Cambridge IGCSE · Mathematics 0580 · Notes

Non-right-angled triangles

Read online at **pastopic.com/cambridge/mathematics-0580/notes/non-right-angled-triangles**

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Read first: **Pythagoras' theorem and right-angled triangles**, **Exact values and trigonometric functions**

What you need to know

Extended only: this whole topic is Extended content (syllabus E6.5). Core candidates do not study it.

By the end of this topic you should be able to:

1. **Use the sine rule and the cosine rule** to find missing sides and angles in any triangle, not just right-angled ones. This includes triangles with an **obtuse angle**, and the **ambiguous case**, where the facts you are given fit two different triangles.
2. **Use the formula** $\text{area} = \frac{1}{2}ab\sin C$ to find the area of a triangle from two sides and the angle between them, and work backwards from a known area to a side or an angle.

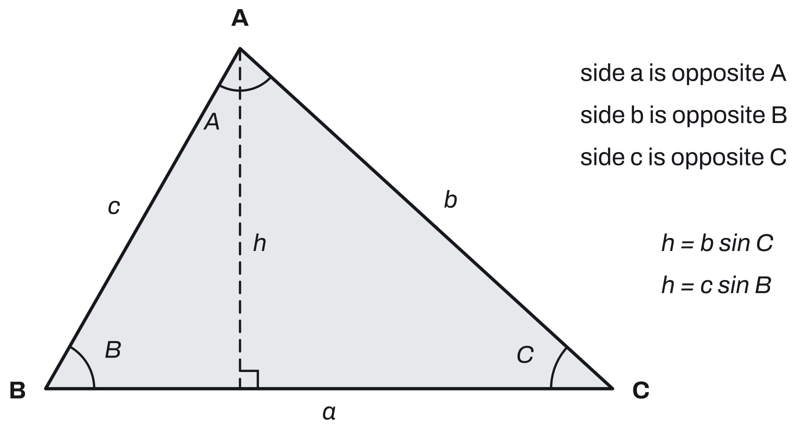
All three formulas are printed in the List of formulas on Paper 2 and Paper 4. Your job is to know **which one to use** and to use it accurately.

Understand it

Labelling a triangle

Name the corners with capital letters, A, B and C, and use the matching **small letter for the side opposite** each corner: side a is opposite angle A, side b is opposite angle B, side c is opposite angle C. Every formula in this topic pairs a side with the angle facing it, so this labelling is the key to using them.

A real question uses its own letters. Relabel in your head: in triangle PQR, the side opposite P is QR, and so on.



Where the rules come from

Drop a perpendicular (a height h) from A to BC, as in the diagram. It splits the triangle into two right-angled triangles, where ordinary trigonometry works:

- in the right-hand one, $\sin C = \frac{h}{b}$, so $h = b\sin C$;
- in the left-hand one, $\sin B = \frac{h}{c}$, so $h = c\sin B$.

Both equal h, so $b\sin C = c\sin B$, which rearranges to $\frac{b}{\sin B} = \frac{c}{\sin C}$. Doing the same with a height from another corner brings in a and A. That is the **sine rule**:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

The same height gives the **area**. Area $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times a \times b\sin C$, so

$$\text{area} = \tfrac{1}{2}ab\sin C.$$

The angle C must be the angle **between** the two sides a and b (the "included" angle).

The sine rule

Use it when you know **a side and the angle opposite it**, plus one more side or angle. You only ever use two of the three fractions.

Finding a side: put the sides on top. In a triangle with $A = 48^\circ$, $B = 67^\circ$ and $a = 9.2$ cm,

$$\frac{b}{\sin 67^\circ} = \frac{9.2}{\sin 48^\circ} \;\Rightarrow\; b = \frac{9.2\sin 67^\circ}{\sin 48^\circ} = 11.4 \text{ cm } (3 \text{ s.f.}).$$

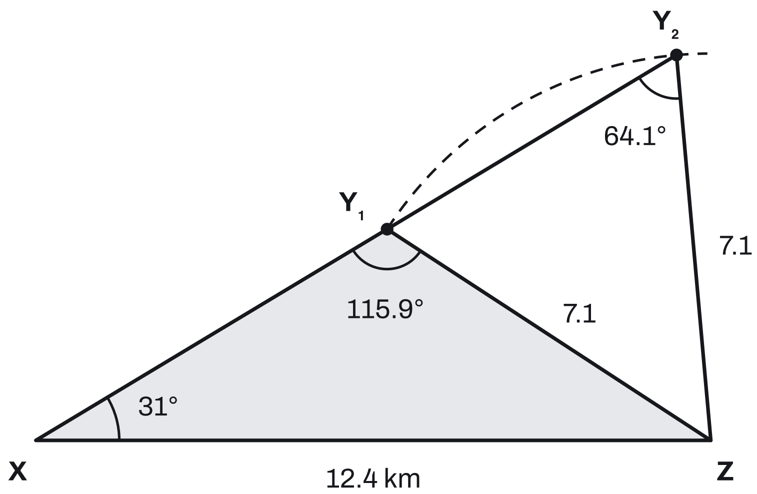
If you are given two angles but not the one opposite your known side, find the third angle first: the angles add up to 180°.

Finding an angle: put the angles on top, $\frac{\sin A}{a} = \frac{\sin B}{b}$, then use $\sin^{-1}$.

Sines of obtuse angles and the ambiguous case

An angle and its partner 180° minus that angle have the **same sine**: $\sin 150^\circ = \sin 30^\circ = 0.5$ and $\sin 115.9^\circ = \sin 64.1^\circ$. So when $\sin^{-1}$ gives you an angle θ, the angle in the triangle could be θ (acute) or $180^\circ - \theta$ (obtuse). Your calculator only ever shows the acute one.

Sometimes both answers make a real triangle. In triangle XYZ with angle $X = 31^\circ$, $XZ = 12.4$ km and $YZ = 7.1$ km, the side YZ can swing to meet the other arm of the 31° angle in two places:



$$\sin Y = \frac{12.4 \sin 31^\circ}{7.1} = 0.8995\ldots \ \Rightarrow\ Y = 64.1^\circ \text{ or } Y = 180^\circ - 64.1^\circ = 115.9^\circ.$$

This is the **ambiguous case**. The question settles it by telling you, for example, "angle Y is obtuse", or by a diagram. **Whenever a question says an angle is obtuse, and you found it with the sine rule, take 180° minus the calculator's answer.** If it doesn't say, the largest angle is opposite the longest side, and a triangle can have at most one obtuse angle.

The cosine rule

The sine rule needs a side with its opposite angle. When you don't have such a pair, use the **cosine rule**:

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

Here A is the angle **between** sides b and c, and a is the side opposite it. If $A = 90^\circ$, then $\cos A = 0$ and the rule becomes Pythagoras' theorem; the extra term corrects for the angle not being a right angle.

Finding a side (two sides and the angle between them): with $b = 7$ cm, $c = 10$ cm and $A = 38^\circ$,

$$a^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 38^\circ = 38.68\ldots, \quad a = 6.22 \text{ cm (3 s.f.)}.$$

Work out $2bc \cos A$ as one number and take it away from $b^2 + c^2$. Don't calculate $(b^2 + c^2 - 2bc)$ first and then multiply by $\cos A$.

Finding an angle (three sides): rearrange to

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

For sides 5, 7 and 9 cm, the largest angle is opposite the 9 cm side:

$$\cos A = \frac{5^2 + 7^2 - 9^2}{2 \times 5 \times 7} = \frac{-7}{70} = -0.1, \quad A = 95.7^\circ \text{ (1 d.p.)}.$$

A **negative** cosine means an obtuse angle, and $\cos^{-1}$ gives it directly. So the cosine rule is never ambiguous: it is the safer choice for an angle that might be obtuse.

Which rule?

You know	You want	Use
Two angles and a side	A side	Sine rule (find the third angle first if you need it)
Two sides and an angle opposite one of them	An angle	Sine rule (watch for obtuse)
Two sides and the angle between them	The third side	Cosine rule
All three sides	An angle	Cosine rule, rearranged
Two sides and the angle between them	The area	$\frac{1}{2}ab\sin C$

If there is a right angle, plain SOHCAHTOA and Pythagoras are quicker. Never assume a right angle that isn't marked.

Area and working backwards

$\frac{1}{2}ab\sin C$ works for any triangle. With sides 6 cm and 9 cm and 40° between them, the area is $\frac{1}{2} \times 6 \times 9 \times \sin 40^\circ = 17.4$ cm² (3 s.f.).

It also works backwards. If you know the area and two sides, solve $\frac{1}{2}ab\sin C = \text{area}$ for $\sin C$, then use $\sin^{-1}$. Because of the obtuse partner, there are **two** possible angles, C and $180^\circ - C$, unless the question says the angle is acute or obtuse. If you know the area and the angle, you can find a missing side the same way.

On the non-calculator Paper 2, the angle will be one with an exact sine, such as $\sin 30^\circ = \frac{1}{2}$ or $\sin 150^\circ = \frac{1}{2}$.

The shortest distance from a point to a line

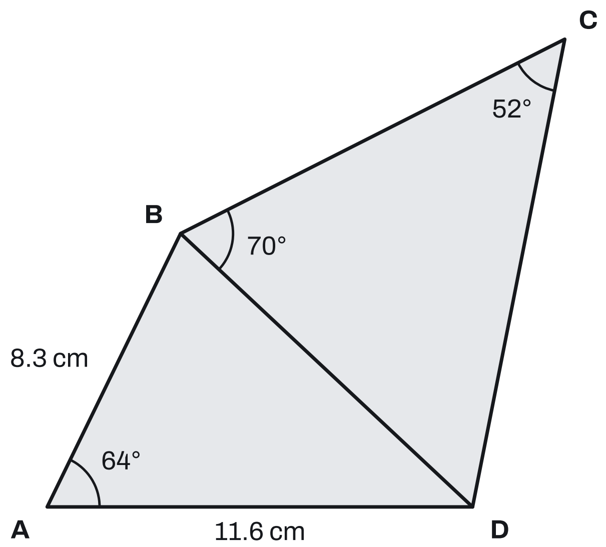
The shortest distance from a point to a line is along the **perpendicular**: the line from the point that meets the other line at 90°. It is **not** the line to the midpoint, and not a line to a corner. Draw it on the diagram and mark the right angle.

Then there are two ways to find it:

- **Right-angled triangle:** the perpendicular is the opposite side of a right-angled triangle whose hypotenuse is a side you know. In the first diagram, the distance from A to BC is $h = c \sin B$ (or $b \sin C$).
- **Area:** the area of the triangle is also $\frac{1}{2} \times \text{base} \times \text{distance}$. Find the area with $\frac{1}{2}ab \sin C$, then distance $= \dfrac{2 \times \text{area}}{\text{base}}$.

Shapes made of several triangles

Most long questions use a quadrilateral split by a diagonal, or a triangle split by a line from a corner. Work one triangle at a time: find the shared side in the first triangle, then use it in the second.



In this quadrilateral, triangle ABD has two sides and the angle between them, so the cosine rule gives BD. Triangle BCD then has BD and all its angles, so the sine rule gives BC or CD. Example 3 works it through. Angles on a straight line (180°) often give the angle you need when a triangle is split by a line from a corner.

Key facts and formulas

"Given" means it is in the List of formulas on Paper 2 and Paper 4. "Learn it" means you must remember it.

Formula	Status
Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$	Given
Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$	Given
Area of a triangle $= \frac{1}{2}ab \sin C$	Given
Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$	Given
Cosine rule for an angle: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	Learn it
Sine rule for an angle: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$	Learn it

Formula	Status
$\sin(180^\circ - \theta) = \sin\theta$: from $\sin^{-1}$, an angle is θ or $180^\circ - \theta$	Learn it
Negative cosine ⇔ obtuse angle	Learn it
Angles in a triangle add up to 180°; on a straight line to 180°	Learn it
Largest angle is opposite the longest side	Learn it
Shortest distance to a line is the perpendicular: $d = \text{side} \times \sin(\text{angle})$ or $d = \dfrac{2 \times \text{area}}{\text{base}}$	Learn it
Exact values $\sin 30^\circ = \sin 150^\circ = \frac{1}{2}$ (for Paper 2)	Learn it

How to do it

In the Paper 2 and Paper 4 questions we have tagged for this topic (43 questions, 2021–2025), the types came up this often. 29 of the 43 questions mix two or more types, so the counts add up to more than 43.

Question type	Questions
Area of a triangle ($\frac{1}{2}ab\sin C$, forwards and backwards)	23
Cosine rule for a side	19
Shortest distance from a point to a line	16
Sine rule for an angle (including obtuse angles)	14
Cosine rule for an angle	13
Sine rule for a side	13
Bearings with the sine and cosine rules	7

1. Area of a triangle (23 questions)

1. **Find two sides and the angle between them.** If the angle you have is not between the two sides, find the one you need first (angle sum, or the sine rule).
2. **Substitute** into $\frac{1}{2}ab\sin C$ and write it down.
3. **Give the units**, cm^2 or m^2.

Variations you will meet:

- **A quadrilateral:** split it into two triangles along a diagonal, find each area and add. Use sides and angles you have already worked out, kept unrounded in your calculator.
- **Working backwards for an angle:** $\frac{1}{2} \times 84 \times 65 \times \sin x = 2000$ gives $\sin x = \dfrac{2000}{2730}$, so $x = 47.1^\circ$ (1 d.p.). If the question says the angle is acute, take the calculator value; if it asks for both possible values, give x and $180^\circ - x$.

- **Working backwards for a side**, for example two equal sides $OA = OB$ with a known area: $\frac{1}{2} \times OA^2 \times \sin\theta = \text{area}$, so $OA = \sqrt{\dfrac{2 \times \text{area}}{\sin\theta}}$.
- **Same area as another triangle:** find the first area, then set it equal to $\frac{1}{2}ab\sin C$ (or $\frac{1}{2} \times \text{base} \times \text{height}$) for the second.
- **Regular polygons:** join the centre to every corner. A regular n-sided polygon splits into n equal isosceles triangles, each with angle $\frac{360^\circ}{n}$ at the centre; area of the polygon $= n \times \frac{1}{2}r^2 \sin\frac{360^\circ}{n}$, where r is the distance from the centre to a corner.
- **Using the area afterwards:** cost per hectare, number of plots, or a percentage of another area. Convert units at the end (1 hectare $= 10\,000$ m²) and round in the direction that makes sense ("greatest number of houses" rounds down).
- **Non-calculator Paper 2:** the angle is 30° or 150°, so $\sin = \frac{1}{2}$ and the arithmetic is exact.

2. Cosine rule for a side (19 questions)

1. **Check you have two sides and the angle between them.** The unknown side is opposite that angle.
2. **Write the rule with the numbers in:** $a^2 = b^2 + c^2 - 2bc\cos A$.
3. **Work out** a^2, writing it down (this value often earns a mark), then square-root.

Variations you will meet:

- **"Show that $XY = 23.6$ m, correct to 1 decimal place."** Show the substitution, then give XY^2 and XY to more accuracy than asked (for example $556.5\ldots$ and $23.59\ldots$) before rounding. Writing just 23.6 loses the last mark.
- **The angle comes from somewhere else:** angles on a straight line ($180^\circ - 65^\circ = 115^\circ$), a bearing difference, or an angle you found in an earlier part.
- **An obtuse angle:** $\cos$ is negative, so $-2bc\cos A$ becomes **added**. Let the calculator handle the sign; type the whole expression in one go.
- **Using the answer:** the side you find is usually a shared side for the next triangle, or part of a perimeter or a speed = distance ÷ time calculation.

3. Shortest distance from a point to a line (16 questions)

1. **Draw the perpendicular** from the point to the line on the diagram and mark the right angle. This alone often earns a mark.
2. **Find a right-angled triangle** that has the perpendicular as one side and a known side as its hypotenuse, with a known angle at the far end.
3. $\text{distance} = \text{hypotenuse} \times \sin(\text{angle})$.

For example, if $PQ = 14.8$ cm and angle $QPR = 36^\circ$, the distance from Q to PR is $14.8\sin 36^\circ = 8.70$ cm (3 s.f.).

Variations you will meet:

- **Area method:** if you know the triangle's area and the length of the line, distance $= \frac{2 \times \text{area}}{\text{base}}$. This works even when there is no convenient angle. For a quadrilateral whose total area you are given, subtract the triangle you can find to get the other one's area first.
- **The distance *along* the line to the foot** (for example "a gate G on AB is at the shortest distance from C; find AG"): the same right-angled triangle, but use $\cos$: $AG = AC \cos A$.
- **The perpendicular from the top of an isosceles triangle** does bisect the base, and the angle at the top. It does **not** bisect the base in any other triangle.

4. Sine rule for an angle (14 questions)

1. **Pair each side with its opposite angle.** You need one complete pair plus the side opposite the angle you want.
2. **Write** $\frac{\sin P}{p} = \frac{\sin Q}{q}$ and rearrange: $\sin P = \frac{p \sin Q}{q}$.
3. **Use** $\sin^{-1}$ and keep the unrounded value.
4. **Decide acute or obtuse.** If the question says the angle is obtuse, or the diagram clearly shows it, the answer is 180° minus the calculator value.

Variations you will meet:

- **"Show that angle $PRQ = 104.3^\circ$."** Often you find another angle with the sine rule first, then subtract both known angles from 180°. Show the unrounded value, such as $104.27\ldots$, before rounding.
- **The angle you want isn't opposite a known side:** find the angle that is, with the sine rule, then use the angle sum. For example, to find obtuse angle ACE, find acute angle AEC first and subtract.
- **Two possible triangles:** if the question asks for "the two possible values", give θ and $180^\circ - \theta$ and check both fit (the angles, including the given one, must add to less than 180°).
- **Inside another shape**, for example a rhombus (all four sides equal) or a triangle drawn inside a circle: work out every length and angle you know from the shape's properties first.

5. Cosine rule for an angle (13 questions)

1. **You need all three sides.** The angle you want is opposite side a.
2. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$: write it with numbers in, and work out the value (write it down, it can earn a mark).
3. **Use** $\cos^{-1}$**.** A negative value gives an obtuse angle automatically.

Variations you will meet:

- **"The largest angle":** it is opposite the longest side, so use that side as a. Similarly the smallest angle is opposite the shortest side.
- **Starting from the implicit form** $9^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \cos A$: collect terms carefully, $2 \times 5 \times 7 \cos A = 5^2 + 7^2 - 9^2$. Most errors come from this rearrangement, so the explicit form above is safer.
- **Angles at the centre of a circle:** two radii and a chord make an isosceles triangle with three known sides; the cosine rule gives the angle at the centre, which then gives an arc length or sector area.

6. Sine rule for a side (13 questions)

1. **Find the angle opposite the side you want**, and a complete side–angle pair. Use the angle sum or angles on a straight line if one is missing.
2. **Write** $\dfrac{x}{\sin X} = \dfrac{p}{\sin P}$ with the numbers in.
3. **Rearrange** by multiplying both sides by $\sin X$: $x = \dfrac{p \sin X}{\sin P}$.

Variations you will meet:

- **A triangle split by a line from a corner:** the angle you need is often 180° minus a given angle on the straight line, then 180° minus two angles in the triangle. Label each angle on the diagram as you find it.
- **Pairing the wrong side and angle** is the commonest error. Before you substitute, check that each side is directly opposite its angle.
- **The radius of a regular polygon's circle:** the triangle from the centre to two neighbouring corners has the side of the polygon opposite the angle at the centre.

7. Bearings with the sine and cosine rules (7 questions)

1. **Draw a North line at every point** involved and mark the given bearings.
2. **Find the angles inside the triangle** from the bearings, using angles on a straight line, angles at a point (360°) and co-interior angles between parallel North lines (they add up to 180°).
3. **Solve the triangle** with the sine or cosine rule.
4. **Turn the angle back into a bearing**: measure clockwise from North at the point the bearing is *from*, and give three figures.

Variations you will meet:

- **Back bearings:** the bearing of A from B is the bearing of B from A plus 180° (or minus 180° if that goes past 360°).
- **"The bearing of B from A":** stand at A. Start from the bearing of a point you already know from A, then add or subtract the angle inside the triangle at A, depending on which way B lies.
- **An obtuse angle from the sine rule** is common here, because triangles in bearing questions are often long and thin.
- **"Bearing and distance"** means two answers: the angle clockwise from North, and the length.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

In triangle PQR, $PQ = 11.5$ cm, $PR = 8.4$ cm and angle $QPR = 57^\circ$.

(a) Calculate QR. **[3]**

(b) Calculate the area of triangle PQR. **[2]**

(a) Two sides and the angle between them, so use the cosine rule. QR is opposite the 57° angle.

$$QR^2 = 11.5^2 + 8.4^2 - 2 \times 11.5 \times 8.4 \times \cos 57^\circ$$

$QR^2 = 97.58\ldots$, so $QR = \sqrt{11.5^2 + 8.4^2 - 2 \times 11.5 \times 8.4 \cos 57^\circ} = 9.88$ cm (3 s.f.).

M2 for $\sqrt{11.5^2 + 8.4^2 - 2 \times 11.5 \times 8.4 \cos 57^\circ}$ (**M1** for the expression alone, or **A1** for $QR^2 = 97.58\ldots$ seen), **A1** for $QR = 9.88$ cm.

(b) $\text{Area} = \frac{1}{2} \times 11.5 \times 8.4 \times \sin 57^\circ = 40.5$ cm² (3 s.f.). **M1 A1**

Example 2

In triangle KLM, $KL = 14.2$ cm, $LM = 9.6$ cm and angle $LKM = 38^\circ$. Angle KML is obtuse.

(a) Calculate angle KML. **[4]**

(b) Calculate KM. **[3]**

(a) $LM = 9.6$ is opposite the 38° angle, and $KL = 14.2$ is opposite angle M: a complete pair plus one side, so use the sine rule for an angle.

$$\frac{\sin M}{14.2} = \frac{\sin 38^\circ}{9.6}$$

M1 for the correct pairing.

$$\sin M = \frac{14.2 \sin 38^\circ}{9.6} = 0.9106\ldots \;\Rightarrow\; \sin^{-1}(0.9106\ldots) = 65.60^\circ$$

M1 for rearranging to $\sin M = \ldots$ (together **M2**). **B1** for 65.6° seen.

Angle KML is obtuse, so $KML = 180^\circ - 65.60^\circ = 114.4^\circ$ (1 d.p.). **A1**

(b) Angle $KLM = 180^\circ - 38^\circ - 114.40^\circ = 27.60^\circ$. **B1**

KM is opposite angle L:

$$\frac{KM}{\sin 27.60^\circ} = \frac{9.6}{\sin 38^\circ} \;\Rightarrow\; KM = \frac{9.6 \sin 27.60^\circ}{\sin 38^\circ} = 7.22 \text{ cm (3 s.f.)}.$$

M1 A1. Using the rounded 27.6° gives the same answer, but keep the calculator value to be safe.

Example 3

The diagram in "Understand it" shows quadrilateral $ABCD$. $AB = 8.3$ cm, $AD = 11.6$ cm and angle $BAD = 64^\circ$. Angle $CBD = 70^\circ$ and angle $BCD = 52^\circ$.

(a) Calculate BD. **[3]**

(b) Calculate CD. **[3]**

(c) Calculate the area of quadrilateral $ABCD$. **[3]**

(d) Calculate the shortest distance from A to BD. **[3]**

(a) In triangle ABD: two sides and the included angle.

$$BD^2 = 8.3^2 + 11.6^2 - 2 \times 8.3 \times 11.6 \times \cos 64^\circ = 119.03\ldots$$

M2 for the substitution and square root, **A1**: $BD = 10.9$ cm (3 s.f.), stored as $10.910\ldots$

(b) In triangle BCD, CD is opposite the 70° angle and BD is opposite the 52° angle.

$$\frac{CD}{\sin 70^\circ} = \frac{10.910\ldots}{\sin 52^\circ} \;\Rightarrow\; CD = \frac{10.910\ldots \times \sin 70^\circ}{\sin 52^\circ} = 13.0 \text{ cm (3 s.f.)}.$$

M2 A1 (**M1** for the implicit form alone).

(c) Angle $BDC = 180^\circ - 70^\circ - 52^\circ = 58^\circ$.

$$\text{Area } ABD = \tfrac{1}{2} \times 8.3 \times 11.6 \times \sin 64^\circ = 43.27\ldots$$
$$\text{Area } BCD = \tfrac{1}{2} \times 10.910\ldots \times 13.010\ldots \times \sin 58^\circ = 60.19\ldots$$

M1 for each triangle. Total $=$ 103 cm² (3 s.f.). **A1**

(d) Draw the perpendicular from A to BD (**M1** for recognising it). Using the area of triangle ABD:

$$\tfrac{1}{2} \times 10.910\ldots \times d = 43.27\ldots \;\Rightarrow\; d = \frac{2 \times 43.27\ldots}{10.910\ldots} = 7.93 \text{ cm (3 s.f.)}.$$

M1 A1

Example 4

A buoy, B, is 7.5 km from a harbour, H, on a bearing of 052°. A lighthouse, L, is 11.2 km from H on a bearing of 125°.

(a) Calculate the distance BL. **[4]**

(b) Calculate the bearing of L from B. **[5]**

(a) Both bearings are measured from North at H, so angle $BHL = 125^\circ - 52^\circ = 73^\circ$. **B1**

$$BL^2 = 7.5^2 + 11.2^2 - 2 \times 7.5 \times 11.2 \times \cos 73^\circ = 132.57\ldots$$

M2, and $BL = 11.5$ km (3 s.f.). **A1**

(b) In triangle BHL, $HL = 11.2$ is opposite angle B, and BL is opposite the 73° angle.

$$\sin HBL = \frac{11.2 \sin 73^\circ}{11.513\ldots} = 0.9302\ldots \ \Rightarrow \ HBL = 68.47^\circ$$

M2 (**M1** for the implicit form). Angle HBL is acute here, because BL is the longest side, so the largest angle is the 73° at H. **A1**

The bearing of H from B is the back bearing $052^\circ + 180^\circ = 232^\circ$. **B1**

A sketch with North lines shows that, from B, H is to the south-west and L is roughly south, so L is 68.47° anticlockwise from the direction of H. So the bearing of L from B is $232^\circ - 68.47^\circ = 163.5^\circ$ (1 d.p.). **A1**

Common mistakes

- **Taking the acute angle when the question says obtuse.** The sine rule's $\sin^{-1}$ only gives acute angles. Examiners see this in almost every paper with the sine rule: if the angle is obtuse, subtract from 180°.
- **Pairing a side with the wrong angle** in the sine rule. The side must be directly opposite its angle. Mark each pair on the diagram before you substitute.
- **Evaluating the cosine rule in the wrong order.** $(b^2 + c^2 - 2bc) \cos A$ is wrong; the $\cos A$ multiplies only the $2bc$. Type $b^2 + c^2 - 2bc \cos A$ into the calculator in one go.
- **Losing the minus sign** when rearranging the cosine rule for an angle, for example getting $\cos A = 0.35$ instead of -0.35, which turns an obtuse angle into an acute one. The explicit form $\cos A = \frac{b^2+c^2-a^2}{2bc}$ avoids this.
- **Treating a triangle as right-angled** when no right angle is marked, and using SOHCAHTOA or Pythagoras on it.
- **Using a slanted side as the height.** In $\frac{1}{2} \times \text{base} \times \text{height}$, the height must be perpendicular to the base. With a slanted side, use $\frac{1}{2}ab \sin C$ instead.
- **Getting the shortest distance wrong.** It is the perpendicular to the line, not the line to the midpoint and not a line to where two lines cross. Draw it and mark the right angle.

- **Rounding too early.** Using 10.9 instead of $10.910\ldots$ in the next part, or 0.91 for a sine, often pushes the final answer out of the accepted range. Keep values in the calculator's memory.
- **"Show that" answers without extra accuracy.** To show a length is 47.6 m to 1 d.p., you must write a more accurate value such as $47.63\ldots$ first.
- **Giving an angle inside the triangle as a bearing.** A bearing is measured clockwise from North at the right point; finish the last step.

How to get the marks

- **Write the formula with the numbers substituted** before calculating. That line earns the method marks even if you slip on the calculator.
- **Accuracy:** give lengths and areas to 3 significant figures and angles to 1 decimal place, unless the question says otherwise. Work with at least 4 significant figures in between, and store values in the calculator.
- **"Show that … correct to 1 decimal place"** means the answer is given. Show every step and a value more accurate than the one printed, then round.
- **Implicit forms earn part marks.** $\frac{\sin M}{14.2} = \frac{\sin 38^\circ}{9.6}$ or $9^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \cos A$ usually gets **M1**; the rearranged form gets the next mark.
- **Follow-through:** later parts using your earlier (wrong) value can still earn their method marks, so always carry on. Use your own earlier answers, unrounded.
- **"Calculate"** means show working that leads to the answer; a value with no working can score nothing if it is out of range.
- **Shortest distance:** a right-angle mark on the diagram, or a sentence such as "the perpendicular from C to AB", earns a method mark on its own.
- **Bearings:** give three figures (076°, not 76°), measured clockwise from North at the point the bearing is from.
- **Units:** write cm, m, km or cm², m² where the answer line doesn't already.

Quick check

1. A triangle has sides 6.5 cm and 8 cm with an angle of 112° between them. Calculate the length of the third side.
2. A triangle has sides 4 cm, 6 cm and 7 cm. Calculate the smallest angle.
3. Without a calculator, find the area of a triangle with sides 12 cm and 15 cm and an angle of 150° between them.
4. A triangle has sides 9 cm and 11 cm and an area of 30 cm². The angle between these sides is acute. Calculate this angle.
5. In triangle DEF, $DE = 10$ cm, angle $D = 42^\circ$ and angle $E = 75^\circ$. Calculate DF, then the shortest distance from F to DE.

Answers

1. $x^2 = 6.5^2 + 8^2 - 2 \times 6.5 \times 8 \times \cos 112^\circ = 145.2\ldots$, so $x = 12.1$ cm (3 s.f.). The cosine of 112° is negative, so the third side is longer than you might expect.
2. The smallest angle is opposite the 4 cm side: $\cos\theta = \dfrac{6^2 + 7^2 - 4^2}{2 \times 6 \times 7} = \dfrac{69}{84}$, so $\theta = 34.8^\circ$ (1 d.p.).
3. $\sin 150^\circ = \sin 30^\circ = \frac{1}{2}$, so the area is $\frac{1}{2} \times 12 \times 15 \times \frac{1}{2} = 45$ cm².
4. $\frac{1}{2} \times 9 \times 11 \times \sin C = 30$, so $\sin C = \dfrac{60}{99} = 0.6060\ldots$ and $C = 37.3^\circ$ (1 d.p.). (The obtuse partner, 142.7°, is ruled out because the angle is acute.)
5. Angle $F = 180^\circ - 42^\circ - 75^\circ = 63^\circ$, and DF is opposite E: $DF = \dfrac{10 \sin 75^\circ}{\sin 63^\circ} = 10.8$ cm (3 s.f.). The perpendicular from F to DE makes a right-angled triangle with hypotenuse DF and angle 42° at D: distance $= 10.84\ldots \times \sin 42^\circ = 7.25$ cm (3 s.f.).

Past questions to try

- **0580/23/M/J/21 Q23**: the largest angle of a triangle from three sides.
- **0580/22/M/J/23 Q17**: working backwards from an area to an angle.
- **0580/23/O/N/22 Q21**: an obtuse angle from the sine rule, then a bearing.
- **0580/43/O/N/21 Q6**: a quadrilateral with the sine rule, cosine rule, area and shortest distance.