Cambridge IGCSE · Mathematics 0580 · Notes

Direct and inverse proportion

Read online at **pastopic.com/cambridge/mathematics-0580/notes/direct-and-inverse-proportion**

Last checked 29 September 2026

Read first: **Ratio, proportion and rates**, **Equations and changing the subject**

What you need to know

Extended only: this whole topic is in the Extended syllabus only (Papers 2 and 4). There is no Core version, so Core students can skip it. By the end of this topic you should be able to:

1. **Read and use the proportion symbol $\propto$.** "$y \propto x$" means "y is directly proportional to x", and "$y \propto \frac{1}{x}$" means "y is inversely proportional to x".
2. **Turn a statement of proportion into an equation** with a constant k, for direct and for inverse proportion, including proportion to a **square**, **cube**, **square root** or **cube root**, and to a bracket such as $(x+2)$.
3. **Use that equation to find unknown quantities**: first the constant k from one pair of values, then any other value of either variable.

The questions also use these ideas in two ways that follow from them: saying what happens to one quantity when the other is multiplied by a factor (or changed by a percentage), and linking two proportion statements into one formula.

Understand it

Direct proportion

Two quantities are **directly proportional** when they grow at the same rate: double one and the other doubles, multiply one by 10 and the other is multiplied by 10. The cost of petrol is directly proportional to the number of litres. At $\$1.50$ a litre, 40 litres cost $\$60$ and 80 litres cost $\$120$.

In symbols, $C \propto L$, and every direct proportion can be written as an equation

$$C = kL,$$

where k is a fixed number called the **constant of proportionality**. Here $k = 1.5$, the price per litre. The ratio $\frac{C}{L}$ is the same for every pair of values: it is always k.

Inverse proportion

Two quantities are **inversely proportional** when one goes **up** as the other goes **down**, in a fixed way: double one and the other **halves**. The time for a 120 km journey is inversely proportional to the speed: at 40 km/h it takes 3 hours, at 60 km/h 2 hours, at 80 km/h 1.5 hours.

In symbols, $t \propto \frac{1}{s}$, and the equation is

$$t = \frac{k}{s},$$

with $k = 120$ here. This time the **product** $t \times s$ is always the same: it is always k.

Squares, cubes and roots

A quantity can be proportional to a **power** or a **root** of another. Put that power or root where x was:

In words	With $\propto$	Equation
y is (directly) proportional to x	$y \propto x$	$y = kx$
y is proportional to the square of x	$y \propto x^2$	$y = kx^2$
y is proportional to the cube of x	$y \propto x^3$	$y = kx^3$
y is proportional to the square root of x	$y \propto \sqrt{x}$	$y = k\sqrt{x}$
y is proportional to the cube root of x	$y \propto \sqrt[3]{x}$	$y = k\sqrt[3]{x}$
y is inversely proportional to x	$y \propto \frac{1}{x}$	$y = \dfrac{k}{x}$
y is inversely proportional to the square of x	$y \propto \frac{1}{x^2}$	$y = \dfrac{k}{x^2}$
y is inversely proportional to the square root of x	$y \propto \frac{1}{\sqrt{x}}$	$y = \dfrac{k}{\sqrt{x}}$

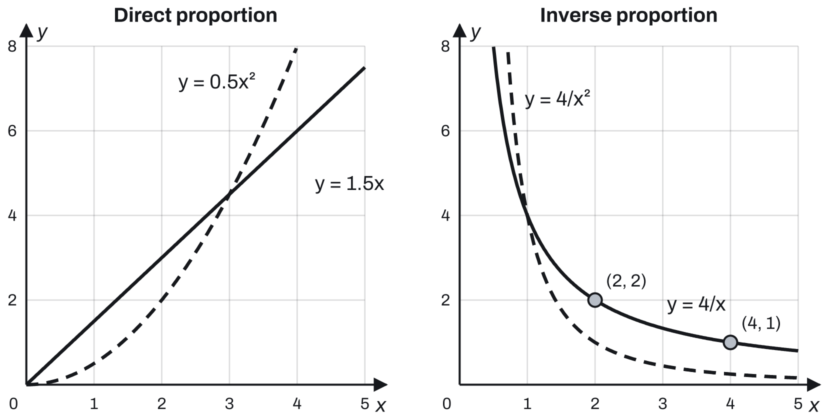
The pattern is always the same: **direct** means $k\times$ (the thing), **inverse** means $k\div$ (the thing). "Proportional to" on its own means directly proportional.

With $y = 3x^2$: $x = 1, 2, 4$ give $y = 3, 12, 48$. Doubling x multiplies y by 4, not 2, because the x is squared.

When the thing is a bracket

"y is proportional to the square of $(x+2)$" means the **whole bracket** is squared: $y = k(x+2)^2$. It is **not** $kx^2 + 2$. If $y = 18$ when $x = 1$, then $18 = k \times 3^2$, so $k = 2$ and $y = 2(x+2)^2$. When $x = 3$, $y = 2 \times 5^2 = 50$.

What the graphs look like



- A **direct** proportion graph always passes through the **origin**: when $x = 0, y = k \times 0 = 0$. Only $y = kx$ is a straight line; $y = kx^2$ curves upwards.
- An **inverse** proportion graph never touches either axis. As x gets bigger, y gets smaller, but never reaches 0. On $y = \frac{4}{x}$, doubling x from 2 to 4 halves y from 2 to 1.

Why you always find k first

The statement "$y \propto \sqrt{x}$" tells you the **shape** of the relationship, but not its size. One pair of values fixes the size. If $y = 12$ when $x = 9$, then $12 = k\sqrt{9} = 3k$, so $k = 4$ and the full rule is $y = 4\sqrt{x}$. Now you can find y for any x, or x for any y.

There is a shortcut that gives the same answers: in a direct proportion, $\dfrac{y}{\text{thing}}$ is the same for every pair; in an inverse one, $y \times \text{thing}$ is. For the bracket example above, $\dfrac{18}{3^2} = \dfrac{50}{5^2} = 2$. Mark schemes accept this "two pairs" method too, but the k method also gives the formula, which some questions ask for.

Multiplying by a factor

If $y = kx^n$ and x is multiplied by a, then y is multiplied by a^n; k does not matter.

- $y \propto x^2$: double x and y is multiplied by $2^2 = 4$.
- $y \propto x^3$: multiply x by 3 and y is multiplied by $3^3 = 27$.
- $y \propto \frac{1}{x^2}$: halve x and y is multiplied by $2^2 = 4$; triple x and y is multiplied by $\frac{1}{3^2} = \frac{1}{9}$.
- $y \propto \sqrt{x}$: multiply x by 4 and y is multiplied by $\sqrt{4} = 2$.

Key facts and formulas

None of these are on the list of formulas printed in the exam paper, so learn them all.

Formula	Status
$y \propto x$ means $y = kx$; "proportional to" means directly proportional	Learn it
$y \propto \dfrac{1}{x}$ means $y = \dfrac{k}{x}$	Learn it
Direct to a power or root: $y = kx^2$, $y = kx^3$, $y = k\sqrt{x}$, $y = k\sqrt[3]{x}$	Learn it
Inverse to a power or root: $y = \dfrac{k}{x^2}$, $y = \dfrac{k}{x^3}$, $y = \dfrac{k}{\sqrt{x}}$, $y = \dfrac{k}{\sqrt[3]{x}}$	Learn it
A bracket stays whole: $y \propto (x+a)^2$ means $y = k(x+a)^2$	Learn it
Direct: $\dfrac{y}{\text{thing}}$ is constant; inverse: $y \times \text{thing}$ is constant	Learn it
Direct, $y = kx^n$: multiplying x by a multiplies y by a^n. Inverse, $y = \dfrac{k}{x^m}$ (m positive): multiplying x by a means y is divided by a^m	Learn it

How to do it

In the questions we have tagged for this topic (23 questions, 2021–2025, all on Extended Papers 2 and 4), the types came up this often. 4 of the 23 questions mix two types, so the counts add up to more than 23.

Question type	Questions
Find a value: k first, then y (or x)	14
Write a formula for y in terms of x	5
Effect of multiplying by a factor, or a percentage change	5
Two linked proportions	3

1. Find a value: k first, then y (or x) (14 questions)

1. **Write the equation** with k, straight from the words: direct or inverse, and which power or root of what. Copy any bracket exactly.
2. **Substitute the pair you are given** and solve for k. Keep k exact (a fraction or a terminating decimal).
3. **Write the equation again with your** k in place.
4. **Substitute the new value** and work out the answer.

For example, y is inversely proportional to the square of x, and $y = 5$ when $x = 2$. Then $y = \frac{k}{x^2}$, so $5 = \frac{k}{4}$ and $k = 20$. When $x = 4$, $y = \frac{20}{16} = 1.25$.

Variations you will meet:

- **A bracket** such as $(x+5)$ or $(t-2)$: work out the bracket first, then the power or root. For $y=\frac{k}{\sqrt{x+5}}$ at $x=11$, the bottom is $\sqrt{16}=4$.
- **Roots**: exam questions are usually set so that the roots come out exactly, such as $\sqrt{81}=9$ or $\sqrt[3]{27}=3$. If yours doesn't, check you added or subtracted inside the bracket correctly.
- **Find x when you know y**: after step 3, substitute y and solve by undoing the operations in reverse order. For $y=3(x-1)^2$ and $y=75$: divide by 3 to get $(x-1)^2=25$, square-root to get $x-1=5$, so $x=6$. For a cube root: $y=\frac{12}{\sqrt[3]{x}}$ gives $y=6$ when $x=8$; and if $y=2$, then $\sqrt[3]{x}=6$ and $x=6^3=216$.
- **Decimals** such as $y=7.2$ or $x=2.25$: use your calculator, but keep the full value of k and don't round until the end.

2. Write a formula for y in terms of x (5 questions)

1. Write the equation with k.
2. Substitute the pair you are given and find k.
3. **Write the full formula with the number in place of k** as your answer, such as $y=\dfrac{36}{(x+1)^2}$.

For example, z is inversely proportional to the square of $(y+1)$, and $z=4$ when $y=2$. Then $4=\frac{k}{3^2}$, so $k=36$ and $z=\dfrac{36}{(y+1)^2}$.

Variations you will meet:

- **Other letters** (z and y, F and d, m and t): the method is the same. Write the answer with the letters the question uses.
- **A real-life context** such as a force and a distance: the formula is still $F=\frac{k}{d^2}$ with the number found for k.
- **The next part uses your formula** to find a value; that is type 1.

3. Effect of multiplying by a factor, or a percentage change (5 questions)

1. **Write the power**: for direct proportion $y=kx^n$, with $n=1,2,3,\frac{1}{2}$ (square root) or $\frac{1}{3}$ (cube root). For inverse proportion write $y=\dfrac{k}{x^m}$, with m one of the same positive numbers.
2. **Multiply x by the factor a**: in direct proportion y is multiplied by a^n; in inverse proportion y is divided by a^m.
3. **Say it in words** the way the question asks: "y is multiplied by 4", or give the factor $p=\frac{1}{2}$.

Variations you will meet:

- **"Halved"** means the factor is $\frac{1}{2}$. If $F\propto\frac{1}{d^2}$ and d is halved, F is multiplied by $\frac{1}{(1/2)^2}=4$.
- **"Doubled"** in an inverse square: the new value is $\frac{1}{4}$ of the old one, so "n times the original" gives $n=\frac{1}{4}$.

- **A percentage change.** An increase of 10% is a factor of 1.1. If $y \propto x^2$, then y is multiplied by $1.1^2 = 1.21$: an increase of 21%. If $y \propto \frac{1}{x^2}$ and x goes up by 25%, y is multiplied by $\frac{1}{1.25^2} = 0.64$: a **decrease** of 36%. Subtract 1 (or take the result from 1) to get the percentage change; don't give the multiplier as the answer.
- **You can also use numbers**: choose $x = 1$ and $x = 2$ (or $x = 100$ and $x = 110$), find both y values with $k = 1$, and compare them.

4. Two linked proportions (3 questions)

1. **Write each statement as its own equation, with a different letter for each constant**, for example $y = \frac{k}{x}$ and $x = cw$.
2. **Find each constant** from the values given. The same set of values usually works for both.
3. **Substitute one equation into the other** to remove the middle letter, and simplify.

For example, $y = \frac{12}{x}$ and $x = 3w$ give $y = \frac{12}{3w} = \frac{4}{w}$.

Variations you will meet:

- **A square on the bracket**: $y = \frac{k}{(x+1)^2}$ with $x = 4w$ becomes $y = \frac{k}{(4w+1)^2}$. Replace x inside the bracket; don't expand it.
- **A link given the "wrong way round"**: if the question says "w is proportional to x" but you need x in terms of w, either write $w = cx$, find c and rearrange to $x = \frac{w}{c}$, or write $x = cw$ straight away (direct proportion works both ways). With $w = 15$ when $x = 90$: $w = cx$ gives $c = \frac{15}{90} = \frac{1}{6}$, so $w = \frac{x}{6}$ and $x = 6w$; or $x = cw$ gives $90 = 15c$, $c = 6$, so $x = 6w$ directly. Then replace x by $6w$: $y = \frac{k}{(x+2)^2}$ becomes $y = \frac{k}{(6w+2)^2}$. The mark scheme gives a mark for the link ($w = \frac{x}{6}$ or $x = 6w$) on its own, so write it down.
- **A root**: $w = c\sqrt{y}$ with $y = \frac{k}{x}$ gives $w = c\sqrt{\frac{k}{x}} = \frac{c\sqrt{k}}{\sqrt{x}}$. Work out $c\sqrt{k}$ as one number.
- **Check** your final formula with the values given in the question: it should give the right answer.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

y is directly proportional to the square of $(x + 1)$. When $x = 3$, $y = 8$.

Find y when $x = 5$. **[3]**

$y = k(x+1)^2$. **M1**

$8 = k \times (3+1)^2 = 16k$, so $k = \frac{1}{2}$. **M1** for substituting their k into the equation again:

$$y = \tfrac{1}{2}(5+1)^2 = \tfrac{1}{2} \times 36 = 18.$$

A1

Example 2

p is inversely proportional to the cube root of $(q-2)$. When $q = 10, p = 6$.

(a) Find p in terms of q. **[2]**

(b) Find q when $p = 4$. **[2]**

(a) $p = \dfrac{k}{\sqrt[3]{q-2}}$. **M1**

$6 = \dfrac{k}{\sqrt[3]{8}} = \dfrac{k}{2}$, so $k = 12$ and

$$p = \frac{12}{\sqrt[3]{q-2}}.$$

A1

(b) $4 = \dfrac{12}{\sqrt[3]{q-2}}$, so $\sqrt[3]{q-2} = 3$. **M1**

Cube both sides: $q - 2 = 27$, so $q = 29$. **A1** (Check: $\sqrt[3]{27} = 3$ and $12 \div 3 = 4$.)

Example 3

The power, P watts, produced by a wind turbine is proportional to the cube of the wind speed.

(a) Write down the number that P is multiplied by when the wind speed doubles. **[1]**

(b) The wind speed increases by 20%. Calculate the percentage increase in P. **[2]**

The brightness, L, of a lamp is inversely proportional to the square of the distance, d m, from the lamp.

(c) The distance increases by 25%. Calculate the percentage decrease in L. **[2]**

(a) $2^3 = 8$. **B1**

(b) The speed is multiplied by 1.2, so P is multiplied by $1.2^3 = 1.728$. **M1**

Percentage increase $= 72.8\%$. **A1**

(c) d is multiplied by 1.25, so L is multiplied by $\dfrac{1}{1.25^2} = 0.64$. **M1**

L is now 64% of what it was: a decrease of 36%. **A1**

Example 4

a is inversely proportional to the square of b. b is directly proportional to the square root of c.

When $c = 16$, $b = 2$ and $a = 5$.

Find a in terms of c. Give your answer in its simplest form. **[3]**

$a = \dfrac{k}{b^2}$: $5 = \dfrac{k}{4}$, so $k = 20$. **M1**

$b = m\sqrt{c}$: $2 = m\sqrt{16} = 4m$, so $m = \frac{1}{2}$. **M1** for the second equation with a **different** constant.

Substitute $b = \frac{1}{2}\sqrt{c}$, so $b^2 = \frac{1}{4}c$:

$$a = \frac{20}{\frac{1}{4}c} = \frac{80}{c}.$$

A1 (Check: $c = 16$ gives $a = 5$.)

Common mistakes

- **Using direct proportion when the question says inverse**, or the other way round. Examiners report this in almost every session. Underline "inversely" when you read the question.
- **Using the wrong power or root**: squaring when it says square root, using $(x+4)$ with no root at all, or reading "cube" as "cube root". Say the phrase to yourself and match it to the table in "Understand it".
- **Leaving out the constant.** Working with $y = \frac{1}{x^2}$ or $y = x^2$, or keeping the $\propto$ sign in every line, gets no method marks. Change $\propto$ into "$= k\times$" at the start.
- **Forgetting the power or the "inverse" in the second step.** Many candidates find k correctly, then use $y = kx$ or forget to square or root when finding the new value. Write the full equation again with your k, then substitute.
- **Rearranging wrongly to find** k, for example $16k = 4$ giving $k = 4$ instead of 0.25. Divide by the number next to k.
- **Splitting the bracket**: $k(x+3)^2$ is not $kx^2 + 3$, and $\sqrt{x+4}$ is not $\sqrt{x} + 4$. Work out the bracket first.
- **Expanding a bracket to solve for** x**.** From $(x+2)^3 = 125$, just take the cube root: $x + 2 = 5$. Expanding the cube makes it much harder.
- **Using** k **twice.** In linked proportions, one letter for two different constants causes confusion. Use k for one and c (or m) for the other.
- **Not giving the full formula.** Working that finds $k = 50$ but an answer line with just "50" loses the final mark. Write $z = \dfrac{50}{(y-3)^2}$.
- **Saying "it doubles" for an inverse square.** If $F \propto \frac{1}{d^2}$ and d is halved, F is **multiplied by** 4. "Increases by 4" suggests adding 4 and is not accepted: say "multiplied by".

How to get the marks

- **The first method mark is for the equation with k**, such as $y = \dfrac{k}{\sqrt{x+4}}$. Write it down even if you then use the two-pairs shortcut.
- **The second method mark is for putting your k back in** with the new value, such as $y = \dfrac{6}{\sqrt{12+4}}$. It is earned even if your k was wrong, so always write this line.
- **"Find y in terms of x"** means the answer is a formula with the number in place of k, such as $y = \dfrac{20}{(x+5)^2}$, on the answer line. Don't write "$k = 20$" alone, and don't expand the bracket.
- **Exact answers**: give fractions such as $\frac{2}{3}$ or exact decimals such as 0.16. Otherwise give 3 significant figures, but keep k exact while you work.
- **Factor questions** want a number with a clear meaning: "multiplied by 4" or "$n = \frac{1}{4}$". For a percentage change, give the percentage, not the multiplier.
- **"Explain what happens to F"** needs the direction and the factor: "F is multiplied by 4".
- **Check your answer makes sense**: in direct proportion both go up together; in inverse proportion one goes up while the other goes down.

Quick check

1. y is directly proportional to the cube of x. When $x = 2$, $y = 20$. Find y when $x = 4$.
2. y is inversely proportional to the square root of $(x + 3)$. When $x = 6$, $y = 4$. Find y in terms of x, and find y when $x = 13$.
3. m is proportional to the square of $(n - 1)$. When $n = 4$, $m = 18$. Find n when $m = 50$, given that $n > 1$.
4. F is inversely proportional to the square of d. What happens to F when d is multiplied by 3?
5. y is proportional to the square of x. x is inversely proportional to w. When $w = 2$, $x = 6$ and $y = 18$. Find y in terms of w.

Answers

1. $y = kx^3$: $20 = 8k$, so $k = 2.5$. When $x = 4$, $y = 2.5 \times 64 = 160$. (Or: x doubles, so y is multiplied by $2^3 = 8$: $20 \times 8 = 160$.)
2. $y = \dfrac{k}{\sqrt{x+3}}$: $4 = \dfrac{k}{\sqrt{9}} = \dfrac{k}{3}$, so $k = 12$ and $y = \dfrac{12}{\sqrt{x+3}}$. When $x = 13$, $y = \dfrac{12}{\sqrt{16}} = 3$.
3. $m = k(n-1)^2$: $18 = 9k$, so $k = 2$. Then $2(n-1)^2 = 50$, so $(n-1)^2 = 25$, $n - 1 = 5$ and $n = 6$.
4. F is multiplied by $\dfrac{1}{3^2} = \dfrac{1}{9}$ (it becomes one ninth of what it was).
5. $y = kx^2$: $18 = 36k$, so $k = \frac{1}{2}$. $x = \dfrac{c}{w}$: $6 = \dfrac{c}{2}$, so $c = 12$. Then $y = \frac{1}{2}\left(\dfrac{12}{w}\right)^2 = \dfrac{72}{w^2}$. (Check: $w = 2$ gives 18.)

Past questions to try

- **0580/23/O/N/22 Q17**: proportional to the square of a bracket; find k, then a new value.
- **0580/42/O/N/22 Q6**: part (f): find x from a cube of a bracket, then the factor for an inverse square when d is halved.
- **0580/42/O/N/21 Q6**: part (e): the percentage increase in energy when speed rises by 30%.
- **0580/21/M/J/22 Q25**: two linked proportions with a square root, combined into one formula.