg on a string of natural length 1 m and modulus 8 N passes the natural-length point at 3 m s^{-1} , moving away from O . At the greatest extension it stops: $\frac{1}{2}(0.5)(3^2) = \frac{8x^2}{2}$, so $x^2 = 0.5625$ and $x = 0.75 \text{ m}$.

Variations you will meet:

- **Two speeds at two lengths** give the modulus, or a speed.
- **A particle on the midpoint of a string** fixed at two points on the table: two halves, so two EPE terms at each position, each with natural length $\frac{l}{2}$.
- **A collision first.** A particle hits and joins one on a spring: use conservation of momentum for the speed just after, then energy for the greatest compression (Quick check 5).

5. Acceleration at an instant (6 questions)

1. Find the extension at that instant ("initial" means at the release point) and the tension from Hooke's law.
2. **Newton's second law** along the motion: resultant force = ma . Include the **weight** when the motion is vertical, and every string that is taut.

For example, a particle of mass 0.2 kg on a smooth table is held against a spring of natural length 0.4 m and modulus 6 N, compressed to 0.3 m. The thrust is $\frac{6 \times 0.1}{0.4} = 1.5$ N, so the acceleration on release is $\frac{1.5}{0.2} = 7.5$ m s⁻².

- With two halves of a string at angle θ to the line of motion, the resultant is $2T \cos \theta$.

6. Inclined plane (5 questions)

1. Distances are measured **along** the plane; a move of d changes the height by $d \sin \alpha$.
2. Find $\sin \alpha$ and $\cos \alpha$ from the given $\tan \alpha$ or $\sin \alpha$ (a right-angled triangle, such as 3, 4, 5).
3. Write the energy equation as in type 1, with $GPE = mgd \sin \alpha$ and, on a rough plane, the work against friction (type 7).
4. For equilibrium on the plane, resolve along it: tension = $mg \sin \alpha$ ($\pm$ friction).

Variations you will meet:

- The particle starts **above** the fixed point with the string stretched, so the string goes slack as it passes through, then taut again below: the EPE at the start and the EPE at the end both count, and nothing in between.
- A particle **projected** down the plane from the natural-length point: include the starting KE.
- **Unknown modulus** from two speeds at two points (type 1).

7. Work done against friction (3 questions)

1. On a rough horizontal surface $R = mg$; on a rough plane $R = mg \cos \alpha$. While sliding, friction = μR (or as given).
2. Energy at the start – energy at the end = friction $\times$ distance moved.
3. Solve. On a horizontal surface, starting from rest at extension x_0 and stopping at extension x (string still taut): $\frac{\lambda(x_0^2 - x^2)}{2l} = \mu mg(x_0 - x)$. Both sides have the factor $(x_0 - x)$. Cancel it (the root $x = x_0$ is just the start) and the remaining equation is linear.

For example, with $m = 0.4$ kg, $\mu = 0.5$, $\lambda = 10$ N, $l = 0.5$ m and $x_0 = 0.3$ m, friction is 2 N and the equation gives $10(0.3 + x) = 2$, so $x = -0.1$. A negative extension is impossible for a string: it reaches the natural length first. There the EPE of 0.9 J has paid for $2 \times 0.3 = 0.6$ J of friction, so P still has 0.3 J of KE and slides a further $\frac{0.3}{2} = 0.15$ m with the string slack.

- **"The string never becomes slack"** means the particle stops before (or just at) the natural-length point: the starting EPE must be no more than the work against friction over the extension, so $\frac{\lambda x_0^2}{2l} \leq \mu mg x_0$. This gives the least μ .
- **Projected with a speed** from the natural length: KE at the start = EPE at the end + friction $\times$ extension.

8. Greatest speed (2 questions)

1. The speed is greatest where the acceleration is zero, so the forces along the motion balance: for example tension = $mg \sin \alpha$ on a smooth plane, or tension = mg hanging.
2. Find the extension there with Hooke's law.
3. Use energy from the start to that point to find the speed.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
 - **A1** accuracy mark: for a correct answer; it needs the M mark before it.
 - **B1** mark for a correct result on its own, with no method needed.
-

Example 1

A particle of mass 1.2 kg hangs in equilibrium from a fixed point O on a light elastic string of natural length 0.6 m and modulus of elasticity 24 N.

(a) Find the extension of the string. **[2]**

The particle is pulled down a further 0.15 m and released from rest.

(b) Find the initial acceleration of the particle. **[3]**

(c) Find its speed when it passes through the equilibrium position. **[4]**

(a) Hooke's law: $T = \frac{24x}{0.6} = 40x$. **B1** In equilibrium $40x = 1.2 \times 10 = 12$, so $x = 0.3$ m. **A1**

(b) The extension is now 0.45 m, so $T = 40 \times 0.45 = 18$ N. **B1**

Newton's second law, upwards: $18 - 12 = 1.2a$. **M1** So $a = 5 \text{ m s}^{-2}$, upwards. **A1**

(c) EPE at the start = $\frac{24 \times 0.45^2}{2 \times 0.6} = 4.05$ J; EPE at equilibrium = $\frac{24 \times 0.3^2}{2 \times 0.6} = 1.8$ J. **B1** for either.

It rises 0.15 m, gaining GPE $1.2 \times 10 \times 0.15 = 1.8$ J. Energy: **M1** for KE, GPE and both EPE terms

$$4.05 = \frac{1}{2}(1.2)v^2 + 1.8 + 1.8 \Rightarrow 0.6v^2 = 0.45.$$

A1. So $v^2 = 0.75$ and $v = 0.866 \text{ m s}^{-1}$ (3 s.f.). **A1**

Example 2

A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $4mg$. The other end is attached to a fixed point O . P is released from rest at O and falls vertically.

(a) Find the greatest extension of the string. **[4]**

(b) Find the greatest speed of P . **[4]**

(a) Let the greatest extension be x . P has then fallen $a + x$ and is momentarily at rest. **B1** for the EPE $\frac{4mgx^2}{2a}$.

Loss of GPE = gain of EPE: **M1**

$$mg(a + x) = \frac{4mgx^2}{2a} \Rightarrow 2x^2 - ax - a^2 = 0.$$

A1. $(2x + a)(x - a) = 0$, and $x > 0$, so the greatest extension is $x = a$. **A1**

(b) The speed is greatest where the acceleration is zero: $\frac{4mge}{a} = mg$, so $e = \frac{a}{4}$. **M1 A1**

P has fallen $\frac{5a}{4}$ there. Energy: **M1**

$$\frac{1}{2}mv^2 = mg \times \frac{5a}{4} - \frac{4mg}{2a} \left(\frac{a}{4}\right)^2 = \frac{5mga}{4} - \frac{mga}{8} = \frac{9mga}{8},$$

so $v^2 = \frac{9ga}{4}$ and the greatest speed is $\frac{3}{2}\sqrt{ga}$. **A1**

The second diagram in "Understand it" shows these positions.

Example 3

A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $2mg$. The other end is attached to a fixed point O on a rough plane inclined at angle α to the horizontal, where $\sin \alpha = \frac{3}{5}$. The coefficient of friction between P and the plane is $\frac{1}{8}$. P is released from rest at O and slides down the line of greatest slope.

(a) Show that P first comes to rest at a distance $2a$ from O . **[5]**

(b) Show that P does not remain at rest in this position. **[3]**

(a) $\cos \alpha = \frac{4}{5}$, so $R = \frac{4}{5}mg$ and friction $= \frac{1}{8} \times \frac{4}{5}mg = \frac{1}{10}mg$. **B1**

Let P stop at distance d from O ; the extension is $d - a$. Loss of GPE = gain of EPE + work done against friction: **M1** for all three terms

$$mgd \times \frac{3}{5} = \frac{2mg(d - a)^2}{2a} + \frac{1}{10}mgd.$$

A1. Multiply by $\frac{2a}{mg}$: $ad = 2(d - a)^2$, so $2d^2 - 5ad + 2a^2 = 0$. **M1**

$(2d - a)(d - 2a) = 0$. The string must be stretched, so $d > a$ and $d = 2a$. **A1** (the root $\frac{a}{2}$ is rejected).

(b) At $d = 2a$ the extension is a , so $T = \frac{2mg \times a}{a} = 2mg$ up the plane. **B1**

Down the plane the weight gives $\frac{3}{5}mg$. Without friction the resultant would be $2mg - \frac{3}{5}mg = \frac{7}{5}mg$ up the plane. **M1**

The most friction can supply is $\mu R = \frac{1}{10}mg < \frac{7}{5}mg$, so P moves back up the plane. **A1**

Example 4

A particle P of mass m is attached to one end of a light elastic string of natural length $3a$ and modulus of elasticity $4mg$. The other end is attached to a fixed point O . P moves in a horizontal circle, whose centre is vertically below O , with constant angular speed $\sqrt{\frac{g}{3a}}$. The string makes an angle θ with the vertical.

(a) Find the extension of the string. **[4]**

(b) Find $\cos \theta$ and the speed of P . **[3]**

(a) Let the extension be x , so the string is $3a + x$ long and the radius is $(3a + x) \sin \theta$. **B1**

Towards the centre: $T \sin \theta = m(3a + x) \sin \theta \times \frac{g}{3a}$, so $T = \frac{mg(3a+x)}{3a}$. **M1**

Hooke's law: $T = \frac{4mgx}{3a}$. **B1** Equating, $4x = 3a + x$, so $x = a$. **A1**

(b) $T = \frac{4mg}{3}$. Vertically, $T \cos \theta = mg$, so $\cos \theta = \frac{3}{4}$. **B1**

$\sin \theta = \frac{\sqrt{7}}{4}$ and the radius is $4a \times \frac{\sqrt{7}}{4} = \sqrt{7}a$. **M1** So $v = r\omega = \sqrt{7}a\sqrt{\frac{g}{3a}} = \sqrt{\frac{7ga}{3}}$. **A1**

Common mistakes

- **Using the stretched length instead of the extension.** In $T = \frac{\lambda x}{l}$ and $E = \frac{\lambda x^2}{2l}$, x is how much longer than natural the string is. A string of natural length a that is $\frac{7}{5}a$ long has extension $\frac{2}{5}a$, not $\frac{7}{5}a$. Examiners see this in almost every session.
- **Mixing up the two formulas.** The EPE has $2l$ underneath; Hooke's law does not. Writing $\frac{\lambda x}{2l}$ for the tension, or $\frac{\lambda x^2}{l}$ for the energy, loses the marks that follow.
- **Leaving out an EPE term.** If the string is already stretched at the start (for example the particle starts in equilibrium), the energy equation needs **two** EPE terms. Find the starting extension first.
- **Combining EPE terms wrongly.** $\frac{\lambda x_1^2}{2l} - \frac{\lambda x_2^2}{2l} = \frac{\lambda(x_1^2 - x_2^2)}{2l}$, not $\frac{\lambda(x_1 - x_2)^2}{2l}$ (the first diagram shows why).
- **Counting EPE for a slack string.** Once the string is back to its natural length it stores nothing. Also, the particle is usually still moving when the string goes slack: don't set the speed to 0 there.
- **Using suvat.** The tension changes with the extension, so the acceleration is not constant. Use energy for speeds and distances, and Newton's second law only for the acceleration at one instant.
- **Signs in the energy equation**, especially the work against friction: it is always energy **lost**, on the same side as the end energies.

- **Using the wrong mass.** When a particle is added or two particles join, the KE and GPE use the combined mass.
- **Forgetting the weight** in Newton's second law for vertical motion, or forgetting that the radius includes the extension in a horizontal circle.
- **Reading the modulus wrongly.** If the modulus is given as λmg , the tension is $\frac{\lambda mgx}{l}$; writing just λ gives a wrong answer with mg in it.
- **Swapping sine and cosine** when a string makes an angle with a ceiling rather than with the vertical. Mark the angle on your diagram.

How to get the marks

- **Energy equations are marked by their terms.** The method mark needs every term that should be there (KE, GPE, each EPE, work done) and nothing else, and it must be dimensionally correct: every term is a force times a length. Write each term separately before simplifying.
- **Each correct EPE term** often earns its own mark, so write $\frac{\lambda x^2}{2l}$ with the numbers in before you combine anything.
- **Quadratics:** divide through by mg (or mg/a) to get a three-term quadratic in x and a , solve it, and **reject** the other root with a reason (the string would be slack, or it is the starting position).
- **"Show that"** means the answer is printed. Write Hooke's law, the equation, and every step of the rearrangement; a jump to the given answer loses the last mark.
- **"In terms of a " or "in terms of m and g "** means an exact expression such as $\frac{7a}{2}$ or $\frac{mg}{8}$. Numerical answers go to 3 significant figures with $g = 10$.
- **State directions** for accelerations and forces ("upwards", "up the plane") when the question asks for a vector quantity.
- **Diagrams help you:** mark natural lengths, extensions and heights. Examiners note that the best answers start this way.
- **Method marks follow through.** A wrong extension used correctly in the energy equation still earns the method mark, so keep going.

Quick check

1. A spring of natural length 0.5 m and modulus 40 N is compressed to a length of 0.4 m. Find the thrust in the spring and the elastic potential energy stored.
2. A string of natural length 1.2 m has modulus 30 N. Find the work done in stretching it from an extension of 0.2 m to an extension of 0.6 m.
3. A particle of mass 0.4 kg lies on a smooth horizontal table, attached to a fixed point O by a light elastic string of natural length 0.5 m and modulus 10 N. It is held with $OP = 0.9$ m and released. Find its speed when the string becomes slack.

4. Fixed points A and B are $5a$ apart with A vertically above B . A particle P of mass m is joined to A by a light elastic string of natural length $2a$ and modulus $4mg$, and to B by a light elastic string of natural length $2a$ and modulus $2mg$. Find AP when P is in equilibrium.
5. A light spring of natural length 0.5 m and modulus 15 N lies on a smooth horizontal surface with one end fixed and a particle B of mass 0.1 kg at the other. A particle A of mass 0.2 kg moving at 3 m s^{-1} along the line of the spring hits B and they join. Find the greatest compression of the spring.

Answers

1. Compression 0.1 m. Thrust = $\frac{40 \times 0.1}{0.5} = 8 \text{ N}$. EPE = $\frac{40 \times 0.1^2}{2 \times 0.5} = 0.4 \text{ J}$.
2. $\frac{30(0.6^2 - 0.2^2)}{2 \times 1.2} = \frac{30 \times 0.32}{2.4} = 4 \text{ J}$.
3. Extension 0.4 m, EPE = $\frac{10 \times 0.4^2}{2 \times 0.5} = 1.6 \text{ J}$. All of it becomes KE: $\frac{1}{2}(0.4)v^2 = 1.6$, so $v^2 = 8$ and $v = 2.83 \text{ m s}^{-1}$.
4. Let $AP = y$, so $PB = 5a - y$. $T_A = \frac{4mg(y-2a)}{2a}$ and $T_B = \frac{2mg(3a-y)}{2a}$. Equilibrium: $T_A = T_B + mg$, so $2(y - 2a) = (3a - y) + a$, giving $3y = 8a$ and $AP = \frac{8a}{3}$. Check: $PB = \frac{7a}{3} > 2a$, so the lower string is taut.
5. Momentum: $0.2 \times 3 = 0.3u$, so $u = 2 \text{ m s}^{-1}$. KE = $\frac{1}{2}(0.3)(2^2) = 0.6 \text{ J} = \frac{15x^2}{2 \times 0.5}$, so $x^2 = 0.04$ and the greatest compression is 0.2 m.

Past questions to try

- **9231/32/M/J/24 Q2**: a particle on the midpoint of a string, equilibrium then energy.
- **9231/33/O/N/23 Q4**: energy and initial acceleration with two halves of a string on a table.
- **9231/33/M/J/23 Q5**: an elastic string as a conical pendulum.
- **9231/32/O/N/21 Q2**: a collision, then the greatest compression of a spring.