Cambridge IGCSE · Mathematics (0580) · Notes

Indices in algebra

Read online at pastopic.com/cambridge/mathematics-0580/notes/algebraic-indices

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Read first: Powers, roots, indices and surds

What you need to know

This topic is the rules of indices used with letters, such as $x^3 \times x^5$ or $(2y^4)^3$. Core and Extended share most of it; fractional indices and the harder equations are **Extended only**. By the end of it you should be able to:

1. **Understand and use positive, zero and negative indices** with letters and numbers, for example $x^0 = 1$ and $x^{-2} = \frac{1}{x^2}$.
2. **Use the rules of indices to simplify** expressions: multiplying and dividing terms (numbers in front included), and raising a term or a bracket to a power, such as $6a^5 \times 2a^{-3}$ or $(3x^2y)^3$.
3. **Find an unknown index**, for example x in $b^x \times b^4 = b^{11}$, or in $2^x = 64$.
4. **Extended only:** understand and use **fractional indices**, such as $x^{\frac{1}{2}} = \sqrt{x}$, and simplify terms such as $(8x^6)^{\frac{2}{3}}$ or $\left(\frac{x^4}{9}\right)^{-\frac{1}{2}}$.
5. **Extended only: solve equations with the unknown in the index** when the two sides use different numbers, such as $4^{x+1} = 8^x$, by writing both sides as powers of the same number.

Logarithms are **not** needed. Working out numerical powers and roots, such as $16^{\frac{3}{4}}$ or $\sqrt[3]{64}$, is in the topic Powers, roots, indices and surds.

Understand it

What an index means

In x^5 the letter x is the **base** and 5 is the **index** (or power): $x^5 = x \times x \times x \times x \times x$. The index only belongs to the thing it is written on: in $3x^2$ only the x is squared, so $3x^2 = 3 \times x \times x$. In $(3x)^2$ the whole bracket is squared, so $(3x)^2 = 3x \times 3x = 9x^2$.

Multiplying: add the indices

Write the powers out and count the letters:

$$x^3 \times x^2 = (x \times x \times x) \times (x \times x) = x^5.$$

So when you multiply powers of the **same letter**, you **add** the indices: $a^m \times a^n = a^{m+n}$.

With numbers in front, multiply the numbers and add the indices of each letter separately. The order doesn't matter when you multiply, so group the numbers together and each letter together:

$$4a^3 \times 6a^5 = (4 \times 6) \times (a^3 \times a^5) = 24a^8, \qquad 2x^3y \times 5xy^4 = 10x^4y^5.$$

A letter on its own has index 1: $x = x^1$, so $x^3 \times x$ is x^4.

Dividing: subtract the indices

Dividing cancels letters from the top and bottom:

$$y^6 \div y^2 = \frac{y \times y \times y \times y \times y \times y}{y \times y} = y^4.$$

So $a^m \div a^n = a^{m-n}$: the top index **minus** the bottom index. Divide the numbers as normal: $30b^7 \div 5b^2 = 6b^5$
.

A power of a power: multiply the indices

$(z^2)^3$ means $z^2 \times z^2 \times z^2 = z^{2+2+2} = z^6$. So $(a^m)^n = a^{mn}$.

When a bracket holds a number and letters, the power applies to **every** part: the number is raised to the power, and each letter's index is multiplied.

$$(3m^4n)^2 = 3^2 \times m^{4\times2} \times n^{1\times2} = 9m^8n^2.$$

Zero and negative indices

Divide a power by itself and you get 1, but the rule says $x^3 \div x^3 = x^{3-3} = x^0$. So

$$x^0 = 1 \quad (x \neq 0).$$

Only the letter becomes 1: $8t^0 = 8 \times 1 = 8$.

Now divide by a bigger power: $w^2 \div w^5 = \frac{w\times w}{w\times w\times w\times w\times w} = \frac{1}{w^3}$, and the rule gives $w^{2-5} = w^{-3}$. So a **negative index means "one over"**:

$$x^{-n} = \frac{1}{x^n}.$$

A negative index does not make the term negative. It only moves the power below the line: $6x^{-2} = \frac{6}{x^2}$ (the 6 has no power, so it stays on top). A whole fraction to a negative power turns upside down: $\left(\frac{2}{x}\right)^{-3} = \left(\frac{x}{2}\right)^3 = \frac{x^3}{8}$.

Fractional indices (Extended only)

Extended only: by the power-of-a-power rule, $\left(x^{\frac{1}{3}}\right)^3 = x^1 = x$. So $x^{\frac{1}{3}}$ is the number that cubes to give x: the **cube root**. In the same way $x^{\frac{1}{2}} = \sqrt{x}$, and in general the bottom of the fraction is a root and the top is a power:

$$x^{\frac{1}{n}} = \sqrt[n]{x}, \qquad x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m.$$

Extended only: for a term with a number and letters, do each part separately. The number: take the root, then the power. Each letter: multiply its index by the fraction.

$$(8x^6)^{\frac{2}{3}}: \quad 8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4, \qquad (x^6)^{\frac{2}{3}} = x^{6\times\frac{2}{3}} = x^4, \qquad \text{so } (8x^6)^{\frac{2}{3}} = 4x^4.$$

Extended only: a negative fractional index is all three ideas at once: the minus sign means one over, then the root, then the power. $(25x^{10})^{-\frac{1}{2}} = \dfrac{1}{(25x^{10})^{\frac{1}{2}}} = \dfrac{1}{5x^5}$.

Unknowns in the index

If two powers of the **same base** are equal, their indices are equal. So to solve $3^x = 81$, write 81 as a power of 3: $81 = 3^4$, so $3^x = 3^4$ and $x = 4$. With a fraction: $2^x = \frac{1}{8} = 2^{-3}$, so $x = -3$.

Extended only: when the two sides use different numbers, first write both as powers of one prime. For $4^{x+1} = 8^x$: $4 = 2^2$ and $8 = 2^3$, so

$$(2^2)^{x+1} = (2^3)^x \;\Rightarrow\; 2^{2x+2} = 2^{3x} \;\Rightarrow\; 2x + 2 = 3x \;\Rightarrow\; x = 2.$$

The indices give an ordinary linear equation. Check: $4^3 = 64$ and $8^2 = 64$.

Key facts and formulas

None of these is on the List of formulas printed in the exam paper, so you must know them all. They work for any base that is not 0.

Formula	Status
$a^m \times a^n = a^{m+n}$	Learn it
$a^m \div a^n = a^{m-n}$	Learn it
$(a^m)^n = a^{mn}$	Learn it
$(ab)^n = a^n b^n$ and $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	Learn it
$a^0 = 1$	Learn it
$a^{-n} = \dfrac{1}{a^n}$ and $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$	Learn it
If $a^x = a^y$ then $x = y$ (same base)	Learn it
Extended only: $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$	Learn it

Powers of small primes that come up again and again: $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $3^3 = 27$, $3^4 = 81$, $3^5 = 243$, $5^3 = 125$, $5^4 = 625$. Also $4 = 2^2$, $8 = 2^3$, $9 = 3^2$, $25 = 5^2$, and $\frac{1}{2} = 2^{-1}$, $\frac{1}{3} = 3^{-1}$.

How to do it

In the questions we have tagged for this topic (57 questions from Papers 1–4, 2021–2025, one of them printed on two papers), the types came up this often. 15 of the 57 questions mix types, so the counts add up to more than 57.

Question type	Questions
Simplifying with the rules of indices	32
Fractional indices (Extended)	16
Finding a missing index with the same base	15
Equations with the unknown in the index	8
Adding fractions with a letter on the bottom (Extended)	1
Reading off indices after differentiating (Extended)	1

1. Simplifying with the rules of indices (32 questions)

1. **Numbers first**: multiply or divide the numbers in front, or raise them to the power outside a bracket.
2. **Then each letter on its own**: add the indices for $\times$, subtract for $\div$, multiply for a power of a power.
3. **Tidy up**: a letter to the power 0 is 1; a letter to the power 1 is written without the index.
4. Write the final answer as **one term**, such as $24a^{10}b^5$.

For example, $(2a^3b)^3 \times 3ab^2 = 8a^9b^3 \times 3ab^2 = 24a^{10}b^5$.

Variations you will meet:

- **Division as a fraction**: $\dfrac{21x^9}{7x^3} = 3x^6$. Divide the numbers ($21 \div 7$); don't subtract them.
- **The answer has a negative index**: $y^3 \div y^8 = y^{-5}$. Mark schemes accept y^{-5} or $\frac{1}{y^5}$.
- **A zero index**: $9h^0 = 9$, because only h^0 becomes 1.
- **Negative indices in the question**: $a^5b^{-3} \times a^2b^4 = a^7b$, and $\dfrac{a^7b^{-1}}{a^2b^4} = a^5b^{-5}$ (take care: $-1-4=-5$).
- **Two letters in a bracket**: $(4c^2d^5)^3 = 64c^6d^{15}$. Every part gets the power, the number included.
- **Simplify inside the bracket first** when it has a product: $(2x \times 3x^2y)^2 = (6x^3y)^2 = 36x^6y^2$.
- **A fraction to a negative power**: $\left(\dfrac{5}{x}\right)^{-2} = \left(\dfrac{x}{5}\right)^2 = \dfrac{x^2}{25}$.
- **Extended only: the square root of a term**: $\sqrt{36x^2} = 6x$. Root the number **and** the letter.
- **Answers as a single power of a number**: $5^4 \times 5^3 = 5^7$, $5^9 \div 5^2 = 5^7$, and $20+5=25=5^2$. Leave the answer as a power when the question asks for one.
- **Write a fraction as a power**: $\dfrac{1}{3\times3\times3\times3} = \dfrac{1}{3^4} = 3^{-4}$.

2. Fractional indices (16 questions)

Extended only.

1. **Split the term** into its number and each letter.
2. **Number**: bottom of the fraction = root, top = power. Take the root first to keep numbers small. If the index is negative, the number ends up underneath.
3. **Letters**: multiply each index by the fraction.
4. Put the parts back together as one term, or as one fraction if there is a negative index.

For example, $(32x^{15})^{\frac{3}{5}}$: $\sqrt[5]{32} = 2$ and $2^3 = 8$; $x^{15\times\frac{3}{5}} = x^9$. The answer is $8x^9$.

Variations you will meet:

- **A huge index**: $(256x^{256})^{\frac{1}{4}} = 4x^{64}$. The fraction multiplies the index ($256 \div 4 = 64$); the number is rooted ($\sqrt[4]{256} = 4$). Don't mix the two up.
- **A negative fractional power of a fraction**: turn the fraction upside down first, then root and power. $\left(\frac{x^{12}}{64}\right)^{-\frac{2}{3}} = \left(\frac{64}{x^{12}}\right)^{\frac{2}{3}} = \frac{16}{x^8}$.
- **Several letters on top and bottom**: $\left(\frac{8x^6y^{12}}{27}\right)^{-\frac{1}{3}} = \left(\frac{27}{8x^6y^{12}}\right)^{\frac{1}{3}} = \frac{3}{2x^2y^4}$.
- **A root written as a power**: $\sqrt[4]{5} = 5^{\frac{1}{4}}$ and $\sqrt{x^3} = x^{\frac{3}{2}}$. So "$\sqrt[4]{5} = 5^h$" gives $h = \frac{1}{4}$.
- **Dividing fractional powers**: $x^{\frac{1}{2}} \div x^{\frac{5}{2}} = x^{\frac{1}{2}-\frac{5}{2}} = x^{-2}$ (or $\frac{1}{x^2}$).
- **The unknown is the base**: $3p^{\frac{1}{2}} = 12$ gives $p^{\frac{1}{2}} = 4$, so $p = 4^2 = 16$. Undo the index with its reciprocal power.

3. Finding a missing index with the same base (15 questions)

1. Write the rule as an equation in the indices: $\times$ means add, $\div$ means subtract, a power of a power means multiply.
2. Solve it.

For example, $b^x \times b^6 = b^{14}$ gives $x + 6 = 14$, so $x = 8$.

Variations you will meet:

- **Dividing, with the unknown second**: $c^{20} \div c^x = c^4$ gives $20 - x = 4$, so $x = 16$. It is **not** $20 \div x = 4$.
- **A negative answer**: $2^9 \times 2^k = 2^{-3}$ gives $9 + k = -3$, so $k = -12$.
- **A power of a power**: $(d^4)^y = d^{28}$ gives $4y = 28$, so $y = 7$.
- **Number bases**: $11^x \div 11^3 = 11^8$ gives $x = 11$. You never need to work out the powers.

4. Equations with the unknown in the index (8 questions)

1. **Choose a prime base** that every number is a power of (2, 3 or 5 usually).

2. **Rewrite every number** as a power of it, in brackets: $9^x = (3^2)^x = 3^{2x}$, $\frac{1}{9} = 3^{-2}$, $3 = 3^1$.
3. **Combine** each side into a single power with the rules.
4. **Set the indices equal** and solve the linear equation.

For example (Core), $5^x = 125$: $125 = 5^3$, so $x = 3$. And $2^x = \frac{1}{16} = 2^{-4}$, so $x = -4$.

Variations you will meet:

- **Extended only: a fraction as the base**: $\left(\frac{1}{5}\right)^x = 25^{x-3}$ becomes $5^{-x} = 5^{2x-6}$, so $-x = 2x - 6$ and $x = 2$.
- **Extended only: a fractional answer**: $32 = 8^k$ becomes $2^5 = 2^{3k}$, so $k = \frac{5}{3}$.
- **Extended only: products and quotients on one side**: $9^x \times 3 = 27^{x-1}$ becomes $3^{2x+1} = 3^{3x-3}$, so $2x + 1 = 3x - 3$ and $x = 4$.
- **Extended only: two letters, "find y in terms of x"**: $8^y = 4^x \times 2$ becomes $2^{3y} = 2^{2x+1}$, so $3y = 2x + 1$ and $y = \dfrac{2x+1}{3}$.
- **Extended only: find a constant n that works for every x**: $8^{nx} = (4^x)^3$ becomes $2^{3nx} = 2^{6x}$, so $3n = 6$ and $n = 2$.

5. Adding fractions with a letter on the bottom (1 question)

Extended only. This question was tagged here because the letter is in the denominator. Make the denominators the same, then add the tops: $\dfrac{5}{3k} + \dfrac{1}{6k} = \dfrac{10}{6k} + \dfrac{1}{6k} = \dfrac{11}{6k}$. The full method is in the topic Algebraic manipulation and algebraic fractions.

6. Reading off indices after differentiating (1 question)

Extended only. Differentiating x^n gives nx^{n-1} (see the topic Differentiation). If $y = px^7 + 2x^q$ and $\frac{dy}{dx} = 21x^6 + 10x^r$, then differentiating gives $7px^6 + 2qx^{q-1}$. Match the parts: $7p = 21$ so $p = 3$; $2q = 10$ so $q = 5$; and $r = q - 1 = 4$.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

Calculators must not be used in this question.

(a) Simplify $m^4 \times m^7$. **[1]**

(b) Simplify $20a^9 \div 4a^3$. **[2]**

(c) Simplify $(2p^3q)^4$. **[2]**

(d) $4^9 \div 4^x = 4^2$. Find the value of x. **[1]**

(e) Simplify $k^2 \div k^6$. **[1]**

(a) $m^{4+7} = m^{11}$. **B1**

(b) $20 \div 4 = 5$ and $a^{9-3} = a^6$, so the answer is $5a^6$. **B2** (**B1** if only the 5 or only the a^6 is right.)

(c) $2^4 = 16$, $(p^3)^4 = p^{12}$ and q^4, so the answer is $16p^{12}q^4$. **B2** (**B1** for two of the three parts right.)

(d) $9 - x = 2$, so $x = 7$. **B1**

(e) $k^{2-6} = k^{-4}$, or $\dfrac{1}{k^4}$. **B1**

Example 2

Extended only. Calculators must not be used in this question.

(a) Simplify $(27x^6)^{\frac{2}{3}}$. **[2]**

(b) Simplify fully $\left(\dfrac{16y^4}{x^{10}}\right)^{-\frac{1}{2}}$. **[3]**

(c) $2^n = \dfrac{1}{\sqrt{2}}$. Find the value of n. **[1]**

(a) $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$ and $x^{6\times\frac{2}{3}} = x^4$. Answer: $9x^4$. **B2** (**B1** for $9x^k$ or kx^4.)

(b) The negative index turns the fraction upside down: $\left(\dfrac{x^{10}}{16y^4}\right)^{\frac{1}{2}}$. **M1**

Square root of each part: $\sqrt{x^{10}} = x^5$, $\sqrt{16} = 4$, $\sqrt{y^4} = y^2$. **B1** for two of these.

Answer: $\dfrac{x^5}{4y^2}$. **A1**

(c) $\sqrt{2} = 2^{\frac{1}{2}}$, so $\dfrac{1}{\sqrt{2}} = 2^{-\frac{1}{2}}$ and $n = -\frac{1}{2}$. **B1**

Example 3

Extended only.

(a) Solve $8^x = 4^{x+2}$. **[3]**

(b) $27^m \times 3 = 9^{2n}$. Find m in terms of n. **[3]**

(a) $8 = 2^3$ and $4 = 2^2$, so $(2^3)^x = (2^2)^{x+2}$. **M1** for writing both sides as powers of 2.

$2^{3x} = 2^{2x+4}$, so $3x = 2x + 4$. **M1** for a correct linear equation.

$x = 4$. **A1** Check: $8^4 = 4096$ and $4^6 = 4096$.

(b) $27^m = (3^3)^m = 3^{3m}$ and $9^{2n} = (3^2)^{2n} = 3^{4n}$. **M1**

$3^{3m} \times 3^1 = 3^{3m+1}$, so $3m + 1 = 4n$. **M1**

$m = \dfrac{4n-1}{3}$. **A1**

Common mistakes

- **Multiplying the indices when you multiply terms.** $x^4 \times x^5 = x^9$, not x^{20}. Examiners see the rules swapped every year: add for $\times$, subtract for $\div$, multiply only for a power of a power.
- **Dividing the indices when you divide terms.** $x^{15} \div x^5 = x^{10}$, not x^3.
- **Adding the indices for a power of a power.** $(x^4)^3 = x^{12}$, not x^7.
- **Forgetting the number, or a letter, when a bracket is raised to a power.** $(2ab^3)^5 = 32a^5b^{15}$. Answers such as $2a^5b^{15}$ (number not raised) or $32ab^{15}$ (a missed) are common and lose marks.
- **Leaving out the letter.** $7w^4 \times 3w^6 = 21w^{10}$, not just 21. Examiners have seen the letter part dropped altogether.
- **Extended only: Treating a fractional power as multiplying the number by the fraction.** In $(64x^{15})^{\frac{1}{3}}$ the 64 is cube-rooted to 4, not multiplied by $\frac{1}{3}$; only the index 15 is multiplied by $\frac{1}{3}$. The answer is $4x^5$.
- **Extended only: Rooting only part of a term.** $\sqrt{49x^2} = 7x$: root the 49 **and** the x^2.
- **Making the whole term 1 with a zero index.** $4y^0 = 4$, not 1 or 0.
- **Dividing instead of subtracting in a missing-index question.** $3^{16} \div 3^t = 3^4$ means $16 - t = 4$, so $t = 12$, not $16 \div 4 = 4$.
- **Writing a fraction as a power wrongly.** $\frac{1}{27} = 3^{-3}$, not 3^3 or -3^3.
- **Extended only: Changing the base wrongly.** $9^x = 3^{2x}$, not 3^x. Put the new power in a bracket, $(3^2)^x$, so the x multiplies the whole index.
- **Stopping part-way with a negative power.** $\left(\frac{x^2}{5}\right)^{-2}$ is not finished at $\left(\frac{x^2}{5}\right)^2$ with the minus dropped; turn the fraction over to get $\frac{25}{x^4}$.
- **Not answering in the form asked.** "As a single power of 5" wants 5^2, not 25.

How to get the marks

- **"Simplify"** means one term, with each letter written once. On a 2-mark simplify question, there is usually a mark for the right number and a mark for the right power: write both, even if you are unsure of one.
- **"Final answer"**: mark schemes often give the mark only if the correct answer is your last line. Don't carry on and spoil it, for example by turning $10a^7$ into $10 + a^7$.
- **Negative indices in answers** are fine unless the question says otherwise: p^{-3} and $\frac{1}{p^3}$ both score.
- **"Find the value of"** a missing index wants just the number. One mark, no working needed, but writing the equation (for example $8 + x = 15$) helps you avoid slips.
- **Extended only: Equations with the unknown in the index**: the method marks are for writing both sides as powers of the **same base** and for the correct linear equation from the indices. Show those two lines.
- **Extended only: "Find m in terms of n"** means $m = \dots$ with only n on the other side. Any correct equivalent form scores, such as $\frac{4n-1}{3}$ or $\frac{4}{3}n - \frac{1}{3}$.
- **Extended only: Exact answers**: give fractions such as $\frac{5}{3}$, not rounded decimals such as 1.67.
- **Papers 1 and 2 are non-calculator**, so write each step of index work by hand.
- **Extended only:** show the root step (for example $\sqrt[4]{81} = 3$) when you work out a fractional power of a number.

Quick check

1. Simplify (a) $c^3 \times c^9$, (b) $24y^{10} \div 6y^5$, (c) $(5x^2y^3)^2$.
2. (a) $g^x \times g^4 = g^{13}$. Find x. (b) $5^k = \dfrac{1}{125}$. Find k.
3. **Extended only.** Simplify $(16x^8)^{\frac{3}{4}}$.
4. **Extended only.** Simplify $\left(\dfrac{x^6}{9}\right)^{-\frac{1}{2}}$.
5. **Extended only.** Solve $9^{x-1} = 27^x$.

Answers

1. (a) c^{12}. (b) $24 \div 6 = 4$ and $y^{10-5} = y^5$, so $4y^5$. (c) $5^2 = 25$, $x^{2\times2} = x^4$, $y^{3\times2} = y^6$, so $25x^4y^6$.
2. (a) $x + 4 = 13$, so $x = 9$. (b) $\frac{1}{125} = \frac{1}{5^3} = 5^{-3}$, so $k = -3$.
3. $16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$ and $x^{8\times\frac{3}{4}} = x^6$, so $8x^6$.
4. Turn it over: $\left(\dfrac{9}{x^6}\right)^{\frac{1}{2}} = \dfrac{3}{x^3}$.
5. $(3^2)^{x-1} = (3^3)^x$, so $3^{2x-2} = 3^{3x}$. Then $2x - 2 = 3x$, so $x = -2$.

Past questions to try

- **0580/11/M/J/21 Q17**: writing one over a product of 2s as a power of 2, a missing index when dividing, and multiplying two terms.
- **0580/22/F/M/22 Q13**: multiplying powers, a fraction to a negative power, and a missing index when dividing.
- **0580/22/O/N/22 Q20**: a fractional power of a term, and a fraction to a negative fractional power.
- **0580/21/O/N/21 Q22**: dividing fractional powers, then two equations with the unknown in the index.