Cambridge IGCSE · Mathematics 0580 · Notes

Graphs of functions and sketching curves

Read online at **pastopic.com/cambridge/mathematics-0580/notes/graphs-of-functions**

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Read first: **Algebraic manipulation and algebraic fractions**, **Equations and changing the subject**

What you need to know

By the end of this topic you should be able to:

1. **Make a table of values and draw the graph** of a function from it, then recognise and interpret the graph. Core uses three kinds: straight lines $y = ax + b$, quadratics $y = x^2 + ax + b$ or $y = -x^2 + ax + b$, and reciprocals $y = \dfrac{a}{x}$ (with $x \neq 0$), where a and b are whole numbers.
2. **Solve equations with a graph**: read off the roots (where the graph crosses the x-axis), where the graph meets a line $y = k$, and where a line and a curve cross, and say what these values mean.
3. **Recognise, sketch and interpret** graphs of straight lines and quadratics, showing their roots and their symmetry. (Core does not need turning points.)
4. **Extended only:** draw graphs of functions made of powers of x, ax^n with $n = -2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2, 3$ (up to three such terms added together, such as $y = x^2 + \frac{2}{x}$), and exponential functions $y = a \times b^x + c$, where a and c are rational numbers and b is a positive whole number.
5. **Extended only:** solve equations by drawing a **suitable straight line** on the graph you have drawn.
6. **Extended only:** draw and interpret graphs of **exponential growth and decay**.
7. **Extended only:** recognise, sketch and interpret graphs of **linear, quadratic, cubic, reciprocal and exponential** functions: $ax + by = c$, $y = ax^2 + bx + c$, $y = ax^3 + d$, $y = ax^3 + bx^2 + cx$, $y = \dfrac{m}{x} + n$ and $y = m^x + n$. Show their **turning points, asymptotes, roots and symmetry**, and find the turning point of a quadratic by **completing the square**.

Finding turning points by differentiation belongs to the Differentiation topic, and estimating a gradient by drawing a tangent is taught in Graphs in practical situations (Extended E2.9). Tangents still turn up in questions on these graphs, so type 8 below shows how to draw one and use it.

Understand it

Tables of values

A graph question usually starts with a table: work out y for each x and fill the gaps. Use your calculator carefully:

- **Negative numbers need brackets.** For $y = x^2 - 2x - 3$ at $x = -2$, type $(-2)^2 - 2 \times (-2) - 3 = 4 + 4 - 3 = 5$. Typing -2^2 gives -4, which is wrong.
- **Watch the sign in front of x^2.** In $y = -x^2 + 4x$, the square is worked out first and then made negative: at $x = -1, y = -(1) - 4 = -5$.
- **Round as the table says**, usually to 1 or 2 decimal places.

Check the pattern: quadratic tables are symmetrical, so repeated y-values should appear either side of the middle.

Drawing the graph

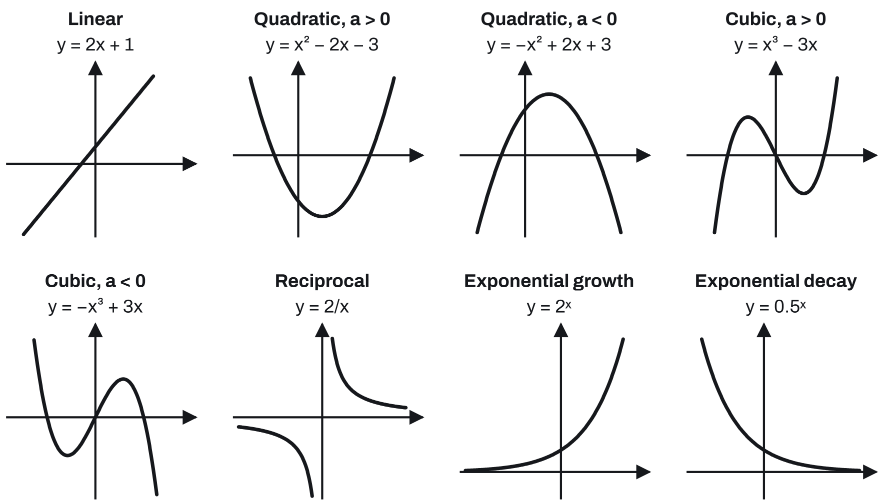
Plot each point as a small cross, accurate to within half a small square. Then:

- **straight line:** join the points with a ruler, across the whole grid you are given;
- **curve:** join the points with **one smooth, thin curve** drawn freehand, passing through every point. Never join curve points with ruled segments, and don't let a quadratic go flat along the bottom between two equal points: it keeps curving down to a turning point below them.

A reciprocal graph has a gap at $x = 0$, so it comes in **two separate pieces**. Never join them across the y-axis.

Shapes to recognise

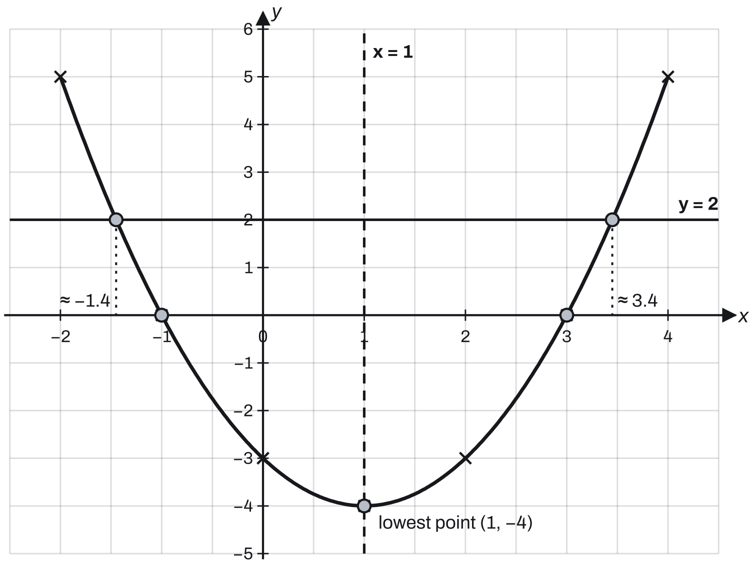
You should know each family of graph on sight, and be able to sketch it.



- **Linear** ($y = mx + c$, or $ax + by = c$): a straight line.

- **Quadratic** ($y = ax^2 + bx + c$): a symmetrical curve called a parabola, a U shape when $a > 0$ and an upside-down U when $a < 0$.
- **Extended only: cubic** ($y = ax^3 + \ldots$): an S-like shape. When $a > 0$ it goes from bottom left to top right; when $a < 0$ from top left to bottom right. It can have two turning points (a hump and a dip) or none, as in $y = x^3$.
- **Reciprocal** ($y = \frac{a}{x}$): two curved pieces that get closer and closer to both axes without touching them. When $a > 0$ they sit in the top-right and bottom-left quarters; when $a < 0$, in the top-left and bottom-right.
- **Extended only: exponential** ($y = b^x$ with $b > 1$): always above the x-axis, rising more and more steeply, and getting closer to the x-axis on the left. With $0 < b < 1$ (or $y = b^{-x}$) it is reflected: falling, and approaching the x-axis on the right.

Quadratic graphs: roots, symmetry and turning point



On the graph of $y = x^2 - 2x - 3$:

- The **roots** of $x^2 - 2x - 3 = 0$ are where the curve crosses the x-axis (where $y = 0$): $x = -1$ and $x = 3$.
- The **line of symmetry** is the vertical line halfway between the roots, or halfway between any two points with the same y-value. Here it is $x = \frac{-1+3}{2} = 1$. It is always written as an equation "$x = \ldots$", never just a number.
- The **turning point** (here the lowest point) is on the line of symmetry: at $x = 1$, $y = 1 - 2 - 3 = -4$, so it is $(1, -4)$. It is usually **not** one of the plotted points.
- The y**-intercept** is the value of y when $x = 0$, here -3.

Extended only: completing the square gives the turning point without a graph. Write $y = x^2 + bx + c$ as $y = \left(x + \frac{b}{2}\right)^2 + q$:

$$x^2 - 2x - 3 = (x-1)^2 - 1 - 3 = (x-1)^2 - 4.$$

A square is never negative, so y is smallest when $(x-1)^2=0$, that is at $x=1$, where $y=-4$. In general $y=(x+p)^2+q$ has its turning point at $(-p,\ q)$ and line of symmetry $x=-p$. With a minus sign in front, $y=q-(x+p)^2$, the turning point $(-p,q)$ is a **maximum**.

Solving equations with a graph

Every point on a graph makes its equation true, so a graph shows the solutions of many equations at once.

- $f(x)=0$**:** read the x-values where the curve crosses the x-axis.
- $f(x)=k$**:** draw the horizontal line $y=k$ and read the x-values where it crosses the curve. On the diagram above, $x^2-2x-3=2$ gives $x\approx-1.4$ and $x\approx3.4$.
- $f(x)=g(x)$ for a line $y=g(x)$: draw the line and read the x-values where it crosses the curve.

Graph readings are estimates, so answers are accepted within a small range (about half a small square either way).

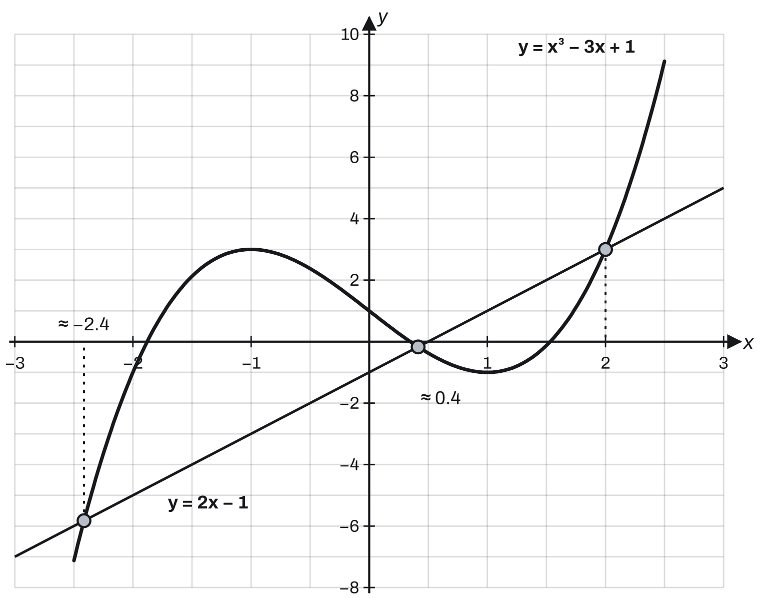
How many solutions? The number of times the line $y=k$ crosses the curve is the number of solutions of $f(x)=k$. For $y=x^2-2x-3$: two solutions when $k>-4$, one when $k=-4$ (the line just touches the lowest point), and none when $k<-4$.

Extended only: solving by drawing a suitable straight line

Suppose you have drawn $y=x^3-3x+1$ and must solve $x^3-5x+2=0$. The curve itself doesn't show this equation, but you can make it into "curve = line":

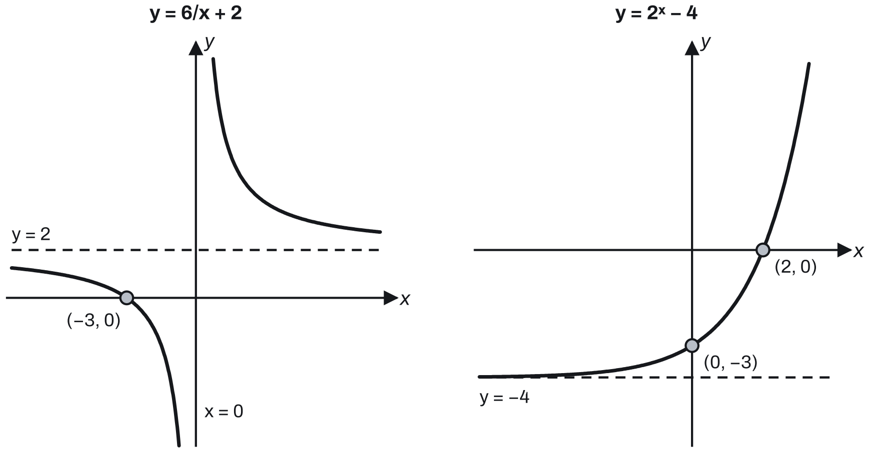
$$x^3-3x+1=(x^3-3x+1)-(x^3-5x+2)=2x-1.$$

So the solutions are where the curve meets the line $y=2x-1$. The quick rule: **if the equation is $E=0$, the line is $y=(\text{graph})-E$.** First multiply or divide the equation so its curved terms match the graph's (sign and size); then the curved terms cancel, leaving a straight line.



Extended only: asymptotes

An **asymptote** is a line that a graph gets closer and closer to but never reaches.



- $y = \dfrac{m}{x} + n$ has a **vertical asymptote** $x = 0$ (you can't divide by 0) and a **horizontal asymptote** $y = n$ (as x gets large, $\frac{m}{x}$ shrinks to nearly 0). It has no y-intercept, and crosses the x-axis where $\frac{m}{x} = -n$.
- $y = m^x + n$ (and $y = a \times b^x + c$) has a **horizontal asymptote** $y = n$ (or $y = c$), because m^x is always positive but shrinks towards 0 on one side. It crosses the y-axis at $m^0 + n = 1 + n$ (or $a + c$).

So $y = \frac{6}{x} + 2$ has asymptotes $x = 0$ and $y = 2$, and crosses the x-axis where $\frac{6}{x} = -2$, at $x = -3$. And $y = 2^x - 4$ has asymptote $y = -4$, crosses the y-axis at $1 - 4 = -3$, and the x-axis where $2^x = 4$, at $x = 2$.

Extended only: exponential growth and decay

A quantity that is multiplied by the same number each time step grows or decays exponentially: $P = 200 \times 1.5^t$ starts at 200 (when $t = 0$) and grows by 50% each step, while $V = 800 \times 0.9^t$ loses 10% each step. Their graphs have the exponential shapes above. Read them like any graph: the value at a given time, or the time when a value is reached (draw the horizontal line and read across).

Extended only: sketching a cubic

A sketch is freehand, not to scale, but it must show the right shape in the right place and have the key values marked on the axes. For a cubic in factorised form, such as $y = (x + 2)(x - 1)(x - 3)$:

1. **Roots:** each bracket gives one: $x = -2$, 1 and 3. A squared bracket, as in $y = x(x - 2)^2$, gives a root where the curve only **touches** the axis and turns back ($x = 2$ here).
2. **y-intercept:** put $x = 0$: $(2)(-1)(-3) = 6$.
3. **Shape:** multiplying out gives $x^3 + \ldots$, so a positive cubic: from bottom left to top right, through each root in turn.

For a form like $y = x^3 - 4x$, factorise first: $x(x^2 - 4) = x(x - 2)(x + 2)$, roots -2, 0 and 2.

Key facts and formulas

"Given" means it is on the List of formulas printed in the exam paper. The quadratic formula is on the Extended list only; everything else must be learnt.

Formula	Status
Straight line: $y = mx + c$, gradient m, y-intercept c	Learn it
y-intercept: put $x = 0$; roots (x-intercepts): put $y = 0$	Learn it
Line of symmetry of a quadratic: $x = \dfrac{x_1 + x_2}{2}$, halfway between two points with equal y (such as the roots)	Learn it
Extended only: $y = (x+p)^2 + q$ has turning point $(-p,\ q)$ and line of symmetry $x = -p$	Learn it
Extended only: $x^2 + bx = \left(x + \dfrac{b}{2}\right)^2 - \left(\dfrac{b}{2}\right)^2$ (completing the square)	Learn it
Extended only: $ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Given
Extended only: $y = \dfrac{m}{x} + n$: asymptotes $x = 0$ and $y = n$	Learn it
Extended only: $y = m^x + n$: asymptote $y = n$, y-intercept $1 + n$; $y = a \times b^x + c$: asymptote $y = c$, y-intercept $a + c$	Learn it
$f(x) = k$: solutions are where the line $y = k$ crosses $y = f(x)$	Learn it
Extended only: to solve $E = 0$ with the graph of $y = f(x)$, first scale E so its curved terms match $f(x)$'s, then draw the line $y = f(x) - E$	Learn it

Symmetry of the reciprocal graph $y = \frac{a}{x}$: rotational symmetry of order 2 about the origin, and two lines of symmetry, $y = x$ and $y = -x$.

How to do it

In the questions we have tagged for this topic (67 questions, 2021–2025), the types came up this often. 46 of the 67 questions have parts of more than one type, so the counts add up to more than 67.

Question type	Questions
Complete a table of values and draw the graph	42
Solve an equation or read values from the graph	30
Line of symmetry and turning point	18

Question type	Questions
Sketch or recognise a graph	17
Solve by drawing a suitable straight line	13
Find constants or equations from a graph's features	6
How many solutions $f(x) = k$ has	6
Draw a tangent to estimate a gradient	3

1. Complete a table of values and draw the graph (42 questions)

1. **Fill each gap** by substituting x into the equation, with brackets round negatives. Round as the table says.
2. **Check with the pattern**: symmetry in a quadratic, y and $-y$ pairs in $y = \frac{a}{x}$ (for example $\frac{12}{-4} = -3$ and $\frac{12}{4} = 3$).
3. **Plot every point** from the table, including the given ones, as small crosses. Read each axis scale first: one small square might be 0.2 or 0.5.
4. **Join with a smooth curve** through every point (or a ruled line for a linear graph), over the whole range of x asked for.

Variations you will meet:

- **Reciprocal graphs** over two ranges, such as $-4 \leqslant x \leqslant -1$ and $1 \leqslant x \leqslant 4$: draw two separate curves and leave the gap at $x = 0$ empty.
- **Extended only: graphs with mixed powers**, such as $y = x^2 - \frac{1}{x}$ or $y = 2^x - 3$: same method; a table often gives decimals to 2 d.p.
- **A graph in context**, such as a volume $V = x^2(9 - x)$: same method, with the variable names from the question.
- **Straight lines**: three points are enough (one is a check); rule the line.

2. Solve an equation or read values from the graph (30 questions)

1. **Match the equation to a line.** $f(x) = 0$: the x-axis. $f(x) = k$: draw the ruled line $y = k$. $f(x) =$ a given straight line: draw that line (plot two or three of its points first).
2. **Read the x-values** of every crossing point, to half a small square. Give as many answers as there are crossings; the answer spaces usually tell you how many.
3. Write the answers as "$x = \ldots$".

Variations you will meet:

- **Where two graphs meet**, such as "write down the coordinates where the line and the curve intersect": give both coordinates of each crossing point.
- **Read y from x** (such as $f(2)$), or x from y: draw across from one axis to the curve and down (or across) to the other.

- **Only part of the range**, such as "for $x > 0$": give only the crossings in that part.

3. Line of symmetry and turning point (18 questions)

1. **Line of symmetry** of a quadratic: find two points with the same y-value (the roots, or equal entries in the table) and take the x halfway between them. Write it as $x = \ldots$.
2. **Lowest or highest point:** it lies **on** the line of symmetry. Its x-coordinate is the line of symmetry's; read y from your curve there, or work it out from the equation.
3. **Extended only:** from the equation, complete the square: $y = (x+p)^2 + q$ gives the turning point $(-p, q)$.

For example, $y = x^2 + 6x - 2 = (x+3)^2 - 9 - 2 = (x+3)^2 - 11$: line of symmetry $x = -3$, lowest point $(-3, -11)$.

Variations you will meet:

- **Use symmetry to find a point.** If $(-5, 7)$ is on a curve with line of symmetry $x = 1$, then so is the point the same distance on the other side: $1 + (1 - (-5)) = 7$, so $(7, 7)$.
- **Reciprocal graph symmetry:** order of rotational symmetry 2; lines of symmetry $y = x$ and $y = -x$ (note that $x = y$ is the same line as $y = x$). One of them misses the graph: for $y = \frac{a}{x}$ with $a > 0$, $y = -x$ doesn't meet it; with $a < 0$, $y = x$ doesn't.

4. Sketch or recognise a graph (17 questions)

1. **Decide the shape** from the equation: linear, quadratic (U or upside-down U), cubic (positive or negative), reciprocal or exponential.
2. **Find the key values:** the y-intercept ($x = 0$), the roots ($y = 0$, factorise if you can), the turning point of a quadratic, and any asymptotes.
3. **Draw freehand** on labelled axes: the right shape in the right quarters, passing through (or touching) the marked values, and bending towards any asymptotes without crossing or curving away from them.
4. **Mark the values** on the axes (-4, 4, -16 and so on) or label points with coordinates.

Variations you will meet:

- **A straight line such as** $2x + 5y = 10$: put $x = 0$ ($y = 2$) and $y = 0$ ($x = 5$), and join these intercepts.
- **Recognise a graph**: name the type (linear, quadratic, cubic, reciprocal, exponential) from its shape.
- **Extended only: a cubic from facts about its gradient**, such as "the gradient is negative for $x < -2$ and $x > 1$, positive between": it falls, turns up at $x = -2$, turns down at $x = 1$. A negative cubic.
- **Extended only: first show the expanded form**, such as $(x-3)(x+2)(x-1) = x^3 - 2x^2 - 5x + 6$, then sketch using the brackets for the roots and the constant for the y-intercept.
- **Extended only: solve $f(x) = g(x)$ from sketches**, such as $\frac{8}{x} = 2x$: sketch both and count or find the crossings ($x^2 = 4$, so $x = \pm 2$).

5. Extended only: solve by drawing a suitable straight line (13 questions)

1. **Write the equation to solve** as $E = 0$.
2. **Find the line**: first make the curved terms of E match the graph's (sign and size), then $y = (\text{the graph's equation}) - E$. Simplify: all the curved terms must cancel. For example, with $y = x^3 - 4x + 2$ and $x^3 - 5x + 1 = 0$, the line is $y = (x^3 - 4x + 2) - (x^3 - 5x + 1) = x + 1$. Decimals are fine: with $y = x^3 - x + 4$ and $x^3 - 1.5x + 3 = 0$, the line is $y = 0.5x + 1$.
3. **Draw it ruled** across the grid (plot two or three points first).
4. **Read the x-values** where it crosses your curve.

Variations you will meet:

- **The equation isn't in the form $E = 0$**, such as $2x^3 + 4x = 3$: move everything to one side first.
- **The leading number is different**, such as the graph of $y = x^3 - 2x + 1$ and the equation $2x^3 - 6x + 1 = 0$: divide the equation by 2 first ($x^3 - 3x + 0.5 = 0$), then use the rule: $y = (x^3 - 2x + 1) - (x^3 - 3x + 0.5) = x + 0.5$.
- **The equation's leading term has the opposite sign**, such as the graph of $y = 2 + 3x - x^3$ and the equation $x^3 - 5x + 1 = 0$: multiply the equation by -1 first ($-x^3 + 5x - 1 = 0$), then use the rule: $y = (2 + 3x - x^3) - (-x^3 + 5x - 1) = 3 - 2x$.
- **Fractions and powers**, such as $y = x^2 - \frac{1}{x}$ and $x^2 - \frac{1}{x} + x + 1 = 0$: same rule: $y = -x - 1$.
- **Given the line, find the equation**: "the line $y = 2x - 1$ was drawn to solve an equation; find it": set the graph's expression equal to the line and rearrange to $\ldots = 0$.

6. Find constants or equations from a graph's features (6 questions)

1. **Write down what each fact gives you**: a point on the curve means its coordinates satisfy the equation; a y-intercept means the value at $x = 0$; a turning point (h, k) means $y = (x - h)^2 + k$ for $y = x^2 + \ldots$.
2. **Substitute and solve** for the unknowns.

Variations you will meet:

- **Extended only: minimum point given, find b and c** in $y = x^2 + bx + c$: minimum at $(2, -7)$ gives $y = (x - 2)^2 - 7 = x^2 - 4x - 3$, so $b = -4$, $c = -3$.
- **Extended only: an exponential through given points**: $y = a \times b^x$ through $(0, 3)$ and $(2, 12)$ gives $a = 3$ (as $b^0 = 1$), then $3b^2 = 12$, so $b = 2$.
- **Extended only: a cubic's y-intercept**: $y = (x - 2)(x + b)(x + 3)$ meets the y-axis at -12: at $x = 0$, $(-2)(b)(3) = -6b = -12$, so $b = 2$.
- **Asymptotes and crossings of a drawn graph**, such as $y = \frac{3}{x} + 1$: asymptotes $x = 0$ and $y = 1$; it crosses the x-axis where $\frac{3}{x} = -1$, at $(-3, 0)$.
- **A reciprocal $y = \frac{k}{x}$ through a point**: $k = xy$ for any point on it.

7. Extended only: how many solutions $f(x) = k$ has (6 questions)

1. **Imagine the horizontal line** $y = k$ sliding up and down the graph.
2. **Count the crossings** for each height. The number changes at the turning points and at the ends of the range drawn.
3. **Give the answer** the question wants: a value of k (often a whole number), or the largest or smallest k in a range.

For $y = x^3 - 3x + 1$ (hump at $(-1, 3)$, dip at $(1, -1)$): three solutions for $-1 < k < 3$; two when $k = 3$ or $k = -1$ (the line touches a turning point); one otherwise. **No solutions** means the line misses the graph completely: below a U-shaped quadratic's lowest point, above an upside-down one's highest point, or beyond an asymptote. For example, $y = 2^x - 4$ never reaches -4 or below, so $2^x - 4 = k$ has no solution for $k \leqslant -4$.

A line $y = mx$ with a set number of solutions. Every line $y = mx$ passes through the origin, so turn your ruler about $(0, 0)$ and count where it crosses the curve within the range drawn. Rearranging checks your choice: on $y = x^3 - 3x + 1$, the line $y = -3x$ gives $x^3 - 3x + 1 = -3x$, so $x^3 = -1$: exactly one solution, $x = -1$ (the point $(-1, 3)$).

8. Extended only: draw a tangent to estimate a gradient (3 questions)

1. **Rule a line that just touches the curve** at the point, lying along its direction there; it must not cut across the curve at that point.
2. **Pick two points far apart on your tangent** and find gradient $= \dfrac{\text{rise}}{\text{run}}$; a downward slope gives a negative gradient.

For example, on $y = x^2 - 2x - 3$ the tangent at $(3, 0)$ passes through $(2, -4)$ and $(4, 4)$: gradient $= \frac{8}{2} = 4$. (The Differentiation topic finds this exactly.)

Variations you will meet:

- **The tangent's equation:** read its gradient m as above and its y-intercept c where it crosses the y-axis, and write $y = mx + c$. The tangent above crosses the y-axis at -12, so it is $y = 4x - 12$.
- **A tangent from a point off the curve:** put the ruler on the point, turn it until it just touches the curve, and read the touching point. From $P(0, -7)$, the tangent to $y = x^2 - 2x - 3$ on the right touches at $(2, -3)$. Its gradient is $\frac{-3-(-7)}{2-0} = 2$ and it passes through P, so it is $y = 2x - 7$.
- **Using symmetry:** if a curve is symmetrical about the y-axis, the tangent at $x = -a$ is the mirror image of the tangent at $x = a$: change the sign of the gradient and keep the y-intercept. On $y = 5 - x^2$ the tangent at $x = 2$ is $y = -4x + 9$, so the tangent at $x = -2$ is $y = 4x + 9$.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

(a) Complete the table of values for $y = x^2 - 2x - 3$. **[3]**

x	-2	-1	0	1	2	3	4
y	5		-3		-3	0	

(b) Draw the graph of $y = x^2 - 2x - 3$ for $-2 \leqslant x \leqslant 4$. **[4]**

(c) Write down the equation of the line of symmetry of the graph. **[1]**

(d) Write down the coordinates of the lowest point of the graph. **[1]**

(e) Use your graph to solve $x^2 - 2x - 3 = 2$. **[2]**

(a) $x = -1$: $(-1)^2 - 2(-1) - 3 = 1 + 2 - 3 = 0$. $x = 1$: $1 - 2 - 3 = -4$. $x = 4$: $16 - 8 - 3 = 5$. **B1 B1 B1**

(b) Plot all seven points and join them with a smooth U-shaped curve, going down to a rounded bottom below $(0, -3)$ and $(2, -3)$. **B4** (**B3** for 7 points plotted correctly with no curve, or fewer marks for fewer points.) The second diagram in "Understand it" shows the finished graph.

(c) $y = -3$ at $x = 0$ and $x = 2$, halfway is $x = 1$. The line of symmetry is $x = 1$. **B1**

(d) On $x = 1$, $y = -4$: the lowest point is $(1, -4)$. **B1**

(e) Draw the ruled line $y = 2$. It crosses the curve at $x = -1.4$ and $x = 3.4$. **B1 B1** (accepted within a small range either side, about -1.5 to -1.4 and 3.4 to 3.5, as the exact answers are $1 \pm \sqrt{6} = -1.45$ and 3.45 to 2 d.p.)

Example 2

(a) Complete the table of values for $y = \dfrac{8}{x}$. **[2]**

x	-8	-4	-2	-1	1	2	4	8
y	-1		-4	-8	8		2	1

(b) Draw the graph of $y = \dfrac{8}{x}$ for $-8 \leqslant x \leqslant -1$ and $1 \leqslant x \leqslant 8$. **[4]**

(c) On the grid, draw the line $y = x + 2$, and use it to solve $\dfrac{8}{x} = x + 2$. **[3]**

(d) Write down the equations of the two lines of symmetry of the graph. **[2]**

(a) $\frac{8}{-4} = -2$ and $\frac{8}{2} = 4$. **B1 B1**

(b) Plot the eight points. Draw one smooth curve through the four points with $x < 0$ and a separate one through the four with $x > 0$; don't join them across $x = 0$. **B4**

(c) The line passes through $(0, 2)$, $(2, 4)$ and $(-4, -2)$: rule it across the grid. **B1**

It crosses the curve at $(2, 4)$ and $(-4, -2)$, so $x = 2$ or $x = -4$. **B1 B1**

(Check: $\frac{8}{2} = 4 = 2 + 2$ and $\frac{8}{-4} = -2 = -4 + 2$.)

(d) $y = x$ and $y = -x$. **B1 B1**

Example 3 (Extended)

The table shows some values of $y = x^3 - 3x + 1$, given correct to 2 decimal places.

x	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
y	-7.13	-1		3	2.38	1		-1	-0.13	3	

(a) Complete the table. **[3]**

(b) Draw the graph of $y = x^3 - 3x + 1$ for $-2.5 \leqslant x \leqslant 2.5$. **[4]**

(c) By drawing a suitable straight line, solve the equation $x^3 - 5x + 2 = 0$. **[4]**

(a) $(-1.5)^3 - 3(-1.5) + 1 = -3.375 + 4.5 + 1 = 2.125 \to 2.13$. $(0.5)^3 - 1.5 + 1 = -0.375 \to -0.38$. $(2.5)^3 - 7.5 + 1 = 9.125 \to 9.13$. **B1 B1 B1**

(b) Plot the 11 points and join them with a smooth curve: up to a hump near $(-1, 3)$, down to a dip near $(1, -1)$, then up again. **B4**

(c) Make the curve's equation appear on one side:

$$y = (x^3 - 3x + 1) - (x^3 - 5x + 2) = 2x - 1.$$

M2 for the line $y = 2x - 1$ ruled on the grid, through $(0, -1)$ and $(2, 3)$ (**M1** if the equation is only written down).

It crosses the curve three times: $x = -2.4$, $x = 0.4$ and $x = 2$. **A2** (**A1** for two of them.) The third diagram in "Understand it" shows this.

(Check: $2^3 - 5(2) + 2 = 0$, so $x = 2$ is exact; the others are $-1 \pm \sqrt{2}$, which are -2.41 and 0.41.)

Example 4 (Extended)

(a) Write $x^2 - 6x + 5$ in the form $(x - p)^2 - q$. **[2]**

(b) Write down the coordinates of the turning point of $y = x^2 - 6x + 5$. **[1]**

(c) Sketch the graph of $y = x^2 - 6x + 5$, showing where it crosses the axes. **[3]**

(d) The equation $x^2 - 6x + 5 = k$ has exactly one solution. Write down the value of k. **[1]**

(a) Half of -6 is -3: $(x - 3)^2 = x^2 - 6x + 9$, which is 4 too many. So $x^2 - 6x + 5 = (x - 3)^2 - 4$. **B1** for $(x - 3)^2$, **B1** for -4.

(b) $(x - 3)^2 \geqslant 0$, so the lowest value -4 is at $x = 3$: the turning point is $(3, -4)$, a minimum. **B1**

(c) Roots: $x^2 - 6x + 5 = (x - 1)(x - 5) = 0$, so $x = 1$ and $x = 5$. y-intercept: 5. Sketch a U shape crossing the x-axis at 1 and 5, the y-axis at 5, with its lowest point at $(3, -4)$. **B1** for the U shape, **B1** for both roots marked, **B1** for the y-intercept and the turning point.

(d) The line $y = k$ meets the curve once only where it just touches the lowest point: $k = -4$. **B1**

Common mistakes

- **Losing negative signs in the table.** Examiners often see $(-3)^2$ worked out as -9 or a minus sign dropped. Use brackets on the calculator, and check the table's pattern.
- **Poor curves.** Joining points with ruled segments, drawing thick, doubled or feathery lines, or missing some of the points all cost the curve mark. Turn the page so your hand is inside the curve, and draw one smooth, thin line through every point.
- **A flat or pointed bottom.** Between two equal lowest points (such as $(0, -3)$ and $(2, -3)$) the curve must dip below them to a rounded turning point.
- **Joining the two pieces of a reciprocal graph.** There is no point at $x = 0$; draw two separate curves, each within its own range.
- **Writing the line of symmetry wrongly.** Reports list answers such as "0.5", "$y = -0.5$" and "$y = mx + c$". It is a vertical line, so it is $x = \ldots$. Drawing it on the graph first helps.
- **Taking a plotted point as the lowest point.** The turning point is usually between plotted points: its x is on the line of symmetry, and its y is a little below the lowest plotted y.
- **Solving by algebra when the question says "use your graph"** and then making slips, or giving answers that don't match your own graph. Read from the graph you drew.

- **Wrong or missing suitable line.** Rearranging carelessly (a sign slip) gives the wrong line; drawing no line loses the method marks even if the answers are right. Subtract the whole equation from the whole graph expression, bracketed.
- **Not knowing asymptotes.** Answers such as the top value on the axis, "∞", or a word like "curved" were common. An asymptote is a line, so give its equation, such as $y = 1$ or $x = 0$.
- **Sketches with the wrong shape or no values.** Drawing a positive cubic for a negative one, putting reciprocal branches in the wrong quarters, curving away from the axes, or leaving off the intercepts all lose marks. Work out the key values first, then sketch.
- **Missing solutions.** If there are three answer spaces, look for three crossings; if you find a different number, check your curve and line.

How to get the marks

- **Tables:** one mark per correct value (or per pair). Keep the accuracy the table uses.
- **Plotting:** each point must be within half a small square. The curve marks are usually **B4** for a correct curve, and partial marks (**B3**, **B2**, **B1**) for the number of points plotted correctly, so plot every point clearly even if you can't draw the curve well. Plotting from your own table earns follow-through (**FT**) marks.
- **The curve:** smooth, a single thin line, through every point, over the whole range asked for; ruled for straight lines.
- **Reading a graph:** accurate to half a small square. Answers from **your** curve earn FT marks, so read carefully from what you drew.
- **"By drawing a suitable straight line"**: the method marks are for the correct line, written as an equation and **ruled** on the grid. Show the rearranging.
- **"Sketch"**: freehand, not to scale, but with the right shape, in the right quarters, with intercepts, turning points and asymptotes marked as values or coordinates, and labelled axes.
- **"Write down"** means no working is needed: read or state it. **"Show that"** means every step must be visible, ending with the expression printed in the question.
- **"Use your graph"** means read the answer from the graph; answers are accepted within a range.
- **Equations of lines**: always a full equation ($x = 1$, $y = 1$), never just a number.

Quick check

1. A quadratic curve crosses the x-axis at $x = -5$ and $x = 1$. Write down the equation of its line of symmetry.
2. **Extended:** you have drawn the graph of $y = x^3 - 2x + 3$. What straight line should you draw to solve $x^3 - 4x - 1 = 0$?
3. **Extended:** the point $(3, 54)$ lies on the curve $y = 2 \times c^x$, where c is a whole number. Find c, and the value of y when $x = -1$.
4. **Extended:** for $y = \dfrac{4}{x} - 2$, write down the equations of the asymptotes and the coordinates of the point where the graph crosses the x-axis.

5. **Extended:** $y = x(x-3)(x+2)$. Write down where the graph crosses the axes, and describe the shape of its sketch.

Answers

1. Halfway between the roots: $x = \frac{-5+1}{2} = -2$, so the line of symmetry is $x = -2$.
2. $y = (x^3 - 2x + 3) - (x^3 - 4x - 1) = 2x + 4$. Draw $y = 2x + 4$ and read where it crosses the curve.
3. $2 \times c^3 = 54$, so $c^3 = 27$ and $c = 3$. At $x = -1$: $y = 2 \times 3^{-1} = \frac{2}{3}$ (or 0.667).
4. Asymptotes $x = 0$ and $y = -2$. Crossing the x-axis: $\frac{4}{x} = 2$, so $x = 2$: the point $(2, 0)$.
5. Roots $x = -2, 0$ and 3, so it crosses the x-axis at $(-2, 0)$, $(0, 0)$ and $(3, 0)$; the y-intercept is 0 (the origin). It multiplies out to $x^3 - x^2 - 6x$, a positive cubic: rising from bottom left through $(-2, 0)$ to a hump, down through the origin to a dip, then up through $(3, 0)$ to the top right.

Past questions to try

- **0580/41/M/J/25 Q14**: table, cubic graph and a suitable straight line.
- **0580/31/M/J/22 Q9**: a reciprocal graph and the line $y = k$.
- **0580/42/F/M/24 Q10**: drawing a cubic, a suitable line and how many solutions $f(x) = k$ has.
- **0580/43/O/N/22 Q9**: sketching a line, a quadratic and an exponential with their intercepts.