Cambridge IGCSE · Mathematics (0580) · Notes

Equations and changing the subject

Read online at pastopic.com/cambridge/mathematics-0580/notes/equations

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Read first: Algebraic manipulation and algebraic fractions

What you need to know

Core and Extended share the first four points. The parts marked **Extended only** are for Papers 2 and 4. By the end of this topic you should be able to:

1. **Construct expressions, equations and formulas** from words or diagrams, for example "a number 3 more than n" is $n+3$, and two facts about prices give a pair of **simultaneous equations**. **Extended only:** the expressions can be harder, such as the product of two consecutive odd numbers, $(2n+1)(2n+3)$, or a time written as a fraction, $\frac{12}{x}$ hours.
2. **Solve linear equations in one unknown**, including ones with brackets and with the unknown on both sides, such as $8-3x=2(x+9)$.
3. **Solve simultaneous linear equations in two unknowns**, such as $3x+2y=16$ and $5x-2y=8$.
4. **Change the subject of a formula.** In Core the new subject appears **once** and is not squared or square-rooted, for example $P=2l+2w$ rearranged to $l=\frac{P-2w}{2}$.
5. **Extended only:** change the subject when the new subject **appears twice**, or is **squared or under a root**, for example $y=\frac{2x+5}{x-3}$ or $A=\pi r^2$.
6. **Extended only:** solve **fractional equations** with numbers or linear expressions in x on the bottom, such as $\frac{x}{3}+\frac{x-1}{4}=5$ or $\frac{6}{x+1}=\frac{4}{x-2}$.
7. **Extended only:** solve **quadratic equations** by factorising, by completing the square and by the quadratic formula, and give answers in surd form when asked.
8. **Extended only:** solve **simultaneous equations where one is linear and one is not** (it has an x^2, y^2 or xy term), such as $y=x^2-4x+1$ and $y=x-3$.

Expanding brackets, factorising and completing the square as algebra are in the topic Algebraic manipulation and algebraic fractions. Inequalities have their own topic, and solving equations by reading graphs is in Graphs of functions and sketching curves.

Understand it

What an equation is

An **equation** says two things are equal, and is true only for certain values of the letter. $4x+7=31$ is true when $x=6$ and false for any other number. **Solving** means finding that value.

Think of the equation as a balance. Whatever you do to one side, you must do to the other, and it stays balanced. To get x on its own, **undo** what has been done to it, in reverse order, using the inverse operations: $+$ undoes $-$, $\times$ undoes $\div$, squaring undoes square-rooting.

$$4x+7=31 \;\Rightarrow\; 4x=24 \;\Rightarrow\; x=6.$$

Here x was multiplied by 4 and then had 7 added, so we subtract 7 first and divide by 4 second. You can **check** any answer by putting it back in: $4\times 6+7=31$.

Linear equations

The unknown on both sides. Gather the x terms on the side with more of them, and the numbers on the other:

$$9x-5=4x+20 \;\Rightarrow\; 5x-5=20 \;\Rightarrow\; 5x=25 \;\Rightarrow\; x=5.$$

If the x terms are negative, add them to both sides: $12-3x=2x-8$ becomes $12=5x-8$, so $20=5x$ and $x=4$.

Brackets. Expand first, then solve:

$$3(2x-5)=4(x+1) \;\Rightarrow\; 6x-15=4x+4 \;\Rightarrow\; 2x=19 \;\Rightarrow\; x=9.5.$$

Answers don't have to be whole numbers. Give 9.5 or $\frac{19}{2}$, not a rounded value.

A fraction. Multiply both sides by the bottom number to clear it: $\frac{x+4}{5}=3$ gives $x+4=15$, so $x=11$.

Extended only: with **two or more fractions**, multiply **every term** by the lowest common multiple of the denominators. For $\frac{x}{3}+\frac{x-1}{4}=5$, multiply by 12:

$$4x+3(x-1)=60 \;\Rightarrow\; 7x-3=60 \;\Rightarrow\; x=9.$$

Forming an equation

Many questions give the facts in words or on a diagram. Let a letter stand for the unknown, write each quantity in terms of it, and use the fact that says what they add up to (or what is equal).

- **"I think of a number, subtract 3, then multiply by 4. The answer is 28."** That is $4(n-3)=28$, so $n-3=7$ and $n=10$. Undoing the steps in reverse order gives the same answer: $28\div 4+3=10$. If the number is **squared** and then 7 added to give 88, then $n^2=81$ and $n=9$ (or -9 if the number can be negative).

- **Perimeter.** A triangle has sides $(x+4)$, $(2x+3)$ and $(3x-2)$ cm, and perimeter 47 cm. Then $6x+5=47$, so $x=7$, and the sides are 11, 17 and 19 cm.
- **Ages, prices, points.** If Ali has x stickers, Bea has twice as many and Cai has 5 fewer than Bea, the total is $x+2x+(2x-5)=5x-5$.

Always answer the question that was asked: if it asks for the length of a side, work out the side from x.

Simultaneous equations

Two unknowns need two equations. **Simultaneous** means both are true at the same time. There are two methods.

Elimination. Make the number in front of one letter the same in both equations, then add or subtract to remove that letter.

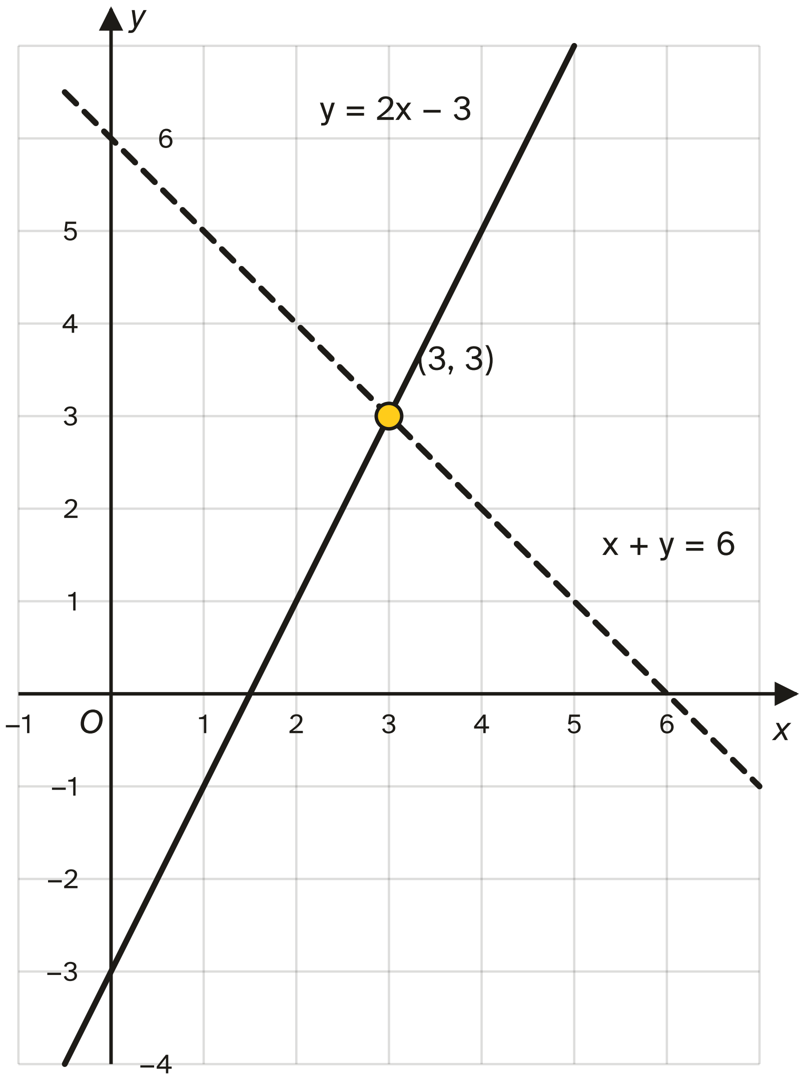
- $3x+2y=16$ and $5x-2y=8$: the y terms are $+2y$ and $-2y$, so **add**: $8x=24$, $x=3$. Substitute: $9+2y=16$, $y=3.5$.
- $2x+3y=12$ and $5x+4y=23$: nothing matches, so multiply the first by 4 and the second by 3: $8x+12y=48$ and $15x+12y=69$. The y terms have the same sign, so **subtract**: $7x=21$, $x=3$, and then $6+3y=12$ gives $y=2$.

The rule: **same signs, subtract; different signs, add.**

Substitution. If one equation already says $y=\ldots$ (or $x=\ldots$), put that expression into the other. For $x+y=6$ and $y=2x-3$:

$$x+(2x-3)=6 \;\Rightarrow\; 3x=9 \;\Rightarrow\; x=3, \quad y=2(3)-3=3.$$

What it means on a graph. Each linear equation is a straight line. The solution is the point where the lines cross, because it is the only point on both lines:



Check your answer in the equation you did **not** use to find the second letter: $3 + 3 = 6$. ✓

Changing the subject

A formula such as $v = u + at$ has v as its **subject**: it is on its own on one side. To make another letter the subject, use exactly the same steps as solving an equation, treating the other letters like numbers. To make t the subject:

$$v = u + at \;\Rightarrow\; v - u = at \;\Rightarrow\; t = \frac{v-u}{a}.$$

Compare solving $30 = 6 + 8t$: subtract 6, then divide by 8. The steps are the same.

Extended only: the subject squared or under a root. Get the square (or root) on its own first, then undo it last. $A = \pi r^2$ gives $r^2 = \frac{A}{\pi}$, so $r = \sqrt{\frac{A}{\pi}}$. For a root, square both sides: $T = 2\pi\sqrt{\frac{l}{g}}$ gives $\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$, then $\frac{T^2}{4\pi^2} = \frac{l}{g}$, so $l = \frac{gT^2}{4\pi^2}$.

Extended only: the subject twice. Collect every term with the subject on one side, **factorise** it out, then divide by the bracket:

$$ax + 3 = bx + 7 \;\Rightarrow\; ax - bx = 4 \;\Rightarrow\; x(a-b) = 4 \;\Rightarrow\; x = \frac{4}{a-b}.$$

If the subject is in a fraction, multiply by the denominator first (Example 4 shows this).

Extended only: quadratic equations

A **quadratic equation** has an x^2 term, such as $x^2 + 2x - 24 = 0$. It usually has **two** solutions. First rearrange it so that one side is 0. Then use one of three methods.

Factorising. If two numbers multiply to give 0, one of them must be 0.

$$x^2 + 2x - 24 = 0 \;\Rightarrow\; (x+6)(x-4) = 0 \;\Rightarrow\; x = -6 \text{ or } x = 4.$$

The quadratic formula works for every quadratic $ax^2 + bx + c = 0$, and it is on the List of formulas:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

For $2x^2 + 3x - 7 = 0$: $a = 2$, $b = 3$, $c = -7$, so

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)} = \frac{-3 \pm \sqrt{65}}{4},$$

giving $x = 1.27$ or $x = -2.77$ (2 d.p.). The $\pm$ gives the two answers.

Completing the square. Write the left side as $(x+p)^2 + q$, then square-root both sides, remembering $\pm$:

$$x^2 + 6x - 3 = 0 \;\Rightarrow\; (x+3)^2 - 12 = 0 \;\Rightarrow\; x + 3 = \pm\sqrt{12} \;\Rightarrow\; x = -3 \pm 2\sqrt{3}.$$

That is the answer in **surd form**; as decimals it is $x = 0.46$ or $x = -6.46$ (2 d.p.).

If a question says "give your answers correct to 2 decimal places", the quadratic will not factorise: use the formula.

Extended only: fractional equations

When x is in a denominator, multiply **every term** by **all** the denominators to clear them. For $\frac{6}{x+1} = \frac{4}{x-2}$, multiply both sides by $(x+1)(x-2)$:

$$6(x-2) = 4(x+1) \;\Rightarrow\; 6x - 12 = 4x + 4 \;\Rightarrow\; x = 8.$$

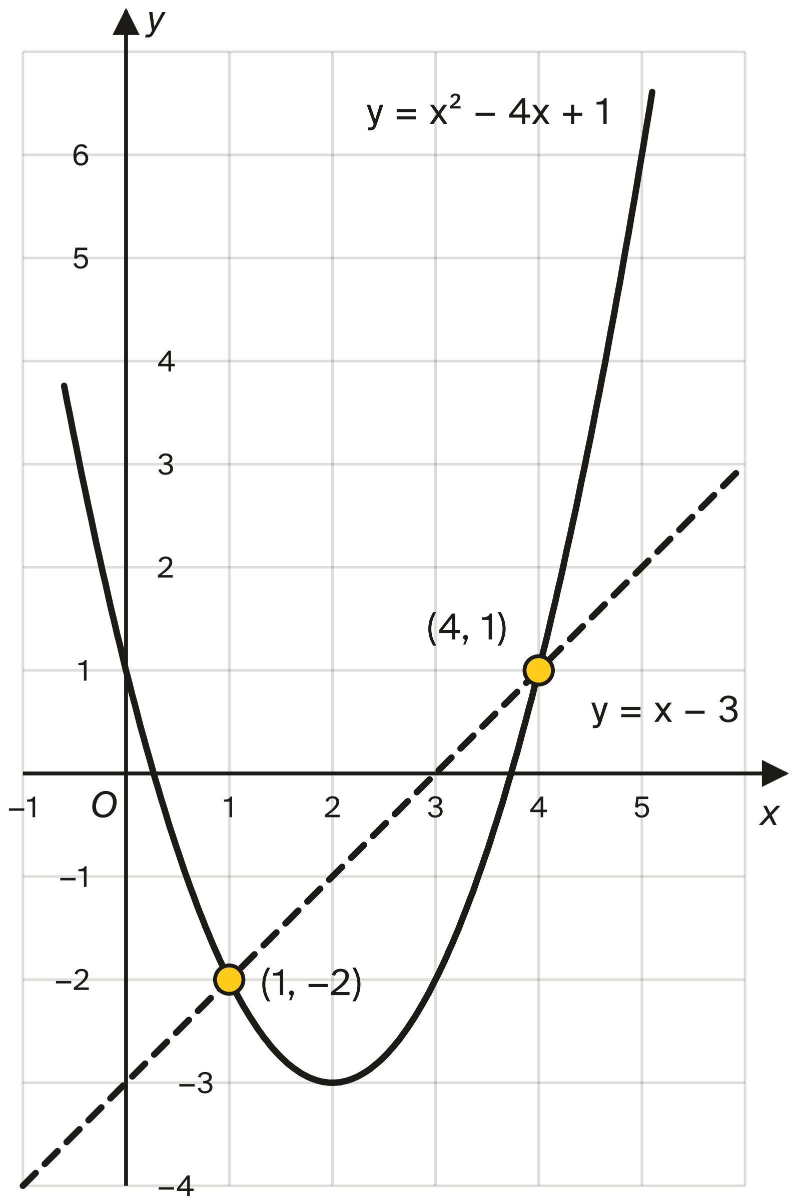
With more terms, the result is often a quadratic (see Example 3).

Extended only: one linear and one non-linear equation

When one equation is a curve, such as $y = x^2 - 4x + 1$, and the other a line, $y = x - 3$, **substitute** the linear one into the other. Elimination does not work here.

$$x^2 - 4x + 1 = x - 3 \ \Rightarrow\ x^2 - 5x + 4 = 0 \ \Rightarrow\ (x-1)(x-4) = 0,$$

so $x = 1$ or $x = 4$. Find each y from the **linear** equation: $y = -2$ and $y = 1$. The line crosses the curve twice, so there are **two pairs** of answers:



Key facts and formulas

"Given" means it is on the List of formulas printed on page 2 of the exam paper. The quadratic formula is on the Extended list (Papers 2 and 4) only. Everything else must be learnt.

Formula	Status
Do the same to both sides; undo operations in reverse order	Learn it

Formula	Status
Simultaneous equations by elimination: same signs subtract, different signs add	Learn it
Changing the subject: the same steps as solving an equation	Learn it
Extended only: $ax + c = bx + d \Rightarrow x(a - b) = d - c \Rightarrow x = \dfrac{d - c}{a - b}$ (factorise out a subject that appears twice)	Learn it
Extended only: if $pq = 0$ then $p = 0$ or $q = 0$	Learn it
Extended only: $ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Given
Extended only: $(x + p)^2 = k \Rightarrow x = -p \pm \sqrt{k}$	Learn it
Time $= \dfrac{\text{distance}}{\text{speed}}$	Learn it
Perimeter of a rectangle $= 2(l + w)$; area of a rectangle $= lw$	Learn it
Area of a triangle $= \frac{1}{2}bh$	Given
Area of a trapezium $= \frac{1}{2}(a + b)h$	Learn it
Angles on a straight line add to 180°; angles in a triangle add to 180°	Learn it

How to do it

In the Paper 1–4 questions we have tagged for this topic (163 questions, 2021–2025, of which 159 are different; the rest appear on two papers), the types came up this often. Counts are of the 159 different questions; 34 of them mix two or more types, so the counts add up to more than 159.

Question type	Questions
Solving linear equations	39
Simultaneous linear equations	36
Changing the subject of a formula	34
Forming equations from words or diagrams	33
One linear and one non-linear equation (Extended)	18
Solving quadratic equations (Extended)	13

Question type	Questions
Forming and solving a quadratic from a context (Extended)	13
Finding an unknown in a formula	11
Fractional equations with x in a denominator (Extended)	8

1. Solving linear equations (39 questions)

1. **Expand any brackets** and clear any fraction by multiplying both sides by its denominator.
2. **Collect the x terms on one side** and the numbers on the other, by adding or subtracting the same thing on both sides.
3. **Divide** by the number in front of x.
4. **Check** by substituting back.

For example, $8 - 3x = 2(x + 9)$: $8 - 3x = 2x + 18$, so $-10 = 5x$ and $x = -2$.

Variations you will meet:

- **One step**, such as $6x = 21$ ($x = 3.5$) or $x - 9 = -4$ ($x = 5$). These are worth one mark each; watch the signs.
- **Two steps**, such as $5x - 8 = 22$: add 8, then divide by 5, so $x = 6$.
- **Division**, such as $\frac{x}{4} = 9$: multiply, so $x = 36$.
- **A number outside a bracket or fraction**, such as $\frac{3(x-2)}{5} = 9$: $3(x - 2) = 45$, $x - 2 = 15$, $x = 17$. You can divide by 3 instead of expanding.
- **Several expressions that are all equal**: pick the two that contain only one letter, set them equal and solve; then use your answer in another pair.
- **Extended only: two or more fractions** with number denominators: multiply every term by the lowest common multiple, with the numerators in brackets.

2. Simultaneous linear equations (36 questions)

1. **Line the equations up** with x under x, y under y and the numbers on the right.
2. **Match** the number in front of one letter by multiplying one or both equations. Pick the letter that needs the least work, or one whose signs differ (then you add).
3. **Add or subtract** to eliminate that letter: same signs subtract, different signs add. Solve for the other letter.
4. **Substitute** that value into either original equation to find the second letter.
5. **Check** both values in the other equation.

For example, $4x + 3y = 7$ and $2x - 5y = 23$: double the second, $4x - 10y = 46$; subtract, $13y = -39$, so $y = -3$ and $4x = 16$, $x = 4$.

Variations you will meet:

- **One equation already gives** y (or x), such as $y = 3x - 1$: substitute it straight into the other.
- **A fraction or decimal answer**, such as $x = 2.5$: that is fine; don't round.
- **"Solve these simultaneous equations" after forming them** from prices or angles (see type 4).
- **Using a graph**: when the two lines are drawn, the solution is the point where they cross; read both coordinates.
- **Extended only: "hence"**: a later part may reuse the same working, for example with $\sin$ or $\cos$ in place of the letters. Solve the letters first, then use them.

3. Changing the subject of a formula (34 questions)

1. **Circle the new subject** in the formula.
2. **Undo the operations around it in reverse order**, doing the same to both sides: remove terms added or subtracted, then multiplication or division.
3. **Write the answer** with the subject on the left, such as $t = \frac{v-u}{a}$.

For example, $W = kT + y$: $W - y = kT$, so $T = \frac{W-y}{k}$.

Variations you will meet:

- **A fraction**: multiply by the denominator first. $M = \frac{3n}{5} - 2$: $M + 2 = \frac{3n}{5}$, $5(M+2) = 3n$, $n = \frac{5(M+2)}{3}$.
- **A bracket**: divide by the number outside, or expand. $P = 4(q - r)$: $\frac{P}{4} = q - r$, so $q = \frac{P}{4} + r$.
- **A negative subject**, such as $v = 20 - 3t$: add $3t$ to both sides first, $3t = 20 - v$, so $t = \frac{20-v}{3}$.
- **The line** $y = mx + c$: $x = \frac{y-c}{m}$, and $m = \frac{y-c}{x}$.
- **Extended only: a square or root**: isolate it, then square-root (or square) as the last step. $E = \frac{1}{2}mv^2$ gives $v = \sqrt{\frac{2E}{m}}$.
- **Extended only: the subject twice**: expand any brackets, collect the subject's terms on one side, factorise, divide. $5k - h = hk + 3$: $5k - hk = 3 + h$, $k(5-h) = 3 + h$, $k = \frac{3+h}{5-h}$.
- **Extended only: the subject in a denominator**, such as $x = \frac{2t}{3-t}$: multiply by the denominator, $x(3 - t) = 2t$, then treat it as "the subject twice": $3x = 2t + xt = t(2 + x)$, so $t = \frac{3x}{2+x}$.

4. Forming equations from words or diagrams (33 questions)

1. **Choose the letter** (the question often chooses it for you) and write every other quantity in terms of it.
2. **Write the equation** from the fact that links them: a total, a perimeter, an angle sum, "is equal to" or "is 3 times as long as".
3. **Solve**, then **answer the question** (a price, an age, a length), with units if needed.

For example, a jacket costs $\$x$, and each of 3 shirts costs $\$15$ less. The total is $\$135$: $x + 3(x - 15) = 135$, so $4x - 45 = 135$ and $x = 45$. The jacket costs $\$45$.

Variations you will meet:

- **Think of a number**: write the steps as an equation, or undo them in reverse. If the number is squared, square-root at the end and check the question: "a positive number" means give only the positive root.
- **Perimeters and angles** from a diagram, often "not to scale": add the sides, or use angles on a straight line (180°) and in a triangle (180°).
- **"Line AB is 3 times as long as line CD"** means $AB = 3 \times CD$, so put the bracket in: $3(\ldots)$.
- **Two unknowns from two facts** (costs of two items bought in two ways, or two angle facts): write two equations, then solve them simultaneously (type 2). For example, 3 coffees and 2 muffins cost \$$13.10$: $3c + 2m = 13.1$.
- **A percentage increase**: 20% more than \$$x$ is \$$1.2x$.
- **Money in two units**: if a price is in cents and the total in dollars, change the total to cents first.
- **A squared term**: a cube with edge x cm and surface area 150 cm² gives $6x^2 = 150$, $x^2 = 25$, $x = 5$ (a length is positive).

5. One linear and one non-linear equation (18 questions)

Extended only.

1. **Make y (or x) the subject of the linear equation**, if it isn't already.
2. **Substitute** it into the non-linear equation.
3. **Rearrange to a quadratic** $= 0$ and solve: factorise, or use the formula if answers to 2 decimal places are asked for.
4. **Find the other letter for each root**, using the linear equation, and **pair them up**: $x = 1, y = -2$ and $x = 4, y = 1$.

Variations you will meet:

- **"The line meets the curve at A and B; find their coordinates"**: the same method; give the answers as (x, y) pairs.
- **Both unknowns squared**, such as $x^2 + y^2 = 25$ with $y = x + 1$: expand $(x+1)^2$ carefully, $x^2 + x^2 + 2x + 1 = 25$, so $2x^2 + 2x - 24 = 0$, $x^2 + x - 12 = 0$, $(x+4)(x-3) = 0$, giving $(3, 4)$ and $(-4, -3)$.
- **The linear equation not in $y = \ldots$ form**, such as $y + 3x = 5$: rearrange to $y = 5 - 3x$ first and keep the signs right.
- **A quadratic with no number term**, such as $2x^2 - 7x = 0$: factorise out x, $x(2x - 7) = 0$, so $x = 0$ or $x = 3.5$. Don't divide by x, or you lose $x = 0$.
- **"Find the values of x"** only: stop after the quadratic.

6. Solving quadratic equations (13 questions)

Extended only.

1. **Rearrange so that one side is** 0: $ax^2 + bx + c = 0$.
2. **Choose the method.** "By factorisation" or whole-number roots: factorise. "Correct to 2 decimal places" or "show all your working": the formula. "In surd form": the formula or completing the square.

3. **Formula**: write a, b and c with their signs, substitute with brackets round negatives, and show the value under the root, such as $\sqrt{65}$.
4. **Give both answers**, rounded as asked.

For example, $2x^2 - 7x - 15 = 0$ factorises to $(2x+3)(x-5) = 0$, so $x = -1.5$ or $x = 5$.

Variations you will meet:

- **Already factorised**, such as $(4x+1)(x-6) = 0$: set each bracket to 0: $x = -\frac{1}{4}$ or $x = 6$.
- **Terms on both sides**, such as $3x^2 = 10 - x$: first rearrange to $3x^2 + x - 10 = 0$.
- **Completing the square first**: if the question gave $x^2 + 10x + 7 = (x+5)^2 - 18$, then solving $x^2 + 10x + 7 = 0$ means $(x+5)^2 = 18$, $x = -5 \pm \sqrt{18}$. If it equals a number other than 0, move that number across too.
- **A cubic that simplifies**: if the x^3 terms cancel when you collect terms, what is left is a quadratic.
- **One solution given**, such as "$x^2 + b = 60$ has one solution $x = 7$": substitute to find b ($49 + b = 60$, $b = 11$); the other solution is $x = -7$, because $(-7)^2 = 7^2$.

7. Forming and solving a quadratic from a context (13 questions)

Extended only. These are long questions that usually go: write an expression, show that it simplifies to a given quadratic, solve it, then use the answer.

1. **Write each quantity**: a time as $\frac{\text{distance}}{\text{speed}}$, an area as length $\times$ width, a number of items as $\frac{\text{total cost}}{\text{cost of one}}$.
2. **Form the equation** from the fact given (total time, difference in times, area).
3. **Clear the fractions** by multiplying every term by every denominator, expand all brackets, and collect everything on one side. The last line must be exactly the quadratic printed in the question.
4. **Solve** it (by factorising if asked, otherwise with the formula) and **reject** any answer that is impossible: a negative length, or a speed that would make another speed negative.
5. **Use the answer** for the final part, such as a time in hours and minutes.

For example, a rectangle is $(2x-1)$ cm by $(x+3)$ cm and has area 60 cm². Then $(2x-1)(x+3) = 60$, so $2x^2 + 5x - 3 = 60$, $2x^2 + 5x - 63 = 0$, $(2x-9)(x+7) = 0$. Since $x + 3$ is a length, $x = -7$ is impossible, so $x = 4.5$.

Variations you will meet:

- **A difference in times**: "the walk takes 1 hour longer than the run" means $\frac{d_1}{x-4} - \frac{d_2}{x} = 1$, with the longer time first.
- **Equal numbers of items**: if \$30 buys the same number of pens at \$$w$ as \$50 buys at \$$(w+2)$, then $\frac{30}{w} = \frac{50}{w+2}$. This one clears to a linear equation: $30w + 60 = 50w$, so $w = 3$.
- **Area of a triangle or trapezium**: use $\frac{1}{2}bh$ or $\frac{1}{2}(a+b)h$ and double both sides first to remove the $\frac{1}{2}$.
- **Converting a time**: 2.4 hours is 2 hours and $0.4 \times 60 = 24$ minutes.

8. Finding an unknown in a formula (11 questions)

1. **Substitute** every value you know into the formula, as written.
2. **Simplify** the numbers.
3. **Solve** the equation that is left for the unknown letter.

For example, $v = u + at$ with $v = 30$, $u = 6$ and $t = 8$: $30 = 6 + 8a$, so $24 = 8a$ and $a = 3$.

Variations you will meet:

- **The letter is in the middle of the formula**, such as $P = 5x - 2y$ with $P = 11$ and $x = 7$: $11 = 35 - 2y$, so $2y = 24$ and $y = 12$. Watch the sign when the unknown term is subtracted.
- **A cost formula in context**, such as $C = 15 + 2.5h$ for hiring something for h hours: substitute the cost paid for C and solve for h.
- **You may rearrange first**, then substitute; either order earns the marks.
- **Extended only: the unknown is squared or under a root**, such as $E = 3mv^2$ with $E = 150$ and $m = 2$: $150 = 6v^2$, $v^2 = 25$, and the positive value is $v = 5$.

9. Fractional equations with x in a denominator (8 questions)

Extended only.

1. **Multiply every term by all the denominators** (their product), for example $(x-1)(x+2)$.
2. **Cancel** each fraction, then **expand every bracket**.
3. **Collect terms**: if x^2 terms remain, rearrange to a quadratic $= 0$ and solve it; otherwise it is linear.
4. **Check** no answer makes a denominator 0.

Variations you will meet:

- **One fraction on each side**: cross-multiply straight away. $\frac{3}{x-4} = \frac{5}{x}$ gives $3x = 5(x-4)$, $x = 10$.
- **Two fractions and a number**, such as $\frac{a}{x+1} + \frac{b}{2x-5} = 3$: the number 3 must be multiplied by **both** denominators too. This usually gives a quadratic, solved by the formula (Example 3 works the same way).
- **Number denominators and an x denominator together**: multiply by all of them at once.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

(a) Solve $7x - 4 = 3(x + 8)$. **[3]**

(b) Solve $\dfrac{2y + 5}{3} = y - 1$. **[3]**

(c) Make h the subject of the formula $V = 3h + 2w$. **[2]**

(a) $7x - 4 = 3x + 24$. **M1** for expanding the bracket correctly.

$4x = 28$ **M1** for collecting the x terms and the numbers correctly, so $x = 7$. **A1**

(b) Multiply both sides by 3: $2y + 5 = 3(y - 1) = 3y - 3$. **M1** for clearing the fraction (the whole of $y - 1$ is multiplied).

$5 + 3 = 3y - 2y$ **M1**, so $y = 8$. **A1** Check: $\frac{21}{3} = 7 = 8 - 1$. ✓

(c) $V - 2w = 3h$ **M1** for moving the $2w$ correctly, so $h = \dfrac{V - 2w}{3}$. **A1**

Example 2

At a café, a coffee costs $\$c$ and a muffin costs $\$m$. Three coffees and two muffins cost $\$13.10$. Five coffees and four muffins cost $\$23.30$.

Write down two equations and solve them to find the value of c and the value of m. You must show all your working. **[6]**

$$3c + 2m = 13.1, \qquad 5c + 4m = 23.3$$

B1 for each equation.

Multiply the first by 2: $6c + 4m = 26.2$. **M1** for making the coefficients of one letter the same.

Subtract the second: $(6c + 4m) - (5c + 4m) = 26.2 - 23.3$, so $c = 2.9$. **M1** for subtracting correctly, **A1** for $c = 2.9$.

Substitute: $3(2.9) + 2m = 13.1$, so $2m = 4.4$ and $m = 2.2$. **A1**

Check in the second equation: $5(2.9) + 4(2.2) = 14.5 + 8.8 = 23.3$. ✓ A coffee costs $\$2.90$ and a muffin $\$2.20$.

Example 3

Extended only.

Priya cycles 18 km at x km/h, then walks 4 km at $(x - 9)$ km/h. The whole journey takes 3 hours.

(a) Show that $3x^2 - 49x + 162 = 0$. **[3]**

(b) Solve $3x^2 - 49x + 162 = 0$, giving your answers correct to 2 decimal places. You must show all your working. **[4]**

(c) Find the time Priya spends walking. Give your answer in hours and minutes, correct to the nearest minute. **[2]**

(a) Time $= \frac{\text{distance}}{\text{speed}}$, so

$$\frac{18}{x} + \frac{4}{x-9} = 3.$$

M1 for this equation. Multiply every term by $x(x-9)$:

$$18(x-9) + 4x = 3x(x-9) \ \Rightarrow\ 18x - 162 + 4x = 3x^2 - 27x.$$

M1 for clearing both fractions and expanding every bracket.

$0 = 3x^2 - 27x - 22x + 162$, so $3x^2 - 49x + 162 = 0$. **A1** with no errors.

(b) $a = 3$, $b = -49$, $c = 162$:

$$x = \frac{-(-49) \pm \sqrt{(-49)^2 - 4(3)(162)}}{2(3)} = \frac{49 \pm \sqrt{457}}{6}.$$

B1 for $\sqrt{457}$ (or $\sqrt{2401 - 1944}$), **B1** for the rest of the formula correct.

$x = 11.73$ or $x = 4.60$. **B1 B1**

(c) The walking speed $x - 9$ must be positive, so $x = 11.73$ (with $x = 4.60$ the speed would be negative). Walking speed $= x - 9 = 2.7296\ldots$ km/h (keep the full calculator value). **M1** for $\frac{4}{\text{their } x-9}$.

Time $= \frac{4}{2.7296\ldots} = 1.4654\ldots$ hours. $0.4654\ldots \times 60 = 27.9$, so 1 hour 28 minutes. **A1**

Example 4

Extended only.

(a) Make x the subject of the formula $y = \dfrac{2x+5}{x-3}$. **[4]**

(b) Solve the simultaneous equations. You must show all your working.

$$y = x^2 - 4x + 1, \qquad y = x - 3$$

[5]

(a) $y(x-3) = 2x + 5$ **M1** for multiplying by the denominator, so $xy - 3y = 2x + 5$.

$xy - 2x = 5 + 3y$ **M1** for collecting the x terms on one side and the rest on the other.

$x(y-2) = 5 + 3y$ **M1** for factorising, so

$$x = \frac{3y+5}{y-2}.$$

A1

(b) $x^2 - 4x + 1 = x - 3$ **M1** for equating the two expressions for y.

$x^2 - 5x + 4 = 0$ **A1**, $(x-1)(x-4) = 0$ **M1**, so $x = 1$ or $x = 4$.

$y = 1 - 3 = -2$ and $y = 4 - 3 = 1$. The solutions are $x = 1, y = -2$ and $x = 4, y = 1$. **B2** (**B1** for one correct pair). The diagram in "Understand it" shows the two crossing points.

Common mistakes

- **Moving a term without changing its sign.** In $6x - 5 = 2x + 7$, the $2x$ becomes $-2x$ and the -5 becomes $+5$ on the other side: $4x = 12$, not $8x = 2$. Examiners see this every year. Write "subtract $2x$ from both sides" in your head, one step per line.
- **Dividing the wrong way round.** $5x = 2$ gives $x = \frac{2}{5} = 0.4$, not $x = 2.5$. Divide the number by the number in front of x.
- **Multiplying only part of a side.** $\frac{x+7}{2} = x - 3$ becomes $x + 7 = 2(x-3) = 2x - 6$; writing $x + 7 = 2x - 3$ loses the mark. Put brackets round the whole side.
- **Adding when you should subtract in elimination.** With $2x + 3y = 13$ and $2x - y = 1$, the $2x$ terms have the same sign, so subtract: $4y = 12$. Adding gives $4x + 2y = 14$ and eliminates nothing. Also subtract **every** term, including the right-hand sides, and take care with $3y - (-y) = 4y$.
- **Stopping after one letter.** Simultaneous equations need both values. Substitute back, and check in the other equation.
- **No working for simultaneous equations.** When the question says "you must show all your working", correct answers found by trial earn very little. Show the multiplied equations and the addition or subtraction.
- **Changing the subject in the wrong order.** For $A = 4b - c$ made b the subject, add c first, then divide by 4: $b = \frac{A+c}{4}$. Writing $b = \frac{A}{4} + c$ divides only part of the side.
- **Extended only: leaving the subject on both sides.** $t = \frac{3x - xt}{2}$ is not an answer, because t is still on the right. Collect the subject's terms and factorise.
- **Extended only: sign slips in the quadratic formula.** With $b = -7$, $-b = +7$ and $b^2 = 49$, not -49. Put brackets round negatives, and make sure the fraction line goes under the whole top, $-b \pm \sqrt{\dots}$.
- **Extended only: losing a solution.** Dividing both sides of $3x^2 = 5x$ by x loses $x = 0$. Rearrange to $3x^2 - 5x = 0$ and factorise instead. Square-rooting $(x-2)^2 = 9$ needs ± 3.
- **Extended only: not multiplying every term.** In $\frac{5}{x} + \frac{2}{x+1} = 3$, the 3 must be multiplied by $x(x+1)$ too.
- **Extended only: the wrong y for each x.** In a line-and-curve question, find y from the linear equation, and keep each y with its own x.
- **Rounding too early.** Keep the full calculator value until the last step, then round to what the question asks.

How to get the marks

- **"Solve"** means find the value of the letter. Show each step: a correct first step (such as $8x = 32$) usually earns a method mark even if you slip later.
- **"You must show all your working"** (always on simultaneous equations and quadratics): without the method, a correct answer may earn only one mark. Write the equations after multiplying, the addition or subtraction, and the substitution. For the formula, write it with the numbers in.
- **Two-letter answers**: give both values, each labelled ($x = \ldots$, $y = \ldots$).
- **Extended only:** for a line and a curve, give two **pairs** of values. Mark schemes often give a mark for one correct pair.
- **Changing the subject**: the final answer must have the subject alone on one side and nowhere on the other. Equivalent forms, such as $\frac{V-2w}{3}$ and $\frac{V}{3} - \frac{2w}{3}$, all score. Each correct step (clearing a fraction, collecting, factorising, dividing) earns a method mark, so write each one.
- **Forming equations**: the equation itself often earns marks before you solve it. Write it down clearly, then solve, then answer in context with units.
- **"Show that"** means the answer is given. Show the equation you start from, clear the fractions, expand every bracket, and end with exactly the printed equation. Any error or missing step loses the last mark.
- **Accuracy**: give exact answers where they are exact ($x = 2.5$, not 2.50 unless asked). Otherwise round to what the question says, or to 3 significant figures if it doesn't say.
- **Extended only: "by factorisation"** means you must show the factorised brackets; answers from the formula alone may not score. **"In surd form"** means leave the root, such as $-3 \pm 2\sqrt{3}$.
- **Extended only: reject impossible answers** in a context, with a reason (a length or speed cannot be negative).

Quick check

1. Solve $5(x-2) = 2x + 11$.
2. Solve the simultaneous equations $3x + 4y = 5$ and $5x - 2y = 17$.
3. Make r the subject of the formula $T = \dfrac{r-4p}{5}$.
4. **Extended only.** Solve $3x^2 + 10x - 8 = 0$ by factorisation.
5. **Extended only.** Make a the subject of the formula $3a + b = ac - 2$.

Answers

1. $5x - 10 = 2x + 11$, so $3x = 21$ and $x = 7$.
2. Double the second: $10x - 4y = 34$. The y terms have different signs, so add it to the first: $13x = 39$, so $x = 3$. Then $9 + 4y = 5$, so $4y = -4$ and $y = -1$. Check: $5(3) - 2(-1) = 15 + 2 = 17$. ✓
3. Multiply by 5: $5T = r - 4p$, so $r = 5T + 4p$.
4. $ac = -24$; 12 and -2 multiply to -24 and add to 10. $3x^2 + 12x - 2x - 8 = 3x(x + 4) - 2(x + 4) = (3x - 2)(x + 4) = 0$, so $x = \frac{2}{3}$ or $x = -4$.
5. Collect the a terms: $3a - ac = -2 - b$. Factorise: $a(3 - c) = -2 - b$. So $a = \dfrac{-2 - b}{3 - c} = \dfrac{b + 2}{c - 3}$.

Past questions to try

- **0580/13/O/N/25 Q6**: a "think of a number" question where the steps are multiplying by 2 and taking a square root.
- **0580/43/O/N/25 Q14**: three expressions that are all equal, giving linear equations to solve one after another.
- **0580/21/O/N/25 Q17**: substituting into a formula, then making k the subject when it is squared.
- **0580/41/O/N/25 Q26**: a quadratic and a linear equation solved simultaneously, where the quadratic has no number term.