

Inequalities

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Read first: [Equations and changing the subject](#)

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What you need to know

Core and Extended share the first point. The parts marked **Extended only** are for Papers 2 and 4. By the end of this topic you should be able to:

1. **Represent and interpret inequalities, including on a number line.** Know the four symbols $<$, $>$, $\leq$ and $\geq$. Draw an inequality such as $-1 < x \leq 3$ on a number line, with an **open circle** for $<$ or $>$ and a **closed (filled) circle** for $\leq$ or $\geq$, and write down the inequality that a number line shows. List the integers (or the odd numbers, primes and so on) that satisfy an inequality.
2. **Extended only: construct, solve and interpret linear inequalities**, such as $4x < x + 9$ or $-5 \leq 2x - 1 < 7$, and turn facts given in words into inequalities.
3. **Extended only: show linear inequalities in two variables on a graph:** a **broken** line for $<$ or $>$, a **solid** line for $\leq$ or $\geq$, and **shading of the unwanted region** (unless the question says otherwise).
4. **Extended only: write down the inequalities that define a given region** on a graph.

Linear programming (finding the largest or smallest value of something such as a profit over a region) is not in the 2025–2027 syllabus. Inequalities for upper and lower bounds, such as $6.5 \leq l < 7.5$, are in the topic Estimation and limits of accuracy.

Understand it

The four symbols

An **inequality** compares two things that need not be equal.

Symbol	Meaning	Example	Values that fit
$<$	is less than	$x < 6$	5.9, 0, -20 , but not 6
$>$	is greater than	$x > -1$	0, 2.5, 100, but not -1
$\leq$	is less than or equal to	$x \leq 4$	4, 3.99, -7
$\geq$	is greater than or equal to	$x \geq 0$	0, 1, 8.2

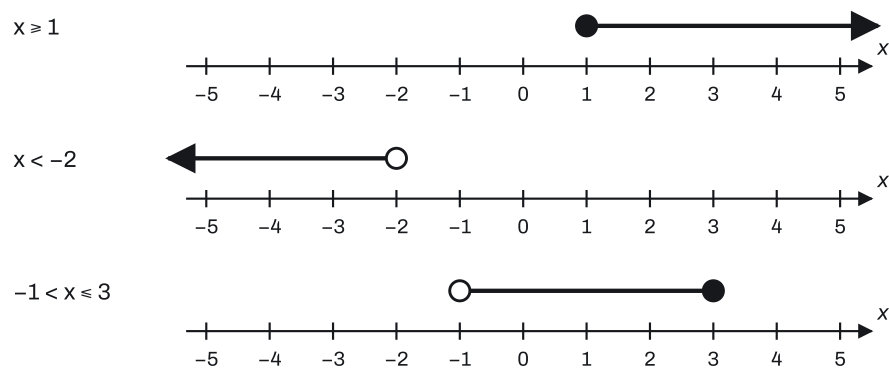
The narrow end of $<$ points at the smaller side. $x < 6$ and $6 > x$ say the same thing, so you can read an inequality from either end. The line under $\leq$ and $\geq$ is half of an equals sign: the end value itself is allowed.

An inequality such as $x < 6$ has infinitely many solutions, not just whole numbers. That is why its answer is itself an inequality, not a single value.

Number lines

A number line shows every value that fits:

- a **closed circle** (filled in) at a number means that number **is** included ($\leq$ or $\geq$);
- an **open circle** (empty) means it is **not** included ($<$ or $>$);
- a line with an **arrow** means "and everything beyond, forever"; a line **between two circles** means every value in between.



The bottom line shows **two** facts at once: $x > -1$ and $x \leq 3$. They are written together, smallest number on the left, as $-1 < x \leq 3$, read as " x is more than -1 and at most 3 ". Both symbols in a double inequality point the same way.

To **read** a number line, look at each end in turn. Open circle at the left end: $-1 < x$. Closed circle at the right end: $x \leq 3$. Then join them: $-1 < x \leq 3$. Always include the letter.

Integers that fit

Many questions ask for the **integers** (whole numbers, which can be negative or zero) that satisfy an inequality. The integers with $-1 < x \leq 3$ are 0, 1, 2, 3: not -1 (open circle), but 3 (closed circle). Check both ends carefully, and don't forget 0.

Sometimes there is an extra condition, such as " n is a multiple of 3 and $3 < n \leq 12$ ", which gives 6, 9, 12, or " p is prime and $30 < p < 40$ ", which gives 31 and 37.

Extended only: solving an inequality

Extended only: you solve a linear inequality almost exactly like an equation: add or subtract the same thing on both sides, and multiply or divide both sides by the same **positive** number. The inequality stays true.

Extended only: there is one extra rule. **Multiplying or dividing by a negative number reverses the inequality sign.** For example, $2 < 5$, but multiplying both sides by -1 gives -2 and -5 , and $-2 > -5$. So

$$4 - 3x < 10 \Rightarrow -3x < 6 \Rightarrow x > -2 \quad (\text{dividing by } -3 \text{ reversed } < \text{ to } >).$$

Extended only: you can avoid dividing by a negative altogether by moving the x terms to the side where they stay positive: $4 - 3x < 10$ becomes $4 < 10 + 3x$, so $-6 < 3x$ and $-2 < x$, which is the same answer, $x > -2$. With x on both sides, as in $3x + 2 > 5x - 6$, collect on the side with more x : $8 > 2x$, so $4 > x$, that is $x < 4$.

Extended only: double inequalities. $1 \leq 2x + 3 < 11$ has three parts. Do the **same thing to all three parts** at once:

$$1 \leq 2x + 3 < 11 \Rightarrow -2 \leq 2x < 8 \Rightarrow -1 \leq x < 4.$$

Extended only: you can also split it into two inequalities, $1 \leq 2x + 3$ and $2x + 3 < 11$, solve each, and then put the answers back together.

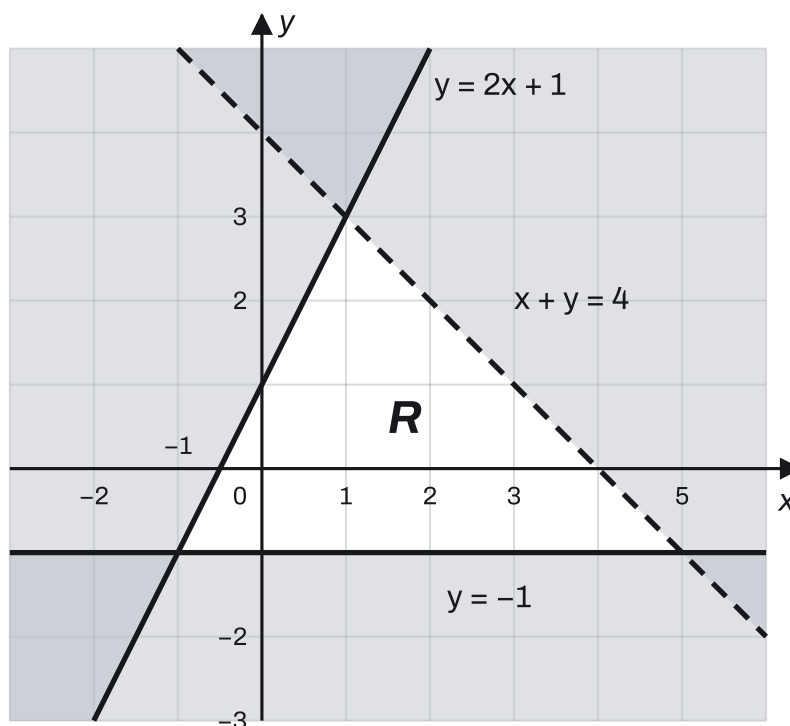
Extended only: regions on a graph

Extended only: an inequality in x and y , such as $x + y < 4$, is true for a whole **region** of the graph. The line $x + y = 4$ is its **boundary**: on one side $x + y < 4$ and on the other $x + y > 4$.

Extended only: to show a region:

1. **Draw the boundary line** as if the sign were $=$. Use a **broken (dashed) line** for $<$ or $>$ (points on the line are not included) and a **solid line** for $\leq$ or $\geq$ (they are).
2. **Choose a test point** that is not on the line; $(0, 0)$ is easiest. Substitute it. If the inequality is true there, that side is wanted; if false, the other side is.
3. **Shade the unwanted side**, so that the region left clear satisfies every inequality. Label it R .

Extended only: the diagram shows $y \geq -1$, $x + y < 4$ and $y \leq 2x + 1$. The origin gives $0 + 0 = 0 < 4$ (true) and $0 \leq 2(0) + 1$ (true), so for both lines the side containing the origin is wanted. For $y \geq -1$, the wanted side is above the line.

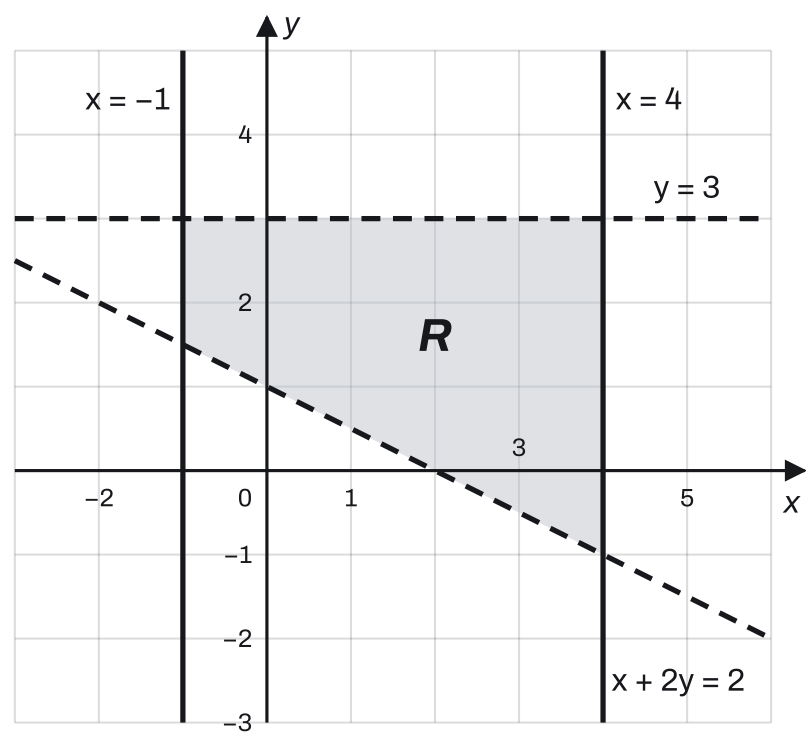


Extended only: lines you will need.

- $x = a$ is **vertical** and $y = b$ is **horizontal**. $x < a$ is left of $x = a$; $y > b$ is above $y = b$.
- $y = mx + c$ crosses the y -axis at c and has gradient m . $y < mx + c$ is **below** the line and $y > mx + c$ is **above** it.
- For a line such as $2x + 3y = 12$, find where it crosses the axes: $x = 0$ gives $y = 4$, and $y = 0$ gives $x = 6$. Join $(0, 4)$ and $(6, 0)$ with a ruler.

Extended only: describing a region

Extended only: this works backwards. For each boundary line, write its equation, decide the sign from the line style (broken: $<$ or $>$; solid: $\leq$ or $\geq$), and pick the direction with a point **inside** the region.



Extended only: R is right of the solid line $x = -1$, left of the solid line $x = 4$ and below the broken line $y = 3$: $x \geq -1$, $x \leq 4$ and $y < 3$. For the broken line $x + 2y = 2$, the point $(0, 2)$ is inside R and gives $0 + 2(2) = 4$, which is more than 2, so $x + 2y > 2$. (The origin gives 0, which is less than 2: the origin is outside R .)

Key facts and formulas

Nothing in this topic is on the List of formulas: learn it all.

Formula	Status
$<$ less than, $>$ greater than, $\leq$ less than or equal to, $\geq$ greater than or equal to	Learn it
Number line: open circle for $<$ and $>$; closed circle for $\leq$ and $\geq$	Learn it
$a < x \leq b$ means $x > a$ and $x \leq b$ (smaller number on the left)	Learn it
Extended only: adding, subtracting, or multiplying or dividing by a positive number keeps the sign	Learn it
Extended only: multiplying or dividing by a negative number reverses the sign ($-3x < 6 \Rightarrow x > -2$)	Learn it
Extended only: graph: broken line for $<$ and $>$; solid line for $\leq$ and $\geq$; shade the unwanted region	Learn it

Formula	Status
Extended only: $x = a$ is vertical, $y = b$ is horizontal; $y = mx + c$ has gradient m and y -intercept c	Learn it
Extended only: $y > mx + c$ is above the line, $y < mx + c$ is below it	Learn it

How to do it

In the Paper 1–4 questions we have tagged for this topic (31 questions, 2021–2025), the types came up this often. 11 of the 31 questions mix two or more types, so the counts add up to more than 31.

Question type	Questions
Integers that satisfy an inequality	11
Reading an inequality from a number line	10
Solving linear inequalities (Extended)	10
Drawing a region on a graph (Extended)	6
Showing an inequality on a number line	5
The best point in a region (Extended, papers before 2025)	4
Writing the inequalities that define a region (Extended)	2
Forming inequalities from words (Extended)	2

1. Integers that satisfy an inequality (11 questions)

1. **Find the two ends** and decide for each whether it is included ($\leq$, $\geq$) or not ($<$, $>$).
2. **List every integer between them**, including an end only if its symbol allows it. Remember 0 and the negative numbers.
3. **Apply any extra condition** (odd, even, prime, multiple of) and **count** the list: $-1 < x \leq 3$ has 4 integers (0, 1, 2, 3).

Variations you will meet:

- **Two separate conditions**, such as " $x \geq -1$ and $x < 2$ ": these are the same as $-1 \leq x < 2$, giving $-1, 0, 1$.
- **The smallest (or largest) integer** that satisfies $x > -3.5$: the smallest is -3 , because $-4 < -3.5$. With $x \geq -3$ the smallest is -3 itself.
- **"Positive integers"** start at 1; 0 is not positive. The positive integers with $x \leq 2.5$ are 1 and 2.
- **Extended only: solve first.** For "the integer values of x with $-2 < 3x + 1 \leq 10$ ", solve to get $-1 < x \leq 3$ (type 3), then list 0, 1, 2, 3.

2. Reading an inequality from a number line (10 questions)

1. **Left end:** open circle at a gives $a < x$; closed circle gives $a \leq x$.
2. **Right end:** open circle at b gives $x < b$; closed circle gives $x \leq b$.
3. **Write one statement** with the letter in the middle: $a < x \leq b$. If the line has an arrow instead of a second circle, there is only one end: for example $x \geq 1$.

Variations you will meet:

- **A different letter**, such as n or y : use the letter the question gives.
- **An end between two marks**, such as halfway between -2 and -1 : the value is -1.5 .
- **"Explain why this inequality is wrong"**: compare each circle with each symbol in the statement given and say which symbol should change and why. For example, two closed circles need $\leq$ at both ends.

3. Solving linear inequalities (10 questions)

Extended only.

1. **Expand brackets** and **clear any fraction** by multiplying **every** part by the denominator.
2. **Collect** the x terms on one side and the numbers on the other, exactly as for an equation. Keeping the x terms positive avoids dividing by a negative.
3. **Divide** by the number in front of x . If it is negative, **reverse the sign**.
4. **Write the answer as an inequality** with x , such as $x > -3$, not $x = -3$ or -3 .

For example, $6 - 2x \geq 14$: $-2x \geq 8$, so $x \leq -4$ (the sign reversed on dividing by -2).

Variations you will meet:

- **A double inequality**, such as $-2 < 3x + 1 \leq 10$: subtract 1 from all three parts, $-3 < 3x \leq 9$, then divide all three by 3: $-1 < x \leq 3$.
- **A fraction**, such as $\frac{x-4}{2} < 3$: multiply both sides by 2, $x - 4 < 6$, so $x < 10$. With a fraction on one side only, multiply **every term on the other side** too (see Quick check 3).
- **Then list the integers** or **show the answer on a number line** (types 1 and 5).

4. Drawing a region on a graph (6 questions)

Extended only.

1. **Draw each boundary line** with a ruler, across the whole grid: solid for $\leq$ or $\geq$, broken for $<$ or $>$.
2. **Test a point** not on the line, usually $(0, 0)$, in each inequality to find the wanted side.
3. **Shade the unwanted side** of every line.
4. **Label** the region left clear with R .

Variations you will meet:

- **Some lines already drawn:** draw only the missing ones, but still check the style (solid or broken) of what you add.
- **A double inequality such as $-3 < y \leq 2$:** two horizontal lines, $y = -3$ broken and $y = 2$ solid; the wanted strip is between them.
- **A line in the form $ax + by = c$:** plot where it crosses the axes (see "Lines you will need" in Understand it).

5. Showing an inequality on a number line (5 questions)

1. **Put a circle at each end:** filled for $\leq$ or $\geq$, open for $<$ or $>$.
2. **Join them with a line,** or, for a single inequality such as $x \geq 1$, draw a line with an arrow from the circle in the right direction ($>$ and $\geq$ go right; $<$ and $\leq$ go left).

Variations you will meet:

- **Extended only: show your answer to the previous part:** after solving, draw your inequality, using a circle, not a dot or a vertical line, at the end value.

6. The best point in a region (4 questions)

Extended only. Some papers up to 2024 ended a region question by asking for the largest profit or the largest value of an expression such as $3x + y$ at a point of R . That is linear programming, which the 2025–2027 syllabus leaves out, so skip that part when you practise those papers. The earlier parts (forming the inequalities, drawing the lines, shading and labelling R) are still in the syllabus, as is deciding whether a given point lies in R (Example 3(b)).

7. Writing the inequalities that define a region (2 questions)

Extended only.

1. **Name each boundary line:** $x = a$, $y = b$, or $y = mx + c$ (read the y -intercept and the gradient from the grid).
2. **Choose the symbol type** from the line: broken gives $<$ or $>$, solid gives $\leq$ or $\geq$.
3. **Pick a point inside R** and substitute it to choose the direction.
4. **Write one inequality per line**, all of them.

Variations you will meet:

- **The lines are labelled with their equations:** put the symbol in place of $=$ and keep the equation's form, for example $x + 2y > 2$. Rearranging first is not needed and invites slips.
- **Four lines** (as in the diagram in Understand it): four inequalities.

8. Forming inequalities from words (2 questions)

Extended only.

1. **Translate each phrase:** "at least 3" is ≥ 3 ; "no more than 9" and "at most" are $\leq$; "fewer than" and "less than" are $<$; "more than" is $>$.
2. **Use the letters given**, such as x for one item and y for the other.
3. **For a "show that"**, write the inequality from the facts (for example a total cost), then simplify it by dividing by a common factor until it matches the printed one exactly.

Variations you will meet:

- **"More cakes than pies"** means $c > p$ (with c cakes and p pies), not $p > c$.
- **A total**, such as "fewer than 20 items in total": $x + y < 20$.
- **Then draw the region** (type 4) and **use it**, for example to decide whether a given number of items is possible (Example 4).

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
 - **A1** accuracy mark: for a correct answer; it needs the M mark before it.
 - **B1** mark for a correct result on its own, with no method needed.
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Example 1

- (a) Show the inequality $-2 \leq x < 2$ on a number line. **[2]**
- (b) x is an integer with $x > -5$ and $x \leq 1$. Write down all the possible values of x . **[2]**
- (c) p is a prime number and $20 < p < 30$. Write down the possible values of p . **[2]**
- (d) Write down the smallest integer that satisfies $y > -4.5$. **[1]**
- (a)** A **closed** circle at -2 (because of $\leq$), an **open** circle at 2 (because of $<$), joined by a straight line. **B1** for both circles right, **B1** for the line between them.
- (b)** $x > -5$ excludes -5 ; $x \leq 1$ includes 1 . So $x = -4, -3, -2, -1, 0, 1$. **B2** (**B1** for five correct and no wrong ones).
- (c)** The numbers from 21 to 29 are 21, 22, ..., 29. The primes among them are 23 and 29 ($21 = 3 \times 7$, $25 = 5^2$, $27 = 3^3$, and the rest are even). **B2** (**B1** for one of them).
- (d)** -4.5 is between -5 and -4 . -5 is less than -4.5 , so it does not fit; the smallest integer that fits is -4 . **B1**

Example 2

Extended only.

(a) Solve $9 - 2x < 3(x + 8)$. **[3]**

(b) (i) Solve $-11 < 4x + 5 \leq 17$. **[3]**

(ii) Write down all the integer values of x that satisfy $-11 < 4x + 5 \leq 17$. **[2]**

(c) Show your answer to part (a) on a number line. **[1]**

(a) $9 - 2x < 3x + 24$. **M1** for expanding the bracket correctly.

Keep the x terms positive by adding $2x$ to both sides and subtracting 24: $9 - 24 < 5x$, so $-15 < 5x$. **M1** for collecting the x terms and the numbers correctly.

$x > -3$. **A1** (the answer must be an inequality).

(b)(i) Subtract 5 from all three parts: $-16 < 4x \leq 12$. **M1**

Divide all three parts by 4: $-4 < x \leq 3$. **M1 A1**

(ii) -4 is not included, 3 is: $x = -3, -2, -1, 0, 1, 2, 3$. **B2** (**B1** for one error or omission)

(c) An **open** circle at -3 with a line and arrow going to the right. **B1**

Example 3

Extended only.

(a) By drawing three suitable lines and shading the unwanted regions, find and label the region R that satisfies these inequalities. **[5]**

$$y \geq -1 \quad x + y < 4 \quad y \leq 2x + 1$$

(b) Does the point $(3, 1)$ lie in R ? Give a reason. **[1]**

(a) The three lines:

- $y = -1$: horizontal, **solid** (because of $\geq$). **B1**
- $x + y = 4$: through $(0, 4)$ and $(4, 0)$, **broken** (because of $<$). **B1**
- $y = 2x + 1$: y -intercept 1, gradient 2, for example through $(0, 1)$ and $(2, 5)$, **solid**. **B1**

Test $(0, 0)$: $0 \geq -1$ ✓, $0 < 4$ ✓, $0 \leq 1$ ✓. The origin is in R , so shade the side of each line **away from** the origin. R is the triangle with corners $(-1, -1)$, $(5, -1)$ and $(1, 3)$, as in the diagram in "Understand it". **B2** for the correct region labelled R (**B1** for a region satisfying two of the inequalities).

(b) No: $3 + 1 = 4$, which is not less than 4. The point lies on the broken line $x + y = 4$, and points on a broken line are not in the region. **B1**

Example 4

Extended only.

A school buys x footballs and y basketballs. A football costs \$15 and a basketball costs \$25.

- It buys at least 6 footballs.
- It buys fewer than 10 basketballs.
- It buys more footballs than basketballs.
- It spends no more than \$450.

(a) Write down three inequalities in x and/or y for the first three statements. [3]

(b) Show that the fourth statement gives $3x + 5y \leq 90$. [1]

(c) Can the school buy 14 footballs and 9 basketballs? Show how you decide. [2]

(a) $x \geq 6$, $y < 10$, $x > y$. **B1** for each.

(b) The cost is $15x + 25y$, and "no more than" means $\leq$: $15x + 25y \leq 450$. Divide every term by 5: $3x + 5y \leq 90$. **B1** (the starting inequality must be seen)

(c) Check all four: $14 \geq 6$ ✓, $9 < 10$ ✓, $14 > 9$ ✓, and $3(14) + 5(9) = 42 + 45 = 87 \leq 90$ ✓ (a cost of $15 \times 14 + 25 \times 9 = 435$ dollars). **M1** for testing the point in the inequalities. Yes, it can. **A1**

Common mistakes

- **Mixing up open and closed circles.** Examiners see this every session, in both directions. Open means the end is **not** included ($<$, $>$); filled means it is. Say it to yourself as you draw or read each end.
- **Leaving out the letter or writing only numbers.** " $-2 < 5$ " or " -2 and 5 " is not an inequality for x . Write $-2 < x < 5$ with the letter in the middle.
- **Symbols pointing opposite ways.** $-2 > x \leq 5$ makes no sense. In a double inequality with the smaller number on the left, both symbols are $<$ or $\leq$.
- **Missing an end or 0 when listing integers.** For $-1 \leq x < 3$ the list is $-1, 0, 1, 2$: include -1 , leave out 3 , and don't forget 0 . Giving the inequality when the question asks for the integers (or the other way round) also loses marks.
- **Extended only: not reversing the sign.** $-5x \geq 20$ gives $x \leq -4$, not $x \geq -4$. Better still, move the x term to the other side so it stays positive: $3 - 4x > 11$ becomes $3 - 11 > 4x$, so $-8 > 4x$ and $x < -2$.
- **Extended only: answering with $=$.** Solving as an equation and writing $x = 3$ loses the final mark. Keep the inequality sign on every line, or put it back at the end.
- **Extended only: treating the three parts of a double inequality differently.** From $-1 \leq 2x - 5 < 7$, add 5 to **all three** parts: $4 \leq 2x < 12$, so $2 \leq x < 6$. Adding to only one side, or combining the two outer numbers, is wrong.

- **Extended only: multiplying only part of a side.** In $2 > \frac{x+1}{3}$, multiplying by 3 gives $6 > x + 1$; every term on both sides is multiplied.
- **Extended only: wrong line style or wrong line.** All lines drawn solid is the most common error in region questions. Also, $y = 2$ is **horizontal** and $x = 2$ is **vertical**; don't swap them. Draw lines with a ruler, right across the grid.
- **Extended only: shading the wrong side.** Test a point every time instead of guessing. If you shade the wanted region instead, the question's "shade the unwanted regions" has not been followed; label R clearly either way.
- **Extended only: all strict signs when listing a region's inequalities.** A solid boundary needs $\leq$ or $\geq$; a broken one needs $<$ or $>$.

How to get the marks

- **Number lines:** one mark is often for the circles and one for the line. Make open circles clearly empty and closed circles clearly filled, drawn at exactly the right values.
- **Writing an inequality:** each end is usually worth a mark ($-3 < x$ and $x \leq 2$ separately), so even if you are unsure of one end, write the other correctly. The final answer must include the letter.
- **Listing integers:** give every value and nothing else. One missing or one extra value usually costs a mark; more costs both.
- **Extended only: "Solve"** an inequality means the answer is an inequality such as $x \geq 3$. The first correct step (expanding, or collecting terms) earns a method mark even if you slip later, so write it. Don't spoil a correct answer by adding a list of values unless the question asks for one.
- **Extended only: double inequalities:** keep all three parts on each line. An answer with only one end right can still earn a mark.
- **Extended only: region questions:** each line is a mark (right position **and** right style), and the region is usually two marks. Rule the lines, shade the unwanted parts, and **write** R in the region.
- **Extended only: "Show that"** for an inequality from words: write the full inequality first (for example $15x + 25y \leq 450$), then the simplifying step, ending with exactly the printed form.
- **"Write down"** means no working is needed, but the answer must be exactly right.

Quick check

1. A number line shows an open circle at -2 and a closed circle at 5 , joined by a line. Write down the inequality for x .
2. n is an even number and $-3 < n \leq 4$. Write down all the possible values of n .
3. **Extended only.** Solve $\frac{2x+1}{3} \geq x-2$.
4. **Extended only.** Find all the integers x that satisfy $2 < 3-x \leq 6$.
5. **Extended only.** A region R is bounded by the broken line $y = 5$, the solid line $x = 1$ and the solid line $y = 2x - 1$, and the point $(2, 4)$ is inside R . Write down the three inequalities that define R .

Answers

1. The open circle gives $-2 < x$ and the closed circle gives $x \leq 5$, so $-2 < x \leq 5$.
2. The even numbers from -2 to 4 (-4 is outside and 4 is included): $-2, 0, 2, 4$.
3. Multiply both sides by 3: $2x + 1 \geq 3(x - 2) = 3x - 6$. Collect on the right, where the x term stays positive: $1 + 6 \geq 3x - 2x$, so $7 \geq x$, that is $x \leq 7$.
4. Subtract 3 from all three parts: $-1 < -x \leq 3$. Multiply all three by -1 and reverse both signs: $1 > x \geq -3$, that is $-3 \leq x < 1$. The integers are $-3, -2, -1, 0$.
5. Test $(2, 4)$ in each. $4 < 5$, and the line is broken: $y < 5$. $2 > 1$, and the line is solid: $x \geq 1$. On $y = 2x - 1$, when $x = 2$ the line is at $y = 3$, and $4 > 3$, so R is above it; the line is solid: $y \geq 2x - 1$. (R is the triangle with corners $(1, 1)$, $(1, 5)$ and $(3, 5)$.)

Past questions to try

- **0580/12/M/J/23 Q16**: listing the integers that satisfy two separate inequalities, one with $\geq$ and one with $<$.
- **0580/41/O/N/22 Q3**: reading a number line, then solving a double inequality and listing its integers (part (c) is on equations).
- **0580/21/O/N/21 Q13**: solving an inequality with a fraction on one side and negative x terms.
- **0580/22/O/N/24 Q13**: drawing a region for a double inequality in y and a sloping line.