Cambridge IGCSE · Mathematics (0580) · Notes

Fractions, decimals, ordering and the four operations

Read online at pastopic.com/cambridge/mathematics-0580/notes/fractions-decimals-and-ordering

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Read first: Types of number and sets

What you need to know

This topic is the same for Core and Extended, except for recurring decimals, which are **Extended only**. By the end of it you should be able to:

1. **Use the words and notation** for proper fractions (such as $\frac{3}{8}$), improper fractions (such as $\frac{11}{4}$), mixed numbers (such as $2\frac{3}{4}$), decimals and percentages, and write any fraction in its **simplest form**.
2. **Recognise equivalent values and convert between them**: fractions, decimals and percentages, improper fractions and mixed numbers, and one quantity as a fraction of another.
3. **Extended only:** read and write **recurring decimal** notation, such as $0.\dot{4} = 0.444\ldots$ and $0.2\dot{1}\dot{7} = 0.2171717\ldots$, and convert a recurring decimal to a fraction and a fraction to a recurring decimal.
4. **Put quantities in order of size**, and use the symbols $=$, $\neq$, $>$, $<$, $\geqslant$ and $\leqslant$.
5. **Add, subtract, multiply and divide** integers, fractions (including improper fractions and mixed numbers) and decimals, with and without a calculator.
6. **Calculate with negative numbers**, including in real situations such as changes in temperature.
7. **Use the correct order of operations and brackets**, and put brackets into a calculation to make it correct.

You will also use place value (the value of each digit in a number) all the way through. Reciprocals, and writing large numbers in words and figures, are in Types of number and sets; percentages of amounts and percentage change are in Percentages.

Understand it

The words

- A **fraction** $\frac{a}{b}$ means a parts out of b equal parts. The top is the **numerator**, the bottom the **denominator**.
- A **proper fraction** is less than 1 (top smaller than bottom): $\frac{5}{8}$.
- An **improper fraction** is 1 or more (top at least as big as bottom): $\frac{13}{5}$.
- A **mixed number** is a whole number and a proper fraction: $2\frac{3}{5}$.

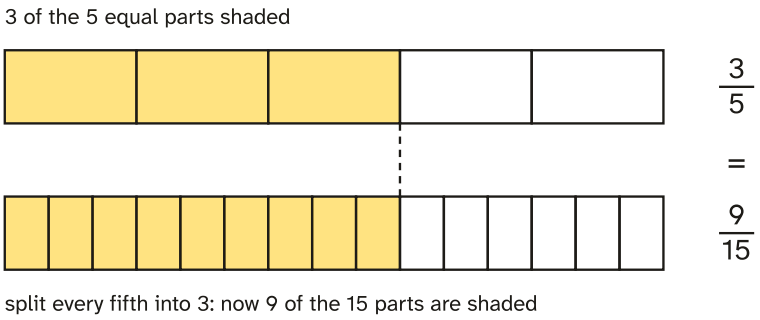
- A **decimal** uses place value after the point: in 3.472 the 4 is 4 tenths, the 7 is $\frac{7}{100}$ and the 2 is $\frac{2}{1000}$.
- A **percentage** is a number of hundredths: $37\% = \frac{37}{100} = 0.37$.

Improper fraction to mixed number: divide the top by the bottom. $\frac{13}{5}$: $13 \div 5 = 2$ remainder 3, so $\frac{13}{5} = 2\frac{3}{5}$.

Mixed number to improper fraction: multiply the whole number by the bottom and add the top. $2\frac{3}{5} = \frac{2\times5+3}{5} = \frac{13}{5}$.

Equivalent fractions and simplest form

Multiplying (or dividing) the top **and** the bottom of a fraction by the same number gives an **equivalent** fraction: the same amount, cut into more (or fewer) pieces.



A fraction is in its **simplest form** when the top and bottom have no common factor except 1. Divide both by their highest common factor (or keep dividing by small common factors until you can't): $\frac{36}{60} = \frac{3}{5}$, dividing both by 12.

To **write one quantity as a fraction of another**, put them in the **same units** first, then simplify: 45 cm as a fraction of 3 m is $\frac{45}{300} = \frac{3}{20}$.

To **shade a fraction of a shape**, count the equal parts. For $\frac{2}{3}$ of a grid of 18 squares, shade $18 \div 3 \times 2 = 12$ squares.

Converting between fractions, decimals and percentages

From	To	How	Example
Percentage	Decimal	Divide by 100	$165\% = 1.65$, $4\% = 0.04$
Decimal	Percentage	Multiply by 100	$0.085 = 8.5\%$
Decimal	Fraction	Write over 10, 100 or 1000, then simplify	$0.35 = \frac{35}{100} = \frac{7}{20}$
Fraction	Decimal	Divide the top by the bottom	$\frac{5}{16} = 5 \div 16 = 0.3125$
Fraction	Percentage	Make the bottom 100, or change to a decimal first	$\frac{9}{25} = \frac{36}{100} = 36\%$
Percentage	Fraction	Write over 100, then simplify	$85\% = \frac{85}{100} = \frac{17}{20}$

These are worth knowing by heart, because they make the non-calculator papers much quicker:

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{3}$	$\frac{2}{3}$
Decimal	0.5	0.25	0.75	0.2	0.125	0.1	0.05	$0.333\ldots$	$0.666\ldots$
Percentage	50%	25%	75%	20%	12.5%	10%	5%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$

Without a calculator, a fraction whose bottom is a factor of 10, 100 or 1000 is easiest: $\frac{7}{40} = \frac{175}{1000} = 0.175$ (multiplying top and bottom by 25).

Recurring decimals (Extended only)

Some fractions give decimals that repeat forever. A dot over a digit shows that it repeats; dots over the first and last digits of a block show that the whole block repeats:

$$0.\dot{4} = 0.4444\ldots, \qquad 0.3\dot{6} = 0.36666\ldots, \qquad 0.\dot{4}\dot{5} = 0.454545\ldots, \qquad 0.\dot{1}3\dot{5} = 0.135135\ldots$$

Fraction to recurring decimal: divide. $\frac{5}{11} = 5 \div 11 = 0.4545\ldots = 0.\dot{4}\dot{5}$.

Recurring decimal to fraction: call the decimal x. Multiply by powers of 10 to make two numbers with **exactly the same digits after the point**, then subtract so the recurring part cancels.

For $x = 0.\dot{4}\dot{5}$, two digits repeat, so multiply by 100:

$$\begin{aligned} 100x &= 45.4545\ldots \\ x &= 0.4545\ldots \\ 99x &= 45 \quad \Rightarrow \quad x = \tfrac{45}{99} = \tfrac{5}{11}. \end{aligned}$$

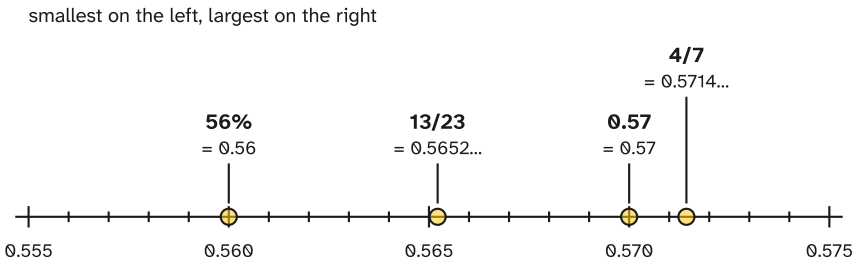
When there is a digit **before** the recurring part, as in $x = 0.3\dot{6} = 0.3666\ldots$, use two multiples that both start the repeating pattern straight after the point:

$$\begin{aligned} 100x &= 36.666\ldots \\ 10x &= 3.666\ldots \\ 90x &= 33 \quad \Rightarrow \quad x = \tfrac{33}{90} = \tfrac{11}{30}. \end{aligned}$$

The check is always the same: the two lines you subtract must have identical tails. $36.666\ldots - 3.666\ldots = 33$ works; $36.666\ldots - 0.3666\ldots$ would not.

Ordering and the symbols

To put numbers of different kinds in order, **change them all to decimals** (to at least 3 or 4 decimal places), compare, then write the **original** numbers in order.



Comparing decimals: line up the points and compare digit by digit from the left. $0.57 > 0.5652$ because at the second decimal place $7 > 6$. The number of digits does not matter: $0.6 > 0.5999$.

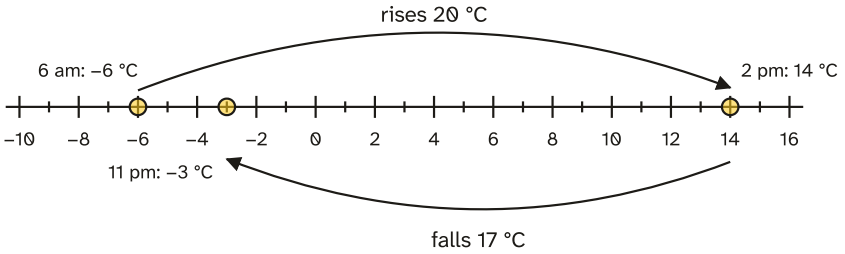
For lengths or masses in different units, change them all to the **same unit** first. 0.4 m, 38 cm and 395 mm in cm are 40, 38 and 39.5.

Symbol	Means	Example
$=$	is equal to	$0.4 = \frac{2}{5}$
$\neq$	is not equal to	$0.3 \neq \frac{1}{3}$
$<$	is less than	$-7 < -2$
$>$	is greater than	$\frac{3}{4} > 0.7$
$\leqslant$	is less than or equal to	$x \leqslant 5$: x can be 5 or anything below
$\geqslant$	is greater than or equal to	$x \geqslant -1$: x can be -1 or anything above

The narrow end of $<$ or $>$ points at the smaller number.

Negative numbers

On a number line, numbers get **bigger to the right**. So $-2 > -7$, even though $7 > 2$: -2 is warmer than -7.



- A **rise** moves right (add); a **fall** moves left (subtract).
- The **difference** between two temperatures is the distance between them, so it is always positive: from -6 to 14 is $14 - (-6) = 20$ degrees. Across zero, add the two parts: 6 to reach 0, then 14 more.

The sign rules:

Calculation	Rule	Example
Adding a negative	same as subtracting	$5 + (-8) = 5 - 8 = -3$
Subtracting a negative	same as adding	$-4 - (-9) = -4 + 9 = 5$
Multiplying or dividing, **same** signs	answer positive	$(-6) \times (-7) = 42$, $(-24) \div (-4) = 6$
Multiplying or dividing, **different** signs	answer negative	$(-6) \times 7 = -42$, $30 \div (-5) = -6$

With three numbers multiplied, count the negatives: an even number of them gives a positive answer, an odd number a negative one. $(-2) \times (-3) \times (-5) = -30$.

The order of operations

Work out a calculation in this order:

1. **B**rackets
2. **I**ndices (powers and roots)
3. **D**ivision and **M**ultiplication, from left to right
4. **A**ddition and **S**ubtraction, from left to right

So $17 - 8 \div 2 \times 3 = 17 - 4 \times 3 = 17 - 12 = 5$. Putting brackets in changes what is done first: $(17 - 8) \div 2 \times 3 = 9 \div 2 \times 3 = 13.5$, and $17 - 8 \div (2 \times 3) = 17 - \frac{8}{6} = 15\frac{2}{3}$.

A fraction line acts like a bracket: $\frac{12+8}{4-2}$ means $(12 + 8) \div (4 - 2) = 10$.

Adding and subtracting fractions

You can only add or subtract fractions with the **same denominator**, because only then are the pieces the same size.

1. Change mixed numbers to improper fractions (or deal with the whole numbers separately).
2. Find a **common denominator**, ideally the lowest common multiple of the denominators.
3. Change each fraction to that denominator, then add or subtract the tops. Keep the bottom.
4. Simplify, and change to a mixed number if asked.

$$\frac{5}{12} + \frac{3}{8} = \frac{10}{24} + \frac{9}{24} = \frac{19}{24}.$$

Multiplying and dividing fractions

Multiplying: change mixed numbers to improper fractions, then multiply the tops and multiply the bottoms. Cancel common factors first (any top with any bottom) to keep the numbers small. You do **not** need a common denominator.

$$1\frac{7}{8} \times 2\frac{2}{5} = \frac{15}{8} \times \frac{12}{5} = \frac{3}{2} \times \frac{3}{1} = \frac{9}{2} = 4\frac{1}{2}$$

(cancelling 15 with 5 and 12 with 8).

Dividing: dividing by a fraction is the same as multiplying by its **reciprocal** (the fraction turned upside down). Change mixed numbers first, then flip only the **second** fraction:

$$2\tfrac{2}{9} \div 1\tfrac{1}{3} = \frac{20}{9} \div \frac{4}{3} = \frac{20}{9} \times \frac{3}{4} = \frac{60}{36} = \frac{5}{3} = 1\tfrac{2}{3}.$$

This is why $n \div 2\frac{1}{4}$ is the same as $n \times \frac{4}{9}$: $2\frac{1}{4} = \frac{9}{4}$, and its reciprocal is $\frac{4}{9}$.

A fraction of an amount: divide by the bottom, multiply by the top. $\frac{4}{9}$ of 162 is $162 \div 9 \times 4 = 72$. "Of" means multiply.

Finding the whole from a part: if 84 tickets are $\frac{7}{12}$ of all the tickets, then $\frac{1}{12}$ is $84 \div 7 = 12$, and the whole is $12 \times 12 = 144$.

Decimals without a calculator

- **Adding and subtracting:** line up the decimal points. $6.4 - 2.75 = 6.40 - 2.75 = 3.65$.
- **Multiplying:** ignore the points, multiply the whole numbers, then give the answer as many decimal places as the two numbers have together. 0.3×0.07: $3 \times 7 = 21$, with $1 + 2 = 3$ decimal places, so 0.021.
- **Dividing:** multiply both numbers by the same power of 10 until you divide by a whole number. $4.2 \div 0.06 = 420 \div 6 = 70$.
- **Using a given result:** if $36 \times 145 = 5220$, then $3.6 \times 14.5 = 52.2$ (each number is $\div 10$, so the answer is $\div 100$), and $5220 \div 14.5 = 360$ (because $360 \times 14.5 = 3.6 \times 1450 = 5220$).

Key facts and formulas

None of these is on the List of formulas printed in the exam paper, so you must know them all.

Formula	Status
$\frac{a}{b} = \frac{a \times k}{b \times k}$ (equivalent fractions)	Learn it
$m\frac{a}{b} = \frac{mb + a}{b}$ (mixed number to improper fraction)	Learn it
$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ (or use the lowest common denominator)	Learn it
$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$	Learn it
$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$	Learn it
$x\% = \frac{x}{100}$; decimal $\times 100$ = percentage	Learn it

Formula	Status
Common equivalents: $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{1}{8}, \frac{1}{10}, \frac{1}{3}, \frac{2}{3}$ as decimals and percentages	Learn it
Extended: recurring decimal to fraction by subtracting two multiples with the same recurring tail	Learn it
Same signs multiply or divide to a positive; different signs to a negative	Learn it
Order of operations: brackets, indices, $\div$ and $\times$, $+$ and $-$	Learn it

How to do it

In the questions we have tagged for this topic (194 questions from Papers 1–4, 2021–2025, 7 of them printed on two papers), the types came up this often. 21 of the 194 questions mix two or three types, so the counts add up to more than 194.

Question type	Questions
Calculating with fractions and mixed numbers	56
Negative numbers and temperature changes	31
Converting between fractions, decimals and percentages	30
Ordering and the symbols $=$, $\neq$, $<$, $>$	21
Place value and numbers in figures	20
Order of operations and inserting brackets	19
Fractions of amounts and number problems in context	18
Recurring decimals (Extended)	15
Decimal calculations and using a given method	8

1. Calculating with fractions and mixed numbers (56 questions)

Almost always "Without using a calculator... show all your working", worth 2 or 3 marks.

1. **Change every mixed number to an improper fraction**, and write it down: $3\frac{1}{6} = \frac{19}{6}$.
2. **Choose by the operation:**
   - $+$ or $-$: rewrite both with a common denominator, then add or subtract the tops.
   - $\times$: cancel, then multiply tops and bottoms.
   - $\div$: flip the second fraction and multiply.
3. **Simplify** by dividing top and bottom by any common factor.

4. **Give the form asked for**: "a fraction in its simplest form" or "a mixed number in its simplest form".

Variations you will meet:

- **Subtracting mixed numbers where the second fraction part is bigger**, such as $3\frac{1}{6} - 1\frac{4}{9}$: going through improper fractions avoids the borrowing problem ($\frac{57}{18} - \frac{26}{18} = \frac{31}{18} = 1\frac{13}{18}$).
- **Three fractions with two operations**, such as $\frac{5}{6} - \frac{1}{4} \times \frac{2}{3}$: the order of operations still applies, so multiply first ($\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$), then subtract ($\frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$).
- **A whole number with a fraction**: write it over 1. $\frac{5}{9} \div 10 = \frac{5}{9} \times \frac{1}{10} = \frac{5}{90} = \frac{1}{18}$.
- **"Draw a ring around the calculation that is equivalent to..."**: dividing by $\frac{a}{b}$ is multiplying by $\frac{b}{a}$, so change any mixed number first, then flip it.
- **"Work out three-fifths of two-thirds"**: "of" is $\times$, so $\frac{3}{5} \times \frac{2}{3} = \frac{6}{15} = \frac{2}{5}$.

2. Negative numbers and temperature changes (31 questions)

1. **Picture a number line** (or a thermometer): right is warmer, left is colder.
2. **"Difference between"** two temperatures: bigger minus smaller, always positive. From -11 to 7 is $7 - (-11) = 18$.
3. **"Rises by" or "higher than"**: add. **"Falls by" or "colder than"**: subtract. -11 rising by 15 is 4; 4 falling by 10 is -6.
4. **Multiplying and dividing**: work out the size, then the sign from the rules.

Variations you will meet:

- **A table of temperatures**: the coldest is the most negative; find differences as above.
- **Two numbers from their sum and product**, such as sum -3 and product -28: the product is negative, so one number is negative and one positive. List factor pairs of 28 and find the pair that adds to -3: -7 and 4.
- **The largest or smallest product from a list**, such as $-8, -5, -1, 0, 3, 6$: two negatives make a large positive, so the largest product of two is $(-8) \times (-5) = 40$. For the smallest product of three, you want a large negative: $(-8) \times 6 \times 3 = -144$.
- **Mixed operations**, such as $(-4) \times 7 - (-18) \div 3$: multiply and divide first (-28 and -6), then $-28 - (-6) = -22$.
- **"The number that is 15 less than -2.5"**: $-2.5 - 15 = -17.5$. **"What must you add to -6 to get 4?"**: $4 - (-6) = 10$.
- **Heights above and below sea level** work the same way: 620 m above and 45 m below are $620 + 45 = 665$ m apart.
- **A chain of changes** (a temperature is so much warmer than one place and so much colder than another): write each step as a calculation, one at a time.

3. Converting between fractions, decimals and percentages (30 questions)

1. **Use the table in "Understand it"**: percentage $\div 100$ gives a decimal; a decimal over 10, 100 or 1000 gives a fraction; top $\div$ bottom gives a decimal.
2. **Simplify** any fraction answer unless the question lets you leave it ("Write 0.35 as a fraction" accepts $\frac{35}{100}$; "in its simplest form" needs $\frac{7}{20}$).

Variations you will meet:

- **Complete a table** of fraction, decimal and percentage: fill each row from the one value you are given.
- **Percentages over** 100 or below 1: $165\% = 1.65$ and $4\% = 0.04$ (not 0.4).
- **"Write $\frac{a}{b}$ as a mixed number"**, or the other way round.
- **Equivalent fractions**: "write down a fraction equivalent to $\frac{4}{7}$" ($\frac{8}{14}$), or "ring the fractions not equivalent to $\frac{3}{5}$": simplify each one and compare.
- **Shade a fraction of a grid or shape**: number of parts $\div$ bottom $\times$ top. Shade whole squares.
- **One quantity as a fraction of another** (people in a group, sectors of a pie chart, 45 cm out of 3 m): part over whole, same units, simplest form. 105 of 180 people is $\frac{105}{180} = \frac{7}{12}$.
- **"Write down the value of the 5 in 0.35 as a fraction"**: it is in the hundredths column, so $\frac{5}{100}$ (see type 5).

4. Ordering and the symbols (21 questions)

1. **Change every value to a decimal** to at least 3 decimal places (percentages $\div 100$, fractions top $\div$ bottom, π as $3.1415\ldots$). Write the decimals down.
2. **Compare** digit by digit from the left.
3. **Write the original values** in order, starting with the one the question asks for (usually the smallest).

Variations you will meet:

- **Values that are very close**, such as $\frac{4}{7}$ and 0.57: go to 4 decimal places (0.5714 and 0.5700).
- **Lengths or masses in mixed units**: change to one unit first.
- **"Write $<$, $>$ or $=$ in each statement"**: convert both sides. 0.09 and $\frac{1}{11} = 0.0909\ldots$ give $0.09 < \frac{1}{11}$.
- **Values that depend on m**, such as $\frac{m}{4}$, 26% of m and $\frac{3}{10}$ of m: compare the multipliers 0.25, 0.26 and 0.3, since m is positive.
- **"Find a fraction between $\frac{4}{9}$ and $\frac{5}{9}$"**: make an equivalent pair with more pieces, $\frac{8}{18}$ and $\frac{10}{18}$, and take $\frac{9}{18} = \frac{1}{2}$.
- **Negative numbers in a list**: the most negative is the smallest. "All the numbers less than -4" means those further left, such as -5 and -9.

5. Place value and numbers in figures (20 questions)

1. **Name each column**: ... ten thousands, thousands, hundreds, tens, units . tenths, hundredths, thousandths.

2. **"The value of the 6 in $406\,815$"**: the 6 is in the thousands column, so its value is 6000 (not just "thousands" or 6).
3. **Words to figures** (a one-mark first question): write one group of three digits for each "million" and "thousand", and fill gaps with zeros. "Three hundred and five thousand and forty" is $305\,040$. Count the digits to check.

The full method for words and figures is in Types of number and sets.

6. Order of operations and inserting brackets (19 questions)

To work out a calculation: brackets, then powers, then $\times$ and $\div$ left to right, then $+$ and $-$ left to right. $9 - 6 \times 2 + 12 \div 4 = 9 - 12 + 3 = 0$.

To put in one pair of brackets:

1. Work out the calculation **as it stands**. If it isn't the target, brackets must change what is done first.
2. **Try brackets around each $+$ or $-$ in turn**, since those are the parts that normally come last. Work each one out.
3. **Check** your chosen version gives exactly the target, and use only **one** pair.

For $4 + 8 \times 5 - 3 = 20$: as it stands it is $4 + 40 - 3 = 41$. Trying $(4 + 8) \times 5 - 3 = 57$, then $4 + 8 \times (5 - 3) = 4 + 16 = 20$. ✓

Variations you will meet:

- **Two parts with the same numbers and different targets**: each target needs a different pair.
- **Negative numbers at the start**, such as $-10 + 6 \div 3 - 5$: keep the minus sign with the 10 and work carefully.
- **Brackets that include the first number**: for $30 - 6 \div 4 + 2 = 8$ the answer is $(30 - 6) \div 4 + 2 = 8$, so try a bracket that starts at the very first number.

7. Fractions of amounts and number problems in context (18 questions)

1. **Decide which of three kinds it is:**
   - **Fraction of an amount**: divide by the bottom, multiply by the top. $\frac{3}{8}$ of \$296 is \$296 ÷ 8 × 3 = \$111.
   - **Whole from a part** ("84 is $\frac{7}{12}$ of the total"): part ÷ top $\times$ bottom $= 84 \div 7 \times 12 = 144$.
   - **What's left** after taking fractions of the **same** whole: $1-$ the fractions. Spending $\frac{2}{5}$ and $\frac{1}{4}$ leaves $1 - \frac{2}{5} - \frac{1}{4} = \frac{20}{20} - \frac{8}{20} - \frac{5}{20} = \frac{7}{20}$.
2. **Check whether the second fraction is of the whole or of what is left.** "$\frac{2}{5}$ of the money, then $\frac{1}{4}$ of **what is left**" is $\frac{1}{4}$ of $\frac{3}{5}$, not $\frac{1}{4}$ of the whole.
3. **Answer the question asked**: the amount spent, or the amount left?

Variations you will meet:

- **How many boxes, buses or coaches**: divide, then **round up**, because a part-full box still needs a box. 500 people in coaches of 52: $500 \div 52 = 9.6\ldots$, so 10 coaches.
- **How many can you buy**: divide, then **round down**, and find the change. \$15 ÷ \$1.85 = 8.1..., so 8 items, and \$15 − 8 × \$1.85 = \$0.20 change.
- **"Does she have enough? Show how you decide"**: do one calculation (such as $500 \div 52$ or 9×52) **and** write a sentence that answers yes or no with the numbers.
- **A fraction with a ratio or percentage in the same question**: find each amount in turn and subtract from the total.

8. Recurring decimals (Extended) (15 questions)

1. **Write $x =$ the decimal** with enough digits to see the pattern: $x = 0.5181818\ldots$
2. **Multiply by powers of** 10 to get two numbers whose digits after the point are identical: $1000x = 518.1818\ldots$ and $10x = 5.1818\ldots$
3. **Subtract**: $990x = 513$.
4. **Divide and simplify**: $x = \frac{513}{990} = \frac{57}{110}$.

A quick way to choose the multipliers: shift the point to just after the first recurring block, and to just before it. With no digits before the recurring part, use $10^n x$ and x, where n is the length of the block.

Variations you will meet:

- **One recurring digit**: $0.\dot{7} = \frac{7}{9}$; $0.1\dot{6} = \frac{16-1}{90} = \frac{15}{90} = \frac{1}{6}$.
- **A three-digit block**: $0.\dot{1}3\dot{5}$: $1000x - x = 135$, so $x = \frac{135}{999} = \frac{5}{37}$.
- **Adding a recurring decimal to a fraction or decimal** ("give your answer as a fraction"): change the recurring decimal to a fraction first, then add with a common denominator. Or add the decimals first, if the result is easy to convert.
- **Read the dots carefully**: $0.5\dot{7} = 0.5777\ldots$ (one digit repeats) but $0.\dot{5}\dot{7} = 0.5757\ldots$ (two digits repeat).

9. Decimal calculations and using a given method (8 questions)

1. **Decimals**: count decimal places for $\times$; make the divisor a whole number for $\div$ (see "Understand it"). $1.4 \times 0.03 = 0.042$ and $57.6 \div 8 = 7.2$.
2. **Using a given fact**, such as $36 \times 145 = 5220$: say how each number has changed ($\times 10$ or $\div 10$) and change the answer to match.
3. **Following someone's method** (for example $46 \times 99 = 46 \times 100 - 46$, or halving both numbers in a division): copy the method's steps exactly with the new numbers and write every line. $46 \times 101 = 4600 + 46 = 4646$; $234 \div 18 = 117 \div 9 = 13$.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

Calculators must not be used in this question.

(a) Write 0.65 as a fraction in its simplest form. **[1]**

(b) Write $\frac{7}{16}$ as a decimal. **[1]**

(c) Write 245% as a decimal. **[1]**

(d) Write these numbers in order, starting with the smallest. **[2]**

$$\frac{5}{9} \qquad 0.55 \qquad 54\% \qquad \frac{6}{11}$$

(e) Write $<$, $>$ or $=$ in each statement. **[2]**

$$0.6 \ \ldots \ \frac{3}{5} \qquad\qquad 0.07 \ \ldots \ \frac{1}{14}$$

(a) $0.65 = \frac{65}{100} = \frac{13}{20}$ (dividing by 5). **B1**

(b) $7 \div 16 = 0.4375$. **B1**

(c) $245 \div 100 = 2.45$. **B1**

(d) As decimals: $\frac{5}{9} = 0.5555\ldots$, 0.55, $54\% = 0.54$, $\frac{6}{11} = 0.5454\ldots$ **M1** for changing all of them to decimals (or all to percentages).

54%, $\frac{6}{11}$, 0.55, $\frac{5}{9}$ **A1**

(e) $\frac{3}{5} = 0.6$, so $0.6 = \frac{3}{5}$. **B1**

$\frac{1}{14} = 0.0714\ldots$, and $0.0700 < 0.0714$, so $0.07 < \frac{1}{14}$. **B1**

Example 2

(a) At 6 am the temperature is $-5\,^{\circ}$C. At 2 pm it is $13\,^{\circ}$C.

(i) Find the difference between these temperatures. **[1]**

(ii) By 11 pm the temperature has fallen by $21\,^{\circ}$C from its value at 2 pm. Find the temperature at 11 pm. **[1]**

(b) Work out $(-5) \times 6 - (-24) \div 4$. **[2]**

(c) Put one pair of brackets into this calculation to make it correct. **[1]**

$$18 - 6 \div 3 \times 2 = 8$$

(d) On Friday a theatre sells 90 tickets. This is $\dfrac{3}{8}$ of the tickets it sells that week. Work out how many tickets it sells that week. **[2]**

(a)(i) $13 - (-5) = 18\,^{\circ}$C. **B1**

(a)(ii) $13 - 21 = -8\,^{\circ}$C. **B1**

(b) $(-5) \times 6 = -30$ and $(-24) \div 4 = -6$. **M1** for either.

$-30 - (-6) = -30 + 6 = -24$. **A1**

(c) As it stands, $18 - 2 \times 2 = 14$. Trying the subtraction first: $(18 - 6) \div 3 \times 2 = 12 \div 3 \times 2 = 8$. ✓ **B1**

(d) $\frac{1}{8}$ of the tickets is $90 \div 3 = 30$. **M1**

$30 \times 8 = 240$ tickets. **A1**

Example 3

Calculators must not be used in this question. You must show all your working.

(a) Work out $4\dfrac{1}{6} - 1\dfrac{5}{8}$. Give your answer as a mixed number in its simplest form. **[3]**

(b) Work out $3\dfrac{1}{3} \div 1\dfrac{7}{9}$. Give your answer as a mixed number in its simplest form. **[3]**

(a) $4\frac{1}{6} = \frac{25}{6}$ and $1\frac{5}{8} = \frac{13}{8}$. **B1** for both improper fractions.

The LCM of 6 and 8 is 24: $\frac{100}{24} - \frac{39}{24}$. **M1** for both with a common denominator and correct tops.

$= \frac{61}{24} = 2\frac{13}{24}$. **A1**

(b) $3\frac{1}{3} = \frac{10}{3}$ and $1\frac{7}{9} = \frac{16}{9}$. **B1**

$\frac{10}{3} \times \frac{9}{16}$ **M1** for turning the second fraction upside down and multiplying.

$= \frac{90}{48} = \frac{15}{8} = 1\frac{7}{8}$. **A1** (Cancelling first: $\frac{5}{1} \times \frac{3}{8}$, since 10 and 16 share 2, and $9 \div 3 = 3$.)

Example 4

This question is Extended. You must show all your working.

(a) Write $0.\dot{6}\dot{3}$ as a fraction in its simplest form. **[2]**

(b) Write $0.3\dot{1}\dot{8}$ as a fraction in its simplest form. **[3]**

(c) Work out $0.\dot{6}\dot{3} + \dfrac{3}{10}$. Give your answer as a fraction. **[2]**

(a) $x = 0.6363\ldots$, $100x = 63.6363\ldots$ **M1** for subtracting to get $99x = 63$.

$x = \frac{63}{99} = \frac{7}{11}$. **A1**

(b) $x = 0.31818\ldots$ The recurring block 18 starts after one digit.

$1000x = 318.1818\ldots$ and $10x = 3.1818\ldots$ **M1** for two multiples with the same recurring tail.

$990x = 315$ **M1**, so $x = \frac{315}{990} = \frac{7}{22}$ (dividing by 45). **A1**

(c) $\frac{7}{11} + \frac{3}{10} = \frac{70}{110} + \frac{33}{110}$ **M1**

$= \frac{103}{110}$. **A1** (Check: $0.6363\ldots + 0.3 = 0.9363\ldots$, and $103 \div 110 = 0.93636\ldots$)

Common mistakes

- **Finding a common denominator to multiply, then not multiplying the bottoms.** Examiners see $\frac{a}{12} \times \frac{b}{12}$ turned into $\frac{ab}{12}$. You never need a common denominator to multiply or divide: multiply tops and bottoms.
- **Using the division rule for addition, or the addition rule for division.** Adding needs a common denominator; dividing needs "flip the second and multiply". Say which operation you are doing before you start.
- **Changing a mixed number wrongly.** $3\frac{1}{6}$ is $\frac{3\times6+1}{6} = \frac{19}{6}$, not $\frac{4}{6}$ or $\frac{18}{6}$.
- **Stopping at an improper fraction when a mixed number is asked for**, or not simplifying the fraction part. "$1\frac{26}{36}$" and "$\frac{31}{18}$" both lose the last mark when the question says "mixed number in its simplest form".
- **Giving a decimal when the question asks for a fraction.** 1.67 is not accepted for $1\frac{2}{3}$, and a rounded decimal is wrong.
- **Recurring decimals: subtracting two multiples whose tails don't match.** For $0.\dot{4}\dot{7}$, $47.47\ldots - 4.74\ldots$ does not cancel. Line up the tails before subtracting.
- **Ignoring the recurring dots**, for example writing $0.\dot{4}\dot{7}$ as $\frac{47}{100}$. That is 0.47, a different number; $0.\dot{4}\dot{7} = \frac{47}{99}$.
- **Ordering with too few decimal places**, or treating 47% as 47. Change every value to a decimal (or every value to a percentage) to at least 3 or 4 places, and write the original values in the final answer.
- **Confusing $\frac{1}{3}$ and 0.3:** $\frac{1}{3} = 0.333\ldots$, so $\frac{1}{3} > 0.3$.
- **Losing the minus sign in a temperature.** A fall of $19\,^\circ$C from $12\,^\circ$C is $-7\,^\circ$C, not $7\,^\circ$C. Check on a number line.
- **Giving a negative difference.** A difference is a distance, so it is positive.
- **Using two pairs of brackets**, or putting a bracket round a power or a single number. Only one pair is allowed; work out your version to check it.
- **Working left to right instead of using the order of operations.** $6 + 4 \times 8$ is 38, not 80.

- **Finding the amount not spent when the question asks for the amount spent**, or taking the second fraction of the whole instead of what is left.
- **Rounding the wrong way in context.** $710 \div 48 = 14.7\ldots$ coaches means 15 coaches, not 14.
- **Writing numbers in figures with the wrong number of zeros.** Count: a number in the millions has 7 digits.

How to get the marks

- **"Without using a calculator... show all your working"**: an answer with no working usually gets **no** marks, even if it is right. Write the improper fractions, the common denominator (or the flipped fraction) and the answer. Each of these steps is usually worth a mark.
- **"Give your answer as a fraction in its simplest form"** or **"as a mixed number in its simplest form"**: the last mark is only for that exact form. When the question just says "as a fraction", any correct equivalent fraction is accepted.
- **Recurring decimals**: show the two multiples of x and the subtraction, such as $100x - x = 45$. That line earns the method mark; an answer copied from a calculator may not.
- **Ordering**: show the decimals you compared. The final answer must be the **original** numbers in the right order; with 2 marks, three in the right order often earns 1.
- **Temperatures**: include the minus sign for a temperature below zero. A difference or an increase is a positive number.
- **Brackets**: write the whole calculation with your brackets in it, on the answer line or in the printed calculation.
- **"Show how you decide"** or **"Explain"**: a correct calculation **and** a conclusion with the numbers, such as "No, 9 coaches carry only 468 people."
- **Place value**: "the value of the 3" is a number, such as 3000 or $\frac{3}{100}$, not a column name.
- **Contexts**: give units ($, °C, cm), and money to 2 decimal places, such as $0.20.

Quick check

1. Write 0.045 as a fraction in its simplest form.
2. Without a calculator, work out (a) $\frac{7}{10} - \frac{4}{15}$ (b) $1\frac{3}{5} \times 2\frac{1}{12}$, giving (b) as a mixed number.
3. At 5 am the temperature is $-9\,^{\circ}$C. By noon it has risen by $16\,^{\circ}$C. (a) Find the temperature at noon. (b) Find the difference between $-9\,^{\circ}$C and $6\,^{\circ}$C.
4. Put one pair of brackets into $3 + 6 \times 4 - 2 = 34$ to make it correct.
5. (Extended) Write $0.2\dot{7}$ as a fraction in its simplest form.

Answers

1. $0.045 = \frac{45}{1000} = \frac{9}{200}$.
2. (a) The LCM of 10 and 15 is 30: $\frac{21}{30} - \frac{8}{30} = \frac{13}{30}$. (b) $\frac{8}{5} \times \frac{25}{12} = \frac{2}{1} \times \frac{5}{3} = \frac{10}{3} = 3\frac{1}{3}$ (cancelling 8 with 12 by 4, and 25 with 5).
3. (a) $-9 + 16 = 7\,^{\circ}\text{C}$. (b) $6 - (-9) = 15\,^{\circ}\text{C}$.
4. As it stands, $3 + 24 - 2 = 25$. $(3 + 6) \times 4 - 2 = 36 - 2 = 34$.
5. $x = 0.2777\ldots$; $100x = 27.777\ldots$ and $10x = 2.777\ldots$, so $90x = 25$ and $x = \frac{25}{90} = \frac{5}{18}$.

Past questions to try

- **0580/23/M/J/25 Q7**: subtracting fractions, then dividing with a mixed number.
- **0580/12/M/J/25 Q8**: putting one pair of brackets into calculations with negative numbers.
- **0580/22/M/J/23 Q16**: changing a recurring decimal with one digit before a repeating two-digit block to a fraction (Extended).
- **0580/11/M/J/22 Q6**: ordering a percentage, a decimal and two fractions that are very close.