Cambridge IGCSE · Mathematics (0580) · Notes

Algebraic manipulation and algebraic fractions

Read online at pastopic.com/cambridge/mathematics-0580/notes/algebraic-manipulation

Last checked 28 September 2026

Read first: Fractions, decimals, ordering and the four operations, Types of number and sets

What you need to know

Core and Extended share the first five points. The parts marked **Extended only** are for Papers 2 and 4. By the end of this topic you should be able to:

1. **Use letters for numbers.** A letter such as n stands for a number you don't know yet, or for any number. Write expressions from words, for example "4 more than n" is $n + 4$, and the cost of t tickets at $18 each is $18t$ dollars.
2. **Substitute numbers into expressions and formulas**, including negative numbers and fractions, and work out the value.
3. **Simplify by collecting like terms**, for example $4a - b + 2a + 6b = 6a + 5b$. **Extended only:** this includes terms such as a^2, ab and numbers in the same expression.
4. **Expand brackets**: a number or term times a bracket, such as $2x(5x - 3y)$, and two brackets multiplied together, such as $(3x - 2)(x + 5)$. **Extended only:** three or more brackets multiplied together, such as $(x + 1)(x - 4)(3x + 2)$, and two brackets with two letters, such as $(2x - y)(x + 3y)$.
5. **Factorise fully by taking out common factors**, for example $8x^2 - 12xy = 4x(2x - 3y)$.
6. **Extended only:** factorise these other kinds of expression:
    - four terms that pair up, such as $ax + bx + kay + kby$ (**grouping**);
    - the **difference of two squares**, $a^2x^2 - b^2y^2$;
    - a **perfect square**, $a^2 + 2ab + b^2$;
    - a **quadratic**, $ax^2 + bx + c$;
    - a **cubic with no number term**, $ax^3 + bx^2 + cx$ (take out x first).
7. **Extended only: complete the square**, writing $ax^2 + bx + c$ in the form $a(x + p)^2 + q$.
8. **Extended only:** add, subtract, multiply and divide **algebraic fractions**, including fractions with letters in the denominator, such as $\frac{2}{x+1} - \frac{x}{x-4}$.
9. **Extended only: simplify algebraic fractions** by factorising the top and bottom and cancelling common factors, such as $\frac{x^2+4x}{x^2+x-12} = \frac{x}{x-3}$.

Solving equations and changing the subject of a formula are in the topic Equations and changing the subject; the rules of indices are in Indices in algebra.

Understand it

Letters, terms and expressions

In algebra a letter stands for a number. $3n$ means $3 \times n$, $\frac{n}{4}$ means $n \div 4$, and n^2 means $n \times n$. We write the number first and leave out the $\times$ sign: $a \times 5 \times b$ is $5ab$.

- A **term** is a number, a letter, or numbers and letters multiplied together: 7, x, $-4xy$, $3a^2$.
- An **expression** is terms added or subtracted, such as $5x - 2y + 8$. It has no $=$ sign.
- An **equation** has an $=$ sign and is true only for certain values, such as $2x + 1 = 9$.
- A **formula** links quantities, such as $A = lw$ for the area of a rectangle.

Writing an expression from words means turning each phrase into operations on the letters. If a bus ticket costs 65 cents and you buy t tickets, the tickets cost $65t$ cents. Paying with \$10, which is 1000 cents, the change is $1000 - 65t$ cents. Check the units: the numbers you combine must all be in cents, or all in dollars.

Substituting

To **substitute**, replace each letter with its number and work out the value using the order of operations (brackets, then powers, then $\times$ and $\div$, then $+$ and $-$). Put **negative numbers in brackets** so you don't lose the sign.

For $P = 3a - b^2$ with $a = 4$ and $b = -5$:

$$P = 3 \times 4 - (-5)^2 = 12 - 25 = -13.$$

$(-5)^2 = 25$ because a negative times a negative is positive, and the minus in front of b^2 stays.

Collecting like terms

Like terms have exactly the same letters with the same powers: $4x$ and $-9x$ are like terms, and so are $2ab$ and $5ba$ (because $ab = ba$). But x and x^2 are **not** like terms, and neither are x and xy.

Each term carries the sign **in front of it**. So in $5a - 3b - 2a + 7b$ the terms are $+5a$, $-3b$, $-2a$ and $+7b$. Adding the a terms gives $5a - 2a = 3a$, and the b terms give $-3b + 7b = 4b$, so the answer is $3a + 4b$.

Expanding brackets

Expanding (or "multiplying out") means removing the brackets. The term outside multiplies **every** term inside:

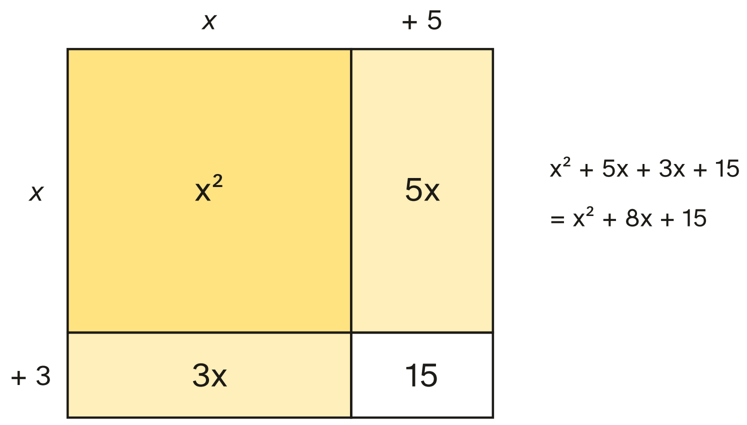
$$4(3x - 5) = 12x - 20, \qquad 2x(x + 7y) = 2x^2 + 14xy.$$

A **negative** term outside changes every sign inside: $-3(x - 5) = -3x + 15$, because $-3 \times -5 = +15$.

To **expand and simplify**, expand each bracket, then collect like terms:

$$5(x+2)-3(2x-1)=5x+10-6x+3=-x+13.$$

Two brackets. Every term in the first bracket multiplies every term in the second bracket, which gives four products. An area diagram shows why:



$$(x+3)(x+5)=x^2+5x+3x+15=x^2+8x+15.$$

The same works with negatives and numbers in front of x:

$$(2x-3)(x+4)=2x^2+8x-3x-12=2x^2+5x-12.$$

A squared bracket means the bracket times itself, so there are **four** products, not two:

$$(x-6)^2=(x-6)(x-6)=x^2-6x-6x+36=x^2-12x+36.$$

Extended only: with **three brackets**, expand any two of them and simplify, then multiply that answer by the third bracket, term by term, and simplify again. For $(x+1)(x-2)(x+5)$: $(x+1)(x-2)=x^2-x-2$, then

$$(x^2-x-2)(x+5)=x^3+5x^2-x^2-5x-2x-10=x^3+4x^2-7x-10.$$

Factorising with a common factor

Factorising is expanding backwards: you put the brackets back in. Find the **highest common factor** of all the terms (the biggest number and the most letters that divide every term), write it outside the bracket, and divide each term by it to get what goes inside:

$$12x^2-8xy=4x(3x-2y).$$

"Factorise" in this syllabus always means **factorise fully**. $2x(6x-4y)$ is correct algebra but not fully factorised, because $6x-4y$ still has a common factor 2. You can always check an answer by expanding it again.

Extended only: more ways to factorise

Grouping. Four terms often split into two pairs that share a bracket:

$$ax+3a+2x+6=a(x+3)+2(x+3)=(x+3)(a+2).$$

The bracket $(x+3)$ is a common factor of both parts, so it comes out. If the second pair starts with a minus, take out a negative factor so the brackets match: $xy-5x-2y+10=x(y-5)-2(y-5)=(y-5)(x-2)$.

The difference of two squares. Expanding $(a-b)(a+b)$ gives $a^2+ab-ab-b^2=a^2-b^2$; the middle terms cancel. So when you see one square **minus** another, factorise it straight away:

$$x^2-49=(x-7)(x+7), \qquad 9x^2-4y^2=(3x-2y)(3x+2y).$$

Look for a common factor first: $2x^2-72=2(x^2-36)=2(x-6)(x+6)$. A **sum** of two squares, such as x^2+49, does not factorise.

Perfect squares. $x^2+10x+25=(x+5)^2$, because $(a+b)^2=a^2+2ab+b^2$. The first and last terms are squares and the middle term is twice their product.

Quadratics x^2+bx+c. Find two numbers that **multiply to** c and **add to** b. For $x^2-2x-15$: the pairs that multiply to -15 include -5 and 3, which add to -2. So $x^2-2x-15=(x-5)(x+3)$.

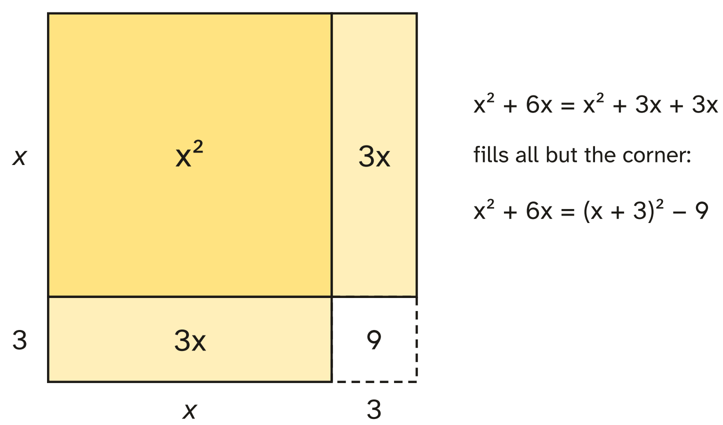
Quadratics ax^2+bx+c with $a\neq 1$. Find two numbers that multiply to $a\times c$ and add to b. Use them to split the middle term, then group. For $2x^2+7x-15$: $a\times c=-30$, and 10 and -3 multiply to -30 and add to 7. So

$$2x^2+10x-3x-15=2x(x+5)-3(x+5)=(x+5)(2x-3).$$

Cubics with no number term. Every term contains x, so take out x (or the highest common factor) first, then factorise the quadratic that is left: $x^3-3x^2-10x=x(x^2-3x-10)=x(x-5)(x+2)$.

Extended only: completing the square

$(x+3)^2=x^2+6x+9$. So x^2+6x is $(x+3)^2$ with 9 taken away:



In general, **halve the number in front of x** to get the number in the bracket, then **subtract its square**:

$$x^2+bx+c=\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c.$$

For example, $x^2-8x+5=(x-4)^2-16+5=(x-4)^2-11$.

When the x^2 term has a number a in front, take a out of the x^2 and x terms first, complete the square inside the bracket, then multiply back:

$$3x^2 + 12x + 1 = 3(x^2 + 4x) + 1 = 3\left[(x+2)^2 - 4\right] + 1 = 3(x+2)^2 - 11.$$

If a is negative, take out the minus sign the same way: $7 + 4x - x^2 = 7 - (x^2 - 4x) = 7 - \left[(x-2)^2 - 4\right] = 11 - (x-2)^2$.

You can always check by expanding your answer.

Extended only: algebraic fractions

Algebraic fractions follow exactly the same rules as number fractions.

- **Multiply**: top times top, bottom times bottom, then cancel. $\frac{2x}{5} \times \frac{10}{x^2} = \frac{20x}{5x^2} = \frac{4}{x}$. Cancelling common factors before multiplying makes it easier.
- **Divide**: turn the second fraction upside down and multiply. $\frac{5a}{6} \div \frac{10a}{9} = \frac{5a}{6} \times \frac{9}{10a} = \frac{45a}{60a} = \frac{3}{4}$.
- **Add or subtract**: use a common denominator. With number denominators, $\frac{x}{4} + \frac{x-2}{3} = \frac{3x}{12} + \frac{4(x-2)}{12} = \frac{7x-8}{12}$. With letters in the denominators, the common denominator is the product of the two denominators:

$$\frac{2}{x} + \frac{3}{x+1} = \frac{2(x+1) + 3x}{x(x+1)} = \frac{5x+2}{x(x+1)}.$$

Put brackets round each numerator when you multiply it, especially when you subtract: $\frac{x+1}{2} - \frac{x-3}{5} = \frac{5(x+1) - 2(x-3)}{10} = \frac{5x + 5 - 2x + 6}{10} = \frac{3x+11}{10}$.

Extended only: simplifying algebraic fractions

You can cancel only **factors**, things that multiply the whole top and the whole bottom, never single terms. So first **factorise the top and the bottom fully**, then cancel any bracket that appears in both:

$$\frac{x^2 + 3x}{x^2 - 9} = \frac{x(x+3)}{(x-3)(x+3)} = \frac{x}{x-3}.$$

Cancelling the x^2 terms, or the 3 in $3x$ against the 9, is wrong, because they are terms added to other terms, not factors.

Key facts and formulas

Nothing in this topic is on the List of formulas in the exam, so learn all of these.

Formula	Status
$a(b+c) = ab + ac$, and $-a(b-c) = -ab + ac$	Learn it

Formula	Status
$(a+b)(c+d) = ac + ad + bc + bd$	Learn it
$(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$	Learn it
Extended only: $a^2 - b^2 = (a-b)(a+b)$	Learn it
Extended only: $x^2 + bx + c = (x+p)(x+q)$ where $pq = c$ and $p + q = b$	Learn it
Extended only: $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$	Learn it
Extended only: $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$	Learn it
Area of a rectangle $=$ length $\times$ width; perimeter $=$ total length of all the sides	Learn it

Also remember: $(-a)^2 = a^2$; $ab = ba$; x and x^2 are not like terms; and $x^2 + a^2$ does not factorise.

How to do it

In the Paper 1–4 questions we have tagged for this topic (155 questions, 2021–2025, of which 149 are different; the rest appear on two papers), the types came up this often. Counts are of the 149 different questions; 47 of them mix two or more types, so the counts add up to more than 149.

Question type	Questions
Expanding two or three brackets	30
Factorising with a common factor	30
Writing expressions from words or diagrams	23
Substituting into expressions and formulas	22
Simplifying by collecting like terms	21
Factorising by grouping, difference of two squares and quadratics (Extended)	21
Algebraic fractions as a single fraction (Extended)	19
Expanding single brackets and simplifying	18
Simplifying algebraic fractions by factorising (Extended)	17
Completing the square (Extended)	9

1. Expanding two or three brackets (30 questions)

1. **Multiply every term in the first bracket by every term in the second.** Write all four products, keeping each sign with its term: $(x-7)(x+3) = x^2 + 3x - 7x - 21.$
2. **Collect the like terms** (usually the two x terms): $x^2 - 4x - 21.$
3. **Give the simplified answer as your final answer**, and don't do anything else to it.

Variations you will meet:

- **A squared bracket**: write it as two brackets first. $(3x-1)^2 = 9x^2 - 6x + 1$.
- **Extended only: two letters**: $(4x-y)(2x+3y) = 8x^2 + 12xy - 2xy - 3y^2 = 8x^2 + 10xy - 3y^2$. xy and yx are like terms.
- **An area**: a rectangle with sides $(x+5)$ and $(x-2)$ has area $(x+5)(x-2) = x^2 + 3x - 10$.
- **Extended only: three brackets.** Expand two brackets and simplify, then multiply by the third and simplify; the answer has up to four terms, such as $ax^3 + bx^2 + cx + d$. Pick the pair that is easiest to multiply first. "Show that" versions give the answer: every line of working must be shown.
- **Extended only:** a difference of two squared brackets, such as $(3x-2)^2 - (x+4)^2$: expand each, put the second in a bracket after the minus sign, then collect. $(9x^2 - 12x + 4) - (x^2 + 8x + 16) = 8x^2 - 20x - 12$.
- **Extended only:** matching coefficients, such as "$(x+a)^2 = x^2 + 14x + b$". Expand the left side, $x^2 + 2ax + a^2$, then match: $2a = 14$ gives $a = 7$, and $b = a^2 = 49$.

2. Factorising with a common factor (30 questions)

1. **Find the highest common factor of the numbers** (the HCF), for example 5 for 20 and 15.
2. **Find the letters in every term**, using the lowest power of each: x^2 and x share x.
3. **Write the HCF outside a bracket** and divide each term by it: $20x^2 - 15x = 5x(4x-3)$.
4. **Check** by expanding, and check that nothing inside the bracket still has a common factor.

Variations you will meet:

- **Two letters**: $21xy - 14y^2 = 7y(3x-2y)$.
- **A term that is the whole HCF** leaves 1 in the bracket: $5x^2 + 5x = 5x(x+1)$, not $5x(x)$.
- **Three terms**: the same method, $4a^2 - 8ab + 12a = 4a(a - 2b + 3)$.
- **"Explain why this is not fully factorised"**, for example $3x(2x-4)$ for $6x^2 - 12x$: say that the bracket still has a common factor of 2, and give $6x(x-2)$.
- **Extended only: a common bracket.** In $5(p+q)^2 - 20(p+q)$ the bracket $(p+q)$ is a common factor: $5(p+q)\left[(p+q)-4\right] = 5(p+q)(p+q-4)$.

3. Writing expressions from words or diagrams (23 questions)

1. **Decide what each letter stands for** and what the answer must measure (cents, dollars, points, lengths).
2. **Turn each phrase into an operation**: "each costs" means multiply, "more than" means add, "fewer than" or "change from" means subtract, "shared equally" means divide.
3. **Put the units right**: if the answer is in cents, change dollars to cents first ($\$20 = 2000$ cents).
4. **Simplify** if asked for the simplest form, and give an **expression**, not an equation: no "$= \ldots$" and no solving.

Variations you will meet:

- **Total cost**: x mangoes at 45 cents and y lemons at 30 cents cost $45x + 30y$ cents.
- **Change**: buying k items at c cents each with a \$10 note gives $1000 - kc$ cents change.
- **Points or scores**: w wins at 2 points and d draws at 1 point give $2w + d$ points.
- **Ages or amounts compared**: if Ana has x stamps and Ben has 4 fewer, Ben has $x - 4$; twice as many as Ben is $2(x - 4)$.
- **A formula**: a fixed charge of \$40 plus \$25 per day for d days gives $C = 25d + 40$ (here it is a formula, so it has "$C =$").
- **Perimeter or area from a diagram**: add the side lengths and collect like terms, or multiply the lengths for a rectangle's area.
- **Extended only: a time as a fraction.** Time $= \dfrac{\text{distance}}{\text{speed}}$, so 15 km at x km/h takes $\dfrac{15}{x}$ hours. These questions often continue by forming an equation (see type 7).

4. Substituting into expressions and formulas (22 questions)

1. **Write the formula with each letter replaced by its number**, putting negatives in brackets: $R = 4P + 7Q$ with $P = 5$, $Q = -2$ gives $R = 4(5) + 7(-2)$.
2. **Work it out in the right order**: powers first, then multiply and divide, then add and subtract. $20 - 14 = 6$.
3. **Give the exact value** unless the question asks for rounding.

Variations you will meet:

- **Squares of negatives**: $(-3)^2 = 9$, but -3^2 means $-(3^2) = -9$. In $2m^2$ with $m = -3$, square first: $2 \times 9 = 18$.
- **Subtracting a negative**: $u - 9.8t$ with $u = 3$ and $t = -5$ is $3 - 9.8 \times (-5) = 3 + 49 = 52$.
- **Fractions and brackets in the formula**, such as $s = \frac{1}{2}(u + v)t$: work out the bracket first.
- **Find a letter that isn't the subject**, such as "find t when $q = 29$ and $r = 5$" in $q = 4r + 3t$. Substitute the numbers you know, $29 = 20 + 3t$, then solve: $3t = 9$, so $t = 3$. (Solving equations is covered in Equations and changing the subject.)

5. Simplifying by collecting like terms (21 questions)

1. **Underline or circle each term with the sign in front of it.**
2. **Group like terms**: same letters, same powers.
3. **Add their numbers**, keeping the signs: $9c - 2d - 4c + 5d = 5c + 3d$.
4. **Write the terms in a sensible order** (usually letters in alphabetical order, then numbers).

Variations you will meet:

- **A term with no number** has a 1: y is $1y$, so $5y - 7y + 3y = y$.
- **An answer of** 0: $4x - 4x = 0$, so the term disappears.

- **Extended only: mixed powers**, such as $4a^2 - 2ab + 3 + a^2 + 5ab - 7 = 5a^2 + 3ab - 4$. Collect a^2 terms, ab terms and numbers separately.
- **Extended only: fractions of a letter** (usually with a common denominator): $\frac{3x}{4} - \frac{x}{6} + \frac{5x}{12} = \frac{9x-2x+5x}{12} = \frac{12x}{12} = x$.

6. Factorising by grouping, difference of two squares and quadratics (21 questions)

Extended only. First **take out any common factor** of all the terms. Then look at what is left:

1. **Two terms, a square minus a square**: use $a^2 - b^2 = (a-b)(a+b)$. $4x^2 - 25y^2 = (2x-5y)(2x+5y)$.
2. **Three terms,** $x^2 + bx + c$: find two numbers with product c and sum b. $x^2 - 9x + 20 = (x-4)(x-5)$.
3. **Three terms,** $ax^2 + bx + c$: find two numbers with product ac and sum b, split the middle term and group. $2x^2 - 5x - 12$: $ac = -24$, numbers -8 and 3; $2x^2 - 8x + 3x - 12 = 2x(x-4) + 3(x-4) = (x-4)(2x+3)$.
4. **Four terms**: group in pairs, take a common factor from each pair, then take out the common bracket.
5. **Check** by expanding.

Variations you will meet:

- **A common factor hiding a difference of two squares**: $12a^2 - 75 = 3(4a^2 - 25) = 3(2a-5)(2a+5)$. Stopping at $3(4a^2 - 25)$ is not fully factorised.
- **A cubic**: $x^3 - 36x = x(x^2 - 36) = x(x-6)(x+6)$.
- **Grouping with the terms in a jumbled order**: rearrange so each pair shares a factor. $3x - 12 - bx + 4b = 3(x-4) - b(x-4) = (x-4)(3-b)$. For $1 - p - 2k + 2kp$: $1(1-p) - 2k(1-p) = (1-p)(1-2k)$.
- **A common bracket already there**: $3(m-5n) + 2m(m-5n) = (m-5n)(3+2m)$. With a squared bracket, such as $4x(x+3y) + 3(x+3y)^2$, take out $(x+3y)$ and simplify what is left: $(x+3y)\big(4x + 3(x+3y)\big) = (x+3y)(7x+9y)$.
- **Using a factorisation next**: a later part may say "hence solve" or "hence find integers" using your factors.

7. Algebraic fractions as a single fraction (19 questions)

Extended only.

Adding or subtracting:

1. **Find the common denominator**: the lowest common multiple of number denominators; the **product** of two different algebraic denominators, such as $(3x-1)(x+2)$.
2. **Multiply each numerator by what its denominator was multiplied by**, in brackets: $\dfrac{2}{3x-1} + \dfrac{5}{x+2} = \dfrac{2(x+2) + 5(3x-1)}{(3x-1)(x+2)}$.

3. **Expand the top and collect like terms**: $\frac{17x-1}{(3x-1)(x+2)}$. Leave the bottom factorised (or expand it; both are accepted), and check whether anything cancels.

Multiplying or dividing: for ÷, turn the second fraction upside down first. Then multiply tops and bottoms and cancel common factors of numbers and letters. **Don't** use a common denominator to multiply.

Variations you will meet:

- **A whole number and a fraction**: write the number over 1. $3 - \frac{2}{x+4} = \frac{3(x+4)-2}{x+4} = \frac{3x+10}{x+4}$.
- **Three fractions with number denominators**: use the lowest common multiple, for example 30 for 5, 10 and 15.
- **A denominator that factorises**: factorise it first; the common denominator may be smaller. $\frac{2}{x+3} + \frac{5}{x^2+3x} = \frac{2x}{x(x+3)} + \frac{5}{x(x+3)} = \frac{2x+5}{x(x+3)}$.
- **Show that an equation simplifies to a quadratic** (in a speed or cost context): write the two fractions, such as $\frac{8}{x} + \frac{6}{x+2} = 2$, multiply every term by the whole common denominator $x(x+2)$, expand every bracket, and collect everything on one side. Here $8(x+2) + 6x = 2x(x+2)$ gives $2x^2 - 10x - 16 = 0$, or $x^2 - 5x - 8 = 0$. Show each step, because the answer is given.

8. Expanding single brackets and simplifying (18 questions)

1. **Multiply every term inside by the term outside**, including its sign.
2. **Collect like terms.**

For example, $4(3d-2) + 5(d+1) = 12d - 8 + 5d + 5 = 17d - 3$.

Variations you will meet:

- **A minus before the second bracket**: $5(t-q) - 3(t-2q) = 5t - 5q - 3t + 6q = 2t + q$. The -3 multiplies **both** terms, and $-3 \times -2q = +6q$.
- **A letter outside**: $h(4-3h) = 4h - 3h^2$; $2x(5x-3) = 10x^2 - 6x$.
- **An expression before a bracket**: $3x - x(4-x) = 3x - 4x + x^2 = x^2 - x$.
- **A bracket with no number in front after a minus**: $3(x-4) - (5-2x) = 3x - 12 - 5 + 2x = 5x - 17$.

9. Simplifying algebraic fractions by factorising (17 questions)

Extended only.

1. **Factorise the numerator fully** (common factor, difference of two squares, quadratic or grouping).
2. **Factorise the denominator fully.**
3. **Cancel the bracket that appears on the top and the bottom**, and write what is left.

For example, $\frac{2x^2 - 9x - 5}{x^2 - 25}$: the top is $(2x+1)(x-5)$ and the bottom is $(x-5)(x+5)$, so the answer is $\frac{2x+1}{x+5}$.

Variations you will meet:

- **A common factor on one line and a difference of two squares on the other:** $\frac{4x - x^2}{16 - x^2} = \frac{x(4-x)}{(4-x)(4+x)} = \frac{x}{4+x}$. Write both lines with the brackets the **same way round** so you can see what cancels.
- **Brackets that are negatives of each other**: $6 - x = -(x-6)$. So $\frac{6x - x^2}{x^2 - 36} = \frac{-x(x-6)}{(x-6)(x+6)} = -\frac{x}{x+6}$.
- **Four terms on top**: factorise by grouping first. $\frac{2xy + 10y - 3x - 15}{x^2 - 25} = \frac{(x+5)(2y-3)}{(x-5)(x+5)} = \frac{2y-3}{x-5}$.
- **Two quadratics**: factorise each, then cancel. If nothing cancels, check your factorising by expanding.

10. Completing the square (9 questions)

Extended only.

1. **With x^2 alone**: halve the number in front of x to get p, then write $(x+p)^2 - p^2 + c$ and work out the number. $x^2 + 8x + 3 = (x+4)^2 - 16 + 3 = (x+4)^2 - 13$.
2. **With ax^2**: take out a from the first two terms, complete the square inside, then multiply back and collect the numbers. $3x^2 + 18x - 4 = 3(x^2 + 6x) - 4 = 3\left[(x+3)^2 - 9\right] - 4 = 3(x+3)^2 - 31$.
3. **Check by expanding.**

Variations you will meet:

- **"Find a and b when $x^2 - 14x + a$ can be written as $(x+b)^2$"**: $(x+b)^2 = x^2 + 2bx + b^2$, so $2b = -14$ gives $b = -7$, and $a = b^2 = 49$.
- **Matching with an unknown in the middle**: $(x-3)^2 + k = x^2 - px + 2$ gives $x^2 - 6x + 9 + k$, so $p = 6$ and $9 + k = 2$, $k = -7$.
- **A negative x^2 term**, such as $10 - 5x - x^2$ in the form $b - (x+a)^2$: take out the minus sign, $10 - (x^2 + 5x) = 10 - \left[(x+2.5)^2 - 6.25\right] = 16.25 - (x+2.5)^2$.
- **Using the completed square**: the turning point of $y = (x+p)^2 + q$ is $(-p, q)$, and solving $(x+p)^2 + q = 0$ gives $x = -p \pm \sqrt{-q}$. These uses belong to the equations and graphs topics, but the completed square is the first step.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

(a) Simplify $4p - 7q - 9p + 2q$. **[2]**

(b) Expand and simplify $5(2x - 3) - 2(x - 4)$. **[2]**

(c) Factorise fully $12x^2y - 18xy$. **[2]**

(d) $T = 3a - b^2$. Find the value of T when $a = -2$ and $b = -4$. **[2]**

(a) $4p - 9p = -5p$ and $-7q + 2q = -5q$, so the answer is $-5p - 5q$. **B2**, or **B1** for one of the two terms right.

(b) $10x - 15 - 2x + 8$ **B1** for the expanded brackets (the -2 gives $-2x + 8$), then $8x - 7$. **B1**

(c) The HCF of 12 and 18 is 6, and both terms contain x and y. So $12x^2y - 18xy = 6xy(2x - 3)$. **B2**, or **B1** for a correct partial factorisation such as $6x(2xy - 3y)$.

(d) $T = 3 \times (-2) - (-4)^2 = -6 - 16 = -22$. **M1** for the correct substitution, **A1** for -22.

Example 2

(a) Bottles of water cost 85 cents each. Ravi buys n bottles and pays with a $\$50$ note. Write an expression, in terms of n, for the change he receives, in cents. **[2]**

(b) A rectangle has length $(2x - 1)$ cm and width $(x + 6)$ cm. Find an expression for its area. Give your answer in the form $ax^2 + bx + c$. **[2]**

(c) Expand and simplify $(x - 4)^2 - (x + 1)(x - 3)$. **[3]**

(a) $\$50 = 5000$ cents, and n bottles cost $85n$ cents. Change $= 5000 - 85n$ cents. **B2**, or **B1** for $85n$ in the answer.

(b) $(2x - 1)(x + 6) = 2x^2 + 12x - x - 6$ **B1** for four correct terms, $= 2x^2 + 11x - 6$ cm². **B1**

(c) $(x - 4)^2 = x^2 - 8x + 16$ **B1**, and $(x + 1)(x - 3) = x^2 - 2x - 3$ **B1**.

$$x^2 - 8x + 16 - (x^2 - 2x - 3) = x^2 - 8x + 16 - x^2 + 2x + 3 = -6x + 19.$$

B1. The bracket round the second expansion makes every sign change.

Example 3

Extended only.

(a) Expand and simplify $(x+4)(x-1)(2x+3)$. **[3]**

(b) Factorise fully $3xy-6x+5y-10$. **[2]**

(c) Write $2x^2-12x+7$ in the form $a(x+b)^2+c$. **[3]**

(a) $(x+4)(x-1)=x^2-x+4x-4=x^2+3x-4$. **B1** for one pair of brackets expanded correctly.

$$(x^2+3x-4)(2x+3)=2x^3+3x^2+6x^2+9x-8x-12=2x^3+9x^2+x-12.$$

B1 for the correct unsimplified expansion, **B1** for the simplified four-term answer.

(b) $3x(y-2)+5(y-2)$ **M1**, $=(y-2)(3x+5)$. **A1**

(c) $2x^2-12x+7=2(x^2-6x)+7$. **B1** for $a=2$.

$$2\left[(x-3)^2-9\right]+7=2(x-3)^2-18+7=2(x-3)^2-11.$$

B1 for $b=-3$, **B1** for $c=-11$. Check: $2(x^2-6x+9)-11=2x^2-12x+7$.

Example 4

Extended only.

(a) Write as a single fraction in its simplest form $\dfrac{3}{x+2}-\dfrac{2}{2x-1}$. **[3]**

(b) Simplify $\dfrac{3x^2-10x-8}{x^2-16}$. **[4]**

(a) Common denominator $(x+2)(2x-1)$. **B1**

$$\frac{3(2x-1)-2(x+2)}{(x+2)(2x-1)}=\frac{6x-3-2x-4}{(x+2)(2x-1)}=\frac{4x-7}{(x+2)(2x-1)}.$$

B1 for $3(2x-1)-2(x+2)$ or better as the numerator, **B1** for the final answer. Watch the $-2\times+2=-4$.

(b) Top: $ac=3\times(-8)=-24$; -12 and 2 multiply to -24 and add to -10.

$$3x^2-12x+2x-8=3x(x-4)+2(x-4)=(3x+2)(x-4).$$

B2 (or **B1** for a partial factorisation such as $3x(x-4)+2(x-4)$).

Bottom: $x^2-16=(x-4)(x+4)$. **B1**

$$\frac{(3x+2)(x-4)}{(x-4)(x+4)}=\frac{3x+2}{x+4}.$$

B1 for the final answer.

Common mistakes

- **Sign errors when a negative multiplies a bracket.** $-3(x-5)$ is $-3x+15$, not $-3x-15$. Examiners see this every year, especially in "expand and simplify" with a minus before the second bracket. Write the multiplied-out terms before collecting.
- **Only multiplying the first term.** $7(x+2)$ is $7x+14$, not $7x+2$. The outside term multiplies everything in the bracket.
- **Collecting like terms but losing the signs.** In $6f+4g-9f+g$, the $9f$ is $-9f$; the answer is $-3f+5g$. Keep each sign attached to the term after it.
- **Squaring a bracket as two terms.** $(x+4)^2$ is $x^2+8x+16$, not x^2+16. Write it as $(x+4)(x+4)$.
- **Partial factorisation.** $2x(4x-6)$ is not "fully" factorised and loses a mark. After taking out a factor, check whether the bracket still has a common factor. **Extended only:** also check for a difference of two squares: $2(9a^2-49)$ should become $2(3a-7)(3a+7)$.
- **Spoiling a correct answer.** Mark schemes often say "final answer": a correct answer that you then change loses marks. Common examples are "solving" an expression by setting it equal to 0, or going on to cancel a correct factorised answer wrongly. **Extended only:** another is writing $x(x^2-36)$ as $x(x-6)^2$.
- **Extended only: cancelling terms instead of factors.** In $\frac{x^2-25}{x^2-3x-10}$ you cannot cancel the x^2s. Factorise the top and bottom completely first, then cancel whole brackets.
- **Extended only: using a common denominator to multiply fractions.** To multiply or divide, work straight across (turning the second fraction over for $\div$). Common denominators are only for adding and subtracting.
- **Extended only: forgetting to multiply the numerator.** When you change $\frac{5}{3x+1}$ to a denominator of $(3x+1)(x-2)$, the top becomes $5(x-2)$. Put the numerators in brackets.
- **Substituting a negative without brackets.** $-(-25)$ is $+25$, and $(-4)^2=16$. Writing -4^2 on a calculator gives -16.
- **Mixing units in an expression.** "Change in cents from \$20" needs 2000, not 20. Check that every number in the expression is in the unit the question asks for.
- **Writing an equation instead of an expression**, such as "$x+(x+5)=23$" when only the expression $2x+5$ was asked for. Read whether the question wants an expression, an equation or a formula.

How to get the marks

- **"Simplify"** means collect like terms (and cancel fractions) until nothing more can be done. **"Expand"** means remove the brackets; **"expand and simplify"** means then collect like terms too.
- **"Factorise"** and **"factorise completely"** both mean fully. A correct but partial factorisation usually earns one mark out of two.
- **Show the unsimplified expansion** before you simplify. With two or three brackets, one mark is often for the four (or more) products with at least three correct, so write them all.

- **Give one final answer.** An answer "seen then spoilt" by further wrong working usually loses the last mark. Once it is simplified or factorised, stop.
- **Extended only: single fractions:** a mark is often for the correct common denominator and one for the correct numerator, in brackets, before simplifying. You can leave the denominator factorised.
- **Extended only: simplifying fractions:** each factorised line earns marks (for example one for the top and one for the bottom), so write the factorised forms before you cancel.
- **Extended only: completing the square:** give each value asked for (a, b, c or p and q) separately, and check by expanding.
- **"Show that"** means the answer is given. Every step must be visible, with no errors, and your last line must match the printed result.
- **Substitution:** writing the substitution with the numbers in (such as $3 \times 6 + 2 \times (-3)$) usually earns a method mark even if you then slip, so always write it.
- **Expressions in context:** give the answer in the units asked for, in terms of the letters asked for, and in its simplest form if the question says so.

Quick check

1. (a) Simplify $6a + 2b - 9a + 5b$. (b) Find the value of $4x - 3y^2$ when $x = -2$ and $y = -3$.
2. Expand and simplify $(x - 6)(x + 4)$.
3. Factorise fully $15m^2n - 20mn^2$.
4. **Extended only.** (a) Factorise fully $2x^2 - 50$. (b) Write $x^2 + 10x + 4$ in the form $(x + p)^2 + q$.
5. **Extended only.** Write as a single fraction in its simplest form $\dfrac{2x+1}{4} - \dfrac{x-3}{6}$.

Answers

1. (a) $6a - 9a = -3a$ and $2b + 5b = 7b$, so $-3a + 7b$. (b) $4(-2) - 3(-3)^2 = -8 - 3 \times 9 = -8 - 27 = -35$.
2. $x^2 + 4x - 6x - 24 = x^2 - 2x - 24$.
3. The HCF is $5mn$: $15m^2n - 20mn^2 = 5mn(3m - 4n)$.
4. (a) $2(x^2 - 25) = 2(x - 5)(x + 5)$. (b) Half of 10 is 5: $(x + 5)^2 - 25 + 4 = (x + 5)^2 - 21$, so $p = 5$ and $q = -21$.
5. The common denominator is 12: $\frac{3(2x+1) - 2(x-3)}{12} = \frac{6x + 3 - 2x + 6}{12} = \frac{4x + 9}{12}$.

Past questions to try

- **0580/42/O/N/25 Q17**: expanding three brackets.
- **0580/41/M/J/25 Q18**: completing the square with a number in front of x^2.
- **0580/21/O/N/25 Q25**: grouping, then a difference of two squares, then cancelling.
- **0580/22/M/J/25 Q19**: multiplying, adding and subtracting algebraic fractions.