Cambridge IGCSE · Mathematics 0580 · Notes

Functions

Read online at **pastopic.com/cambridge/mathematics-0580/notes/functions**

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Read first: **Algebraic manipulation and algebraic fractions**, **Equations and changing the subject**

What you need to know

Extended only: this whole topic is Extended content (syllabus E2.13). Core candidates do not study functions.

By the end of this topic you should be able to:

1. **Understand what a function is and use function notation.** Read $f(x) = 3x - 2$ as a rule, work out values such as $f(4)$ or $f(-1)$, and substitute expressions such as $f(2x)$ or $f(x + 1)$.
2. **Understand domain and range.** The domain is the set of inputs; the range is the set of outputs you get from them. You may meet this as a list of numbers or as a mapping diagram.
3. **Find inverse functions**, written $f^{-1}(x)$: the function that undoes f. Find $f^{-1}(x)$ as an expression and use it with numbers.
4. **Form composite functions**, where one function is applied after another: $gf(x) = g(f(x))$ means "do f first, then g".

You are **not** asked for the domain or range of a composite function. Any functions can appear: linear ones such as $3x - 5$, fractions such as $\frac{3(x+4)}{5}$ or $\frac{12}{x+3}$, quadratics such as $2x^2 + 3$, and exponentials such as 2^x.

Understand it

A function is a rule

A **function** takes an input, does something to it, and gives exactly one output. The rule $f(x) = 3x - 2$ says "multiply the input by 3, then subtract 2". The letter x just marks the place where the input goes.

- $f(4) = 3(4) - 2 = 10$
- $f(-1) = 3(-1) - 2 = -5$
- $f(0.5) = 3(0.5) - 2 = -0.5$

The input can be an expression, not just a number. Replace **every** x with the whole expression, in brackets:

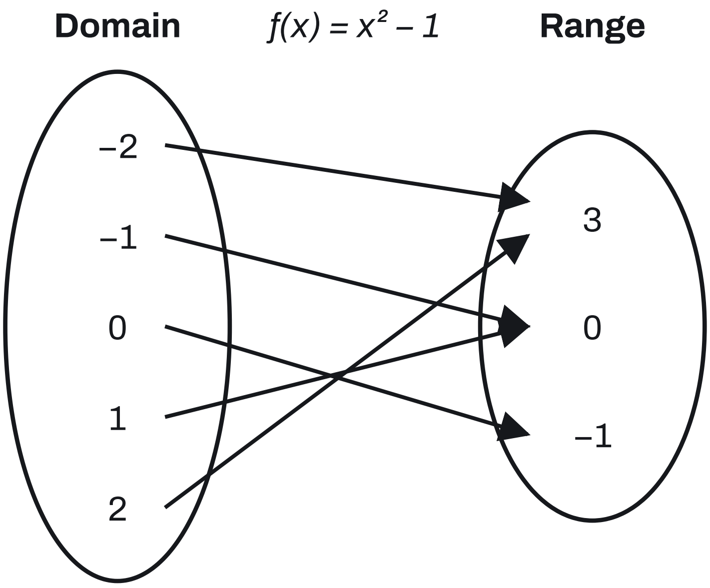
- $f(2x) = 3(2x) - 2 = 6x - 2$

- $f(a+1) = 3(a+1) - 2 = 3a + 1$

$f(x)$ is not "f times x". It is the name of the output when the input is x.

Domain and range

The **domain** is the set of inputs the function is allowed to take. The **range** is the set of outputs it actually produces. A **mapping diagram** shows each input with an arrow to its output.



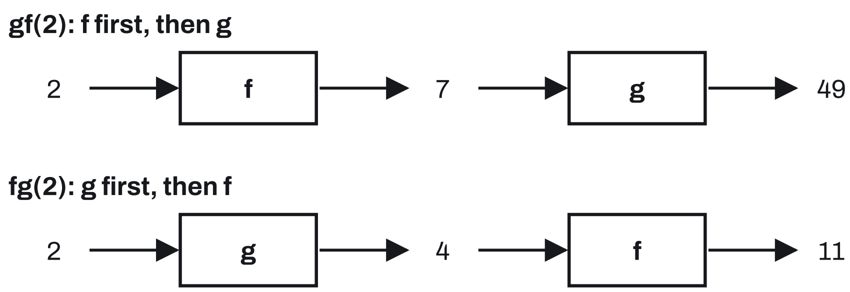
For $f(x) = x^2 - 1$ with domain $\{-2, -1, 0, 1, 2\}$, the outputs are $3, 0, -1, 0, 3$. The range lists each output **once**: $\{-1, 0, 3\}$. Two inputs can share an output, but one input never has two outputs.

Some functions leave out a value because it would make them meaningless. $h(x) = \dfrac{4}{x-3},\ x \neq 3$ cannot take $x = 3$, since that means dividing by 0. The "$x \neq 3$" is part of the definition; it is not an equation to solve.

Composite functions

A **composite function** applies one function and then feeds its output into another. Take $f(x) = 2x + 3$ and $g(x) = x^2$.

$gf(x)$ means $g(f(x))$: the function written **next to the x acts first**. So $gf(2)$: first $f(2) = 7$, then $g(7) = 49$.



The order matters. $fg(2)$ means g first: $g(2) = 4$, then $f(4) = 11$. So $gf(2) = 49$ but $fg(2) = 11$.

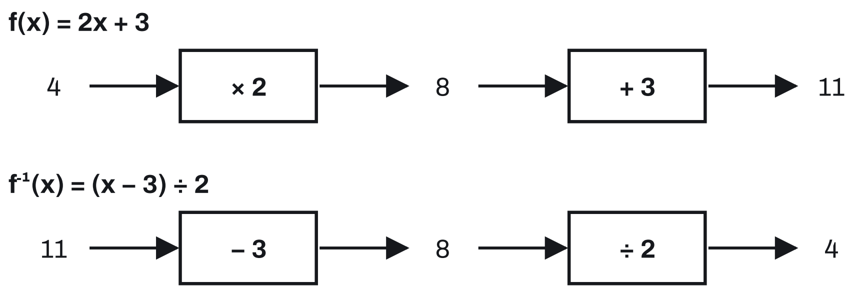
To find a composite **in terms of** x, put the whole inner function into the outer one:

- $gf(x) = g(2x + 3) = (2x + 3)^2 = 4x^2 + 12x + 9$
- $fg(x) = f(x^2) = 2x^2 + 3$
- $ff(x) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 9$

A composite is **not** a product: $gf(x)$ is not $g(x) \times f(x)$.

Inverse functions

The **inverse function** f^{-1} undoes f: whatever f does, f^{-1} takes you back. For $f(x) = 2x + 3$, f multiplies by 2 then adds 3. To undo it, do the opposite steps in the opposite order: subtract 3, then divide by 2.



So $f^{-1}(x) = \dfrac{x-3}{2}$, and $f^{-1}(11) = \dfrac{11-3}{2} = 4$, back where we started.

The reliable algebra method is to **swap x and y and rearrange**:

$$y = 2x + 3 \xrightarrow{\text{swap}} x = 2y + 3 \Rightarrow x - 3 = 2y \Rightarrow y = \frac{x-3}{2}.$$

Three facts follow from "undoes":

- $f^{-1}f(x) = x$ and $ff^{-1}(x) = x$. For example $ff^{-1}(7) = 7$, with no working needed.
- $f^{-1}(x) = k$ **means $x = f(k)$.** If $f^{-1}(x) = 5$, then $x = f(5) = 13$. You don't need to find f^{-1} at all.
- The -1 is not a power. $f^{-1}(x)$ is **not** $\dfrac{1}{f(x)}$.

A function can be its own inverse. $p(x) = \dfrac{1}{x}$ gives $pp(x) = \dfrac{1}{1/x} = x$, so $p^{-1}(x) = p(x)$.

Exponential functions

Functions such as $h(x) = 3^x$ appear too. Use index rules to solve them:

- $h(x) = 81$: $3^x = 3^4$, so $x = 4$.
- $h(x) = \frac{1}{9}$: $3^x = 3^{-2}$, so $x = -2$.
- $h^{-1}(x) = 2$: $x = h(2) = 3^2 = 9$.

Key facts and formulas

There are no function formulas on the List of formulas printed in Papers 2 and 4, so everything here must be learnt. The quadratic formula (on the Extended list) is often needed when solving.

Formula	Status
$f(a)$: replace every x in $f(x)$ with a, in brackets	Learn it
$gf(x) = g(f(x))$: do f first, then g	Learn it
$f^{-1}(x)$: write $y = f(x)$, swap x and y, make y the subject	Learn it
$f^{-1}f(x) = ff^{-1}(x) = x$	Learn it
$f^{-1}(x) = k \iff x = f(k)$	Learn it
Range: the set of outputs from the domain, each listed once	Learn it
$ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Given
$a^{-n} = \dfrac{1}{a^n}$, $a^0 = 1$ (for exponential functions)	Learn it

How to do it

In the Paper 2 and Paper 4 questions we have tagged for this topic (31 questions, 2021–2025), the types came up this often. Function questions come in several parts, so 29 of the 31 questions mix types and the counts add up to more than 31.

Question type	Questions
Inverse functions	29
Solving equations involving functions	23
Values of functions and composite functions	19
Composite functions in terms of x	17
Combining functions: products, squares and single fractions	10
Domain and range	1

1. Inverse functions (29 questions)

1. Write $y = f(x)$.
2. **Swap** x and y. This is your first step, and it earns a method mark on its own.
3. **Rearrange** to make y the subject, doing the opposite operation at each step.
4. Write the answer as $f^{-1}(x) = \ldots$, **in terms of** x.

For example, $f(x) = 9 - 4x$: $x = 9 - 4y \Rightarrow 4y = 9 - x \Rightarrow f^{-1}(x) = \dfrac{9-x}{4}$.

Variations you will meet:

- **A fraction with x on the bottom**, such as $f(x) = \dfrac{6}{x+1},\ x \neq -1$. Swap: $x = \dfrac{6}{y+1}$. Multiply up: $x(y+1) = 6$. Divide: $y + 1 = \dfrac{6}{x}$. So $f^{-1}(x) = \dfrac{6}{x} - 1$, which is also $\dfrac{6-x}{x}$.
- **A power**, such as $f(x) = x^3 - 2$: $x = y^3 - 2 \Rightarrow y = \sqrt[3]{x+2}$.
- **"Find x when $f^{-1}(x) = k$"**: $x = f(k)$. Just substitute k into f.
- **$ff^{-1}(\ldots)$ or $f^{-1}f(\ldots)$**: they cancel, so $ff^{-1}(x-5) = x - 5$ and $f^{-1}f(20) = 20$.
- **"Which function is its own inverse?"** (the one with $pp(x) = x$). Try each: $p(x) = \frac{a}{x}$ and $p(x) = c - x$ are their own inverses; $2x - 1$, $3x + 2$ and x^2 are not.

2. Solving equations involving functions (23 questions)

1. **Write the equation out in x**, replacing each function by its expression. For a composite, build it first.
2. **Solve** as usual:
   - linear: rearrange;
   - quadratic: collect everything on one side, then factorise or use the formula;
   - a fraction: multiply both sides by the denominator first;
   - a square such as $(5 - 3x)^2 = 49$: square-root **both** ways, $5 - 3x = \pm 7$.
3. Give **every** solution, unless the question says "positive solution" or gives a condition such as $x > 0$.

For example, with $f(x) = 2x + 3$ and $g(x) = x^2$: $gf(x) = 25$ means $(2x+3)^2 = 25$, so $2x + 3 = 5$ or $2x + 3 = -5$, giving $x = 1$ or $x = -4$.

Variations you will meet:

- **"Find x when $f(x) = 15$"**: solve $f(x) = 15$. Do not work out $f(15)$.
- $f(x) = g(x)$, where one of them is a fraction: multiply up to get a quadratic. With $f(x) = \frac{6}{x+1}$ and $g(x) = x + 2$: $6 = (x+2)(x+1) = x^2 + 3x + 2$, so $x^2 + 3x - 4 = 0$, $(x+4)(x-1) = 0$, $x = -4$ or $x = 1$.
- **A number on the other side that needs working out first**, such as "$f(x) = g(3)$" or "$h(x) = ff(2)$". Find that number, then solve.
- **An exponential function**, such as $h(x) = 2^x$ with $h(x) = \frac{1}{8}$: write $\frac{1}{8} = 2^{-3}$, so $x = -3$.
- **An unknown constant**, such as $f(x) = kx + 5$ with $f(3) = 17$: $3k + 5 = 17$, so $k = 4$.
- **Equations with composites on both sides**, such as $fg(x+1) = gf(x)$: build each side separately (the first one needs $x + 1$ in place of x), expand, and solve.

3. Values of functions and composite functions (19 questions)

1. Substitute the number, in brackets, especially if it is negative: $f(-3)$ with $f(x) = 6 - 4x$ is $6 - 4(-3) = 18$.
2. For a composite such as $gf(-1)$, work **from the inside out**: find $f(-1)$ first, then put that answer into g.

Variations you will meet:

- $ff(2)$ **or** $hh(1)$: apply the same function twice.
- **Fractions and exponentials**: with $h(x) = \frac{1}{x+2}$, $hh(1) = h\left(\frac{1}{3}\right) = \frac{1}{7/3} = \frac{3}{7}$. Give exact fractions.
- **Very large answers**, such as $jj(2)$ with $j(x) = 4^x$: $j(2) = 16$, then $4^{16} = 4\,294\,967\,296$, which is 4.29×10^9 in standard form.

4. Composite functions in terms of x (17 questions)

1. Write the composite as a function of a function: $fg(x) = f(g(x))$.
2. **Replace every x in the outer function with the whole inner function, in brackets.**
3. Expand and simplify. "Simplest form" or "in the form $ax + b$" means collect like terms.

For example, $f(x) = 5 - 4x$ and $g(x) = x + 2$: $fg(x) = 5 - 4(x + 2) = -4x - 3$, and $gf(x) = (5 - 4x) + 2 = 7 - 4x$.

Variations you will meet:

- **A function of an expression**, such as $g(8x)$ or $f(3 - 2x)$: the same idea, with the expression as the input.
- $fg(x) = ax + b$**, find a and b**: build $fg(x)$, simplify, and read off the numbers.
- **A squared bracket**, such as $gf(x)$ with $g(x) = x^2 + 3$ and $f(x) = 4 - 5x$: $(4 - 5x)^2$ is $16 - 40x + 25x^2$, never $16 + 25x^2$. Write it as $(4 - 5x)(4 - 5x)$ and expand all four terms.
- **A fraction inside a fraction**, such as $gf(x)$ with $g(x) = \frac{6}{x-1}$ and $f(x) = 3x + 4$: $\frac{6}{3x+4-1} = \frac{6}{3x+3} = \frac{2}{x+1}$.

5. Combining functions: products, squares and single fractions (10 questions)

Here the functions are combined with ordinary arithmetic, not composed.

1. Replace each function by its expression **in brackets**.
2. **Products and squares**: $f(x)g(x)$ means multiply; $f(x)f(x) = (f(x))^2$. Expand carefully.
3. **Fractions**: find a common denominator (usually the two denominators multiplied), write one numerator over it, and simplify the numerator. You may leave the denominator factorised.

For example, $f(x) = x + 2$ and $g(x) = x - 1$:

$$\frac{3}{f(x)} - \frac{1}{g(x)} = \frac{3(x-1) - (x+2)}{(x+2)(x-1)} = \frac{2x - 5}{(x+2)(x-1)}.$$

Variations you will meet:

- $(f(x))^2 - ff(x) = ax^2 + bx + c$: the square is a product; the $ff(x)$ is a composite. Work each out separately, then subtract with brackets.
- **Mixing both**, such as $f(x)g(x) + fg(x) + 1$: two separate pieces added together.
- **A product of three**, such as $f(x)\,g(x)\,f(x)$: multiply two brackets, then multiply the result by the third.

6. Domain and range (1 question)

Substitute each value in the domain, then write the outputs as a set in curly brackets, each once. With $g(x) = 7 - 3x$ and domain $\{-1, 0, 2\}$: $g(-1) = 10$, $g(0) = 7$, $g(2) = 1$, so the range is $\{1, 7, 10\}$.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

$f(x) = 4x - 7 \qquad g(x) = x^2 + 2$

(a) Find $gf(3)$. **[2]**

(b) Find $f^{-1}(x)$. **[2]**

(c) Find $fg(x)$, giving your answer in its simplest form. **[2]**

(d) Find x when $f^{-1}(x) = 2$. **[1]**

(a) $f(3) = 12 - 7 = 5$. **M1**. Then $g(5) = 25 + 2 = 27$. **A1**

(b) $x = 4y - 7$. **M1** for swapping x and y (or any correct first step). Then $4y = x + 7$, so

$$f^{-1}(x) = \frac{x + 7}{4}.$$

A1

(c) $fg(x) = f(x^2 + 2) = 4(x^2 + 2) - 7$. **M1**. So $fg(x) = 4x^2 + 1$. **A1**

(d) $f^{-1}(x) = 2$ means $x = f(2) = 8 - 7 = 1$. **B1**

Example 2

$f(x) = x^2 - 4x \qquad g(x) = 2x + 1 \qquad h(x) = 8^x$

(a) Solve $fg(x) = 12$. **[4]**

(b) Find $hg\left(-\frac{1}{2}\right)$ and $hg(1)$. **[2]**

(c) Find x when $h(x) = 2$. **[1]**

(d) The domain of $f(x)$ is $\{0, 1, 3, 4\}$. Find the range of $f(x)$. **[2]**

(a) $fg(x) = (2x+1)^2 - 4(2x+1)$. **M1** for putting $2x+1$ into both xs of f.

$4x^2 + 4x + 1 - 8x - 4 = 12$ **B1**, so $4x^2 - 4x - 15 = 0$.

$(2x-5)(2x+3) = 0$ **M1**, so $x = \dfrac{5}{2}$ or $x = -\dfrac{3}{2}$. **A1**

(b) $g\left(-\frac{1}{2}\right) = 0$, so $hg\left(-\frac{1}{2}\right) = 8^0 = 1$. **B1**. $g(1) = 3$, so $hg(1) = 8^3 = 512$. **B1**

(c) $2 = \sqrt[3]{8} = 8^{1/3}$, so $x = \dfrac{1}{3}$. **B1**

(d) $f(0) = 0$, $f(1) = -3$, $f(3) = -3$, $f(4) = 0$. Two pairs of inputs share outputs, so the range is $\{-3, 0\}$. **B2** (**B1** for one correct value)

Example 3

$f(x) = \dfrac{12}{x+3},\ x \neq -3 \qquad g(x) = 2x - 1$

(a) Find $f^{-1}(x)$. **[3]**

(b) Solve $f(x) = g(x)$, giving your answers correct to 2 decimal places. **[5]**

(a) $x = \dfrac{12}{y+3}$ **M1**, so $x(y+3) = 12$ **M1**, so $y + 3 = \dfrac{12}{x}$ and

$$f^{-1}(x) = \frac{12}{x} - 3 \quad \left(\text{or } \frac{12-3x}{x}\right).$$

A1

(b) $\dfrac{12}{x+3} = 2x - 1$, so $12 = (2x-1)(x+3)$. **M1** for multiplying up.

$12 = 2x^2 + 6x - x - 3$ **B1**, so $2x^2 + 5x - 15 = 0$. **A1**

This does not factorise, so use the formula with $a = 2$, $b = 5$, $c = -15$: **M1**

$$x = \frac{-5 \pm \sqrt{25+120}}{4} = \frac{-5 \pm \sqrt{145}}{4}.$$

$x = 1.76$ or $x = -4.26$. **A1**

Example 4

$f(x) = x + 2 \qquad g(x) = 3x - 1$

(a) $(g(x))^2 - gg(x) = ax^2 + bx + c$. Find the values of a, b and c. **[4]**

(b) Write $\dfrac{3}{f(x)} - \dfrac{2}{g(x)}$ as a single fraction in its simplest form. **[3]**

(a) $(g(x))^2 = (3x-1)^2 = 9x^2 - 6x + 1$. **B1**

$gg(x) = 3(3x-1) - 1 = 9x - 4$. **M1**

$(9x^2 - 6x + 1) - (9x - 4) = 9x^2 - 15x + 5$. **M1** for subtracting with brackets.

$a = 9, b = -15, c = 5$. **A1**

(b) Common denominator $(x+2)(3x-1)$. **B1**

$$\frac{3(3x-1) - 2(x+2)}{(x+2)(3x-1)} = \frac{9x - 3 - 2x - 4}{(x+2)(3x-1)}$$

B1 for the numerator before simplifying.

$$= \frac{7x - 7}{(x+2)(3x-1)} = \frac{7(x-1)}{(x+2)(3x-1)}.$$

A1

Common mistakes

- **Doing a composite in the wrong order.** $fg(x)$ means g first. Say it to yourself as "f of g of x" and start with the function nearest the x.
- **Multiplying instead of composing.** $fg(x)$ is not $f(x) \times g(x)$. Examiners see this every session. Only a product written as $f(x)g(x)$, with both (x)s, means multiply.
- **Treating $f^{-1}(x)$ as $\frac{1}{f(x)}$.** The -1 means "inverse", not a power. The inverse of $3x - 2$ is $\frac{x+2}{3}$, not $\frac{1}{3x-2}$.
- **Wrong signs or half-divided terms when rearranging.** From $x = 4y + 3$, subtract 3 (don't add it), and divide the **whole** of $x - 3$ by 4. Simply "reversing the signs" of $f(x)$ is not an inverse.
- **Leaving the inverse in terms of** y, or writing it ambiguously. The final answer must use x, and $7 - x/2$ is not the same as $\frac{7-x}{2}$: draw one full fraction line.
- **Finding $f(15)$ when asked for x with $f(x) = 15$.** Read which one you are asked for.
- **Squaring a bracket wrongly.** $(x+4)^2 = x^2 + 8x + 16$, not $x^2 + 16$. Also keep the rest of the expression: $(x+4)^2 - 9$ is $x^2 + 8x + 7$, not $x^2 + 8x + 16$.
- **Dropping part of the input.** In $fg(x+1)$, the $+1$ goes into g first. Write $g(x+1)$ out before going on.
- **Not simplifying.** "Simplest form" loses the last mark if you leave $2(3x-1) + 5$ or $\frac{6}{3x+4-1}$ unsimplified.
- **Missing that ff^{-1} or $f^{-1}f$ cancel.** Long algebra for $ff^{-1}(x-4)$ usually goes wrong; the answer is simply $x - 4$.
- **Treating "$x \neq 0$" as part of the rule.** It only tells you which input is not allowed.

How to get the marks

- **Inverse functions** are usually **[2]**: one mark for a correct first step (swapping x and y, or starting to rearrange), one for the final answer in terms of x. Show that first line.
- **Composites**: write the substitution before simplifying, for example $2(5x-3)+1$. That line earns the method mark even if a later slip spoils the answer.
- **"Simplest form"**, "in the form ax^2+bx+c" and "as a single fraction" are part of the answer. Collect like terms; for fractions give one numerator over one denominator (factorised or expanded both count).
- **Single fractions** usually give marks for the common denominator and for the numerator separately, so show both.
- **"Solve"** means give every solution. "The positive solution" or "$x>0$" means leave out the other.
- **Accuracy**: give exact answers (fractions) where they are easy; otherwise 3 significant figures, or the decimal places the question asks for.
- **Follow-through**: $gf(2)$ after a wrong $f(2)$ can still earn its mark if you apply g correctly, so always show which value you put in.
- **Range** answers are sets: curly brackets, each value once.

Quick check

1. $f(x)=7-2x$. Find $f(-4)$ and $f^{-1}(x)$.
2. $f(x)=x+3$ and $g(x)=x^2-1$. Find $gf(x)$ and $fg(x)$ in their simplest forms.
3. $f(x)=3x-4$ and $h(x)=5^x$. Find x when (a) $h(x)=\frac{1}{25}$, (b) $f^{-1}(x)=6$.
4. $g(x)=10-x^2$ has domain $\{-2,0,1,3\}$. Find the range of $g(x)$.
5. $f(x)=2x+1$ and $g(x)=\dfrac{1}{x-1},\ x\neq 1$. Write $f(x)+g(x)$ as a single fraction in its simplest form.

Answers

1. $f(-4) = 7 - 2(-4) = 15$. Swap: $x = 7 - 2y$, so $2y = 7 - x$ and $f^{-1}(x) = \dfrac{7 - x}{2}$.
2. $gf(x) = (x + 3)^2 - 1 = x^2 + 6x + 8$. $fg(x) = (x^2 - 1) + 3 = x^2 + 2$.
3. (a) $\frac{1}{25} = 5^{-2}$, so $x = -2$. (b) $x = f(6) = 18 - 4 = 14$.
4. $g(-2) = 6$, $g(0) = 10$, $g(1) = 9$, $g(3) = 1$, so the range is $\{1, 6, 9, 10\}$.
5. $\dfrac{(2x + 1)(x - 1) + 1}{x - 1} = \dfrac{2x^2 - x - 1 + 1}{x - 1} = \dfrac{2x^2 - x}{x - 1} = \dfrac{x(2x - 1)}{x - 1}$.

Past questions to try

- **0580/42/F/M/24 Q11**: composite values, an inverse, and solving with a fraction and an exponential.
- **0580/23/O/N/22 Q19**: a composite with a fraction inside, the inverse of a fraction, and ff^{-1}.
- **0580/22/M/J/21 Q18**: composites on both sides of an equation.
- **0580/41/M/J/21 Q12**: values, $ff(x)$, a quadratic from functions and a cubic from combining them.