Cambridge IGCSE · Mathematics (0580) · Notes

Powers, roots, indices and surds

Read online at pastopic.com/cambridge/mathematics-0580/notes/powers-roots-and-surds

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Read first: Types of number and sets

What you need to know

Core and Extended share most of this topic. Fractional indices and surds are **Extended only**. By the end of it you should be able to:

1. **Know squares and cubes by heart**: the squares of 1 to 15 and their square roots, and the cubes of $1, 2, 3, 4, 5$ and 10 and their cube roots. You must be able to write these down without a calculator.
2. **Work out other powers and roots**, such as 23^3, $\sqrt{2.89}$ or $\sqrt[4]{0.0081}$, using a calculator on Papers 3 and 4, and give the answer to the accuracy asked for.
3. **Understand and use indices** that are positive whole numbers, zero or negative whole numbers, for example $5^0 = 1$ and $5^{-2} = \frac{1}{25}$.
4. **Extended only:** understand and use **fractional indices**, for example $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ and $32^{-\frac{2}{5}} = \frac{1}{4}$.
5. **Use the rules of indices** with numbers: multiplying and dividing powers of the same number, and a power of a power, such as $3^5 \times 3^{-2}$, $(5^2)^3$ and $2^4 \div 2^7$.
6. **Extended only:** understand and use **surds**: simplify one, such as $\sqrt{18} = 3\sqrt{2}$, add and subtract them, multiply them and expand brackets that contain them.
7. **Extended only: rationalise the denominator** of a fraction, whether the bottom is a single surd, such as $\frac{12}{\sqrt{3}}$, or a number plus or minus a surd, such as $\frac{1}{2+\sqrt{5}}$.

Papers 1 and 2 are non-calculator papers, so most index and surd questions must be done by hand. Indices with letters, such as $(2x^3)^4$, are in the topic Indices in algebra, and numbers such as 4.2×10^{-3} are in Standard form.

Understand it

Powers

A **power** is repeated multiplication. In 5^3, the 5 is the **base** and the small 3 is the **index** (or power, or exponent):

$$5^3 = 5 \times 5 \times 5 = 125.$$

5^2 is read "5 squared" and 5^3 "5 cubed". A power is **not** the base times the index: 5^3 is 125, not 15.

Squares, cubes and their roots

A **square root** undoes squaring: $\sqrt{81} = 9$ because $9^2 = 81$. A **cube root** undoes cubing: $\sqrt[3]{64} = 4$ because $4^3 = 64$. Learn these, because the non-calculator papers expect you to write them down straight away.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n^2	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

n	1	2	3	4	5	10
n^3	1	8	27	64	125	1000

Some facts worth knowing:

- Every positive number has two square roots, one positive and one negative ($7^2 = 49$ and $(-7)^2 = 49$), but the sign $\sqrt{}$ means the **positive** one, so $\sqrt{49} = 7$.
- A negative number has a cube root: $(-2)^3 = -8$, so $\sqrt[3]{-8} = -2$. It has no square root that you will meet.
- Other roots work the same way: $\sqrt[4]{81} = 3$ because $3^4 = 81$, and $\sqrt[5]{32} = 2$ because $2^5 = 32$.
- The square root of a number with an even number of decimal places can be found from a known square: $\sqrt{1.44} = 1.2$ because $1.2^2 = 1.44$.

Powers of negative numbers and the order of operations

Powers are worked out **before** multiplying, dividing, adding and subtracting, and a minus sign in front counts as "multiply by -1". So

$$(-3)^2 = (-3) \times (-3) = 9, \qquad \text{but} \qquad -3^2 = -(3 \times 3) = -9.$$

With brackets, the power is applied to the whole bracket. A negative number to an even power is positive; to an odd power it is negative: $(-2)^4 = 16$ and $(-2)^3 = -8$.

Using a calculator

On Papers 3 and 4 you use the calculator's power key (x^2, x^3 or $x^{\blacksquare}$) and root keys ($\sqrt{}$, $\sqrt[3]{}$ or $\sqrt[\blacksquare]{}$). Two things go wrong:

- **Put the root sign over everything it should cover.** $\sqrt{6.4 + 1.2}$ needs brackets or the calculator's template so that it gives $\sqrt{7.6}$, not $\sqrt{6.4} + 1.2$.
- **Round only at the end, and only if asked.** Write down the full calculator display first, then round. For example, $7^6 = 117\,649$, which is $118\,000$ to 3 significant figures. Don't write 118: rounding must keep the number the same size.

A quick check: square your square root (or cube your cube root) and you should get back to the number you started with.

The rules of indices

When you multiply powers of the **same base**, you are just counting how many times the base is multiplied:

$$2^3 \times 2^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2^7.$$

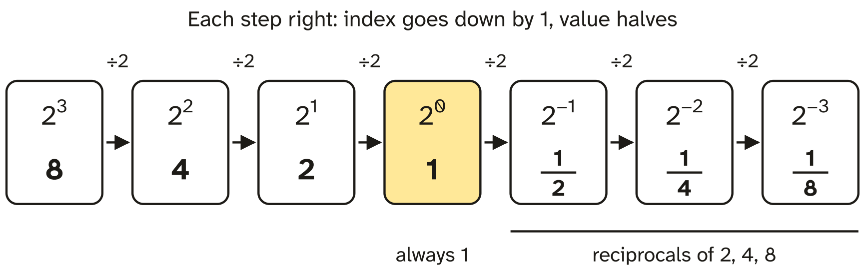
So you **add** the indices. Dividing cancels factors, so you **subtract**: $2^6 \div 2^2 = \frac{2\times2\times2\times2\times2\times2}{2\times2} = 2^4$. A power of a power repeats the whole power: $(2^3)^2 = 2^3 \times 2^3 = 2^6$, so you **multiply** the indices.

$$a^m \times a^n = a^{m+n}, \qquad a^m \div a^n = a^{m-n}, \qquad (a^m)^n = a^{mn}.$$

These rules only work when the **bases are the same.** $2^3 \times 3^2$ can't be combined into one power; work it out as $8 \times 9 = 72$. When the bases are different but related, rewrite them as powers of the same prime first: $8 = 2^3$ and $32 = 2^5$, so $8 \times 32 = 2^3 \times 2^5 = 2^8$.

Zero and negative indices

Follow the pattern down. Each time the index goes down by 1, you divide by the base once more.



This gives two rules that fit the rules of indices exactly:

- **Zero index:** any number (except 0) to the power 0 is 1. For example $5^0 = 1$ and $12^0 = 1$. This agrees with the division rule: $5^3 \div 5^3 = 5^0$, and anything divided by itself is 1.
- **Negative index:** a negative index means "one over", the **reciprocal**: $a^{-n} = \frac{1}{a^n}$. So $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ and $10^{-2} = \frac{1}{100} = 0.01$. A negative index never makes the answer negative.

Dividing by a negative power multiplies: $\frac{4}{5^{-2}} = 4 \times 5^2 = 100$. A fraction to a negative power turns upside down: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$.

Fractional indices (Extended only)

What should $9^{\frac{1}{2}}$ mean? By the rules, $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^1 = 9$. The number that multiplies by itself to make 9 is 3, so $9^{\frac{1}{2}} = \sqrt{9} = 3$. In the same way:

$$a^{\frac{1}{2}} = \sqrt{a}, \qquad a^{\frac{1}{3}} = \sqrt[3]{a}, \qquad a^{\frac{1}{n}} = \sqrt[n]{a}.$$

For a fraction such as $\frac{m}{n}$, the **bottom is the root** and the **top is the power**:

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m.$$

Take the root first, because it keeps the numbers small: $125^{\frac{2}{3}} = (\sqrt[3]{125})^2 = 5^2 = 25$. Squaring first ($125^2 = 15\,625$) gives the same answer but is much harder without a calculator.

A **negative fractional index** has three parts, and you can do them in this order: the minus means reciprocal, the bottom means root, the top means power.

$$16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{2^3} = \frac{1}{8}.$$

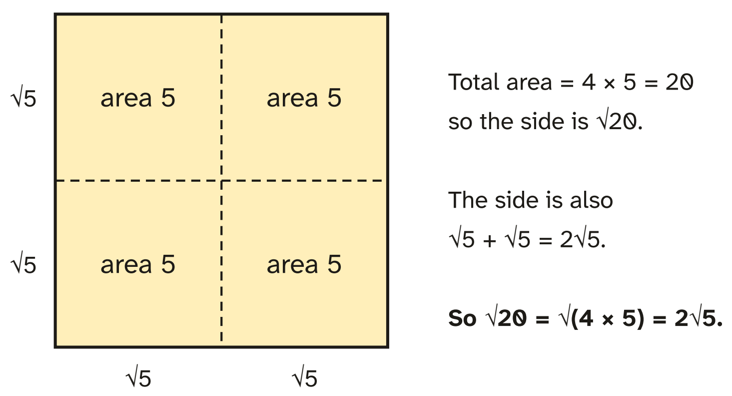
Surds (Extended only)

A **surd** is a root that isn't a whole number or fraction, such as $\sqrt{2}$, $\sqrt{5}$ or $\sqrt{20}$. It is irrational, so the only way to write its **exact** value is with the root sign. Questions that say "exact" or that are on the non-calculator paper want the surd, not a decimal.

The key fact is that roots multiply and divide nicely:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}, \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}, \qquad \sqrt{a} \times \sqrt{a} = a.$$

Simplifying a surd means taking out the largest square factor. $20 = 4 \times 5$ and 4 is a square, so $\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$. The picture shows why this is true.



A surd is in its simplest form when the number under the root has no square factor except 1: $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$. If you take out a smaller square, such as $\sqrt{72} = 2\sqrt{18}$, keep going: $2\sqrt{18} = 2 \times 3\sqrt{2} = 6\sqrt{2}$.

Adding and subtracting. Surds behave like letters: $5\sqrt{3} + 2\sqrt{3} = 7\sqrt{3}$, just as $5x + 2x = 7x$. You can only collect **the same surd**, so simplify each one first: $\sqrt{50} - \sqrt{8} = 5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$. But $\sqrt{a} + \sqrt{b}$ is **not** $\sqrt{a+b}$: $\sqrt{9} + \sqrt{16} = 7$, while $\sqrt{25} = 5$.

Multiplying. Multiply the whole numbers together and the roots together: $2\sqrt{3} \times 5\sqrt{12} = 10\sqrt{36} = 60$.

Expanding brackets. Multiply every term in the first bracket by every term in the second, then collect the whole numbers and the surds:

$$(3 - \sqrt{2})(1 + 2\sqrt{2}) = 3 + 6\sqrt{2} - \sqrt{2} - 2 \times 2 = -1 + 5\sqrt{2}.$$

The last term is where marks are lost: $-\sqrt{2} \times 2\sqrt{2} = -2 \times (\sqrt{2} \times \sqrt{2}) = -2 \times 2 = -4$.

Rationalising the denominator (Extended only)

To **rationalise the denominator** means to rewrite a fraction so that there is no surd on the bottom. You multiply the top and bottom by the same thing, which doesn't change the value of the fraction (it is the same as multiplying by 1).

- **A single surd on the bottom:** multiply top and bottom by that surd, because $\sqrt{a} \times \sqrt{a} = a$:

$$\frac{14}{\sqrt{7}} = \frac{14 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{14\sqrt{7}}{7} = 2\sqrt{7}.$$

- **A number plus or minus a surd on the bottom:** multiply top and bottom by the same two terms with the **sign in the middle changed**. The bottom becomes a difference of two squares, and the surd disappears: $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$.

$$\frac{6}{4 - \sqrt{10}} = \frac{6(4 + \sqrt{10})}{(4 - \sqrt{10})(4 + \sqrt{10})} = \frac{6(4 + \sqrt{10})}{16 - 10} = \frac{6(4 + \sqrt{10})}{6} = 4 + \sqrt{10}.$$

Always finish by simplifying: cancel a common factor of the **whole** top and the bottom, and simplify any surd.

Key facts and formulas

None of these is on the List of formulas printed in the exam paper, so you must know them all.

Formula	Status
Squares 1^2 to 15^2, cubes 1^3 to 5^3 and 10^3, and their roots	Learn it
$a^m \times a^n = a^{m+n}$	Learn it
$a^m \div a^n = a^{m-n}$	Learn it
$(a^m)^n = a^{mn}$	Learn it
$a^0 = 1$ (for $a \neq 0$)	Learn it
$a^{-n} = \dfrac{1}{a^n}$ and $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$	Learn it
Extended: $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$	Learn it
Extended: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$, $\sqrt{a} \times \sqrt{a} = a$	Learn it
Extended: $\dfrac{k}{\sqrt{a}} = \dfrac{k\sqrt{a}}{a}$	Learn it
Extended: $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$	Learn it

How to do it

In the questions we have tagged for this topic (47 questions from Papers 1–4, 2021–2025), the types came up this often. 15 of the 47 questions mix two types, so the counts add up to more than 47. All the surd questions are from 2025, one on every Extended Paper 2 that year.

Question type	Questions
Working out powers and roots	29
Zero, negative and fractional indices and the rules of indices	20
Rationalising the denominator	7
Simplifying, multiplying and expanding surds	6

1. Working out powers and roots (29 questions)

These are usually one-mark parts of a long first question on Paper 3, or a short question on Paper 1.

1. **Read the symbol carefully**: $\sqrt{}$ is a square root, $\sqrt[3]{}$ a cube root, $\sqrt[4]{}$ a fourth root, and a small raised number is a power.
2. **Without a calculator**, use the squares and cubes you know: $\sqrt{144} = 12$, $\sqrt[3]{27} = 3$, $5^3 = 125$, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$.
3. **With a calculator**, type the whole calculation in one go and write down the full display.
4. **Round only if the question asks**, and check the answer is a sensible size.

Variations you will meet:

- **Roots of decimals**: $\sqrt{19.36} = 4.4$, $\sqrt{0.0049} = 0.07$ (check: $0.07^2 = 0.0049$). A fourth root such as $\sqrt[4]{0.0625} = 0.5$ is **not** 4×0.0625.
- **Large roots on a calculator**: $\sqrt[3]{9261} = 21$, $\sqrt{2209} = 47$.
- **"Give your answer correct to 3 significant figures"**: $7^6 = 117\,649 \approx 118\,000$. Write the full value too, because it earns a mark on its own.
- **A zero index in a list of parts**: "Find the value of 9^0" is 1 (see type 2).
- **Negative numbers and powers**: "the sum of 5^2 and -5^2" is $25 + (-25) = 0$, because -5^2 means $-(5^2)$.
- **Picking a square or cube number from a list**, or finding a number from its root ("I square root a number and get 12"): $12^2 = 144$.
- **Combinations**: $\sqrt{16} \times \sqrt{25} = 4 \times 5 = 20$, or $6^3 \div 2^3 = 216 \div 8 = 27$. Work out each power first, then combine.

2. Zero, negative and fractional indices and the rules of indices (20 questions)

Finding a value:

1. **If the bases are the same, combine first** with the rules: add for $\times$, subtract for $\div$, multiply for a power of a power.
2. **Zero index**: the answer is 1.
3. **Negative index**: write the reciprocal, $a^{-n} = \frac{1}{a^n}$.
4. **Fractional index (Extended)**: bottom = root, top = power. Take the root first.
5. Give the answer as a whole number or a fraction, such as $\frac{1}{64}$, unless the question asks for a decimal.

For example, $5^3 \div 5^{-1} = 5^{3-(-1)} = 5^4 = 625$, and $32^{-\frac{2}{5}} = \frac{1}{(\sqrt[5]{32})^2} = \frac{1}{2^2} = \frac{1}{4}$.

Variations you will meet:

- **"Write in the form 3^n"**: turn every number into a power of 3 first. $9 \times 27^2 = 3^2 \times (3^3)^2 = 3^2 \times 3^6 = 3^8$. The answer is the power, not its value.
- **Dividing by a negative power**: $\frac{5}{2^{-3}} = 5 \times 2^3 = 40$.
- **Find the missing index**: "$2^6 \times 2^x = 2^{10}$" gives $6 + x = 10$, so $x = 4$.
- **Write a number as a power**: $0.001 = \frac{1}{1000} = 10^{-3}$, and $\frac{1}{64} = \frac{1}{4^3} = 4^{-3}$.
- **Solve an equation with the unknown in the index**: write both sides as powers of the same number and compare the indices. $25^x = \frac{1}{125}$ becomes $5^{2x} = 5^{-3}$, so $2x = -3$ and $x = -\frac{3}{2}$.
- **Powers of 10 with a root**: simplify inside the root with the rules, then halve the index for a square root. $$\sqrt{\frac{10^{500}}{10^{120} \times 10^{80}}} = \sqrt{10^{300}} = 10^{150}.$$
- **The unknown is the base (Extended)**: collect the powers of the letter on one side, then undo the index with its reciprocal power. $m^{-\frac{1}{2}} = 4m^{-1}$ gives $m^{-\frac{1}{2}} \div m^{-1} = 4$, so $m^{\frac{1}{2}} = 4$ and $m = 4^2 = 16$.
- **An expression with a letter in the index**: $32 \times 4^n = 2^5 \times 2^{2n} = 2^{5+2n}$.

3. Rationalising the denominator (7 questions)

1. **Look at the bottom.** A single surd $\sqrt{a}$ (or $k\sqrt{a}$): multiply top and bottom by $\sqrt{a}$. A number and a surd, $p + \sqrt{q}$: multiply top and bottom by $p - \sqrt{q}$ (and by $p + \sqrt{q}$ if the bottom is $p - \sqrt{q}$).
2. **Multiply out the bottom**: $\sqrt{a} \times \sqrt{a} = a$, or $(p + \sqrt{q})(p - \sqrt{q}) = p^2 - q$. Leave the top in brackets for now.
3. **Simplify**: cancel any common factor of the number outside the bracket and the bottom, and simplify any surd.

For example, $\frac{12}{\sqrt{6}} = \frac{12\sqrt{6}}{6} = 2\sqrt{6}$, and $\frac{9}{5+\sqrt{7}} = \frac{9(5-\sqrt{7})}{25-7} = \frac{9(5-\sqrt{7})}{18} = \frac{5-\sqrt{7}}{2}$.

Variations you will meet:

- **"Rationalise the denominator and simplify"**: the final cancelling step is a mark on its own. Answers such as $\frac{15(6-\sqrt{11})}{25}$ are not finished: cancel by 5 to get $\frac{3(6-\sqrt{11})}{5}$.
- **Top of 1**: $\frac{1}{\sqrt{5}+2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5} - 2$.
- **A surd on the top as well**: $\frac{6}{\sqrt{10}} = \frac{6\sqrt{10}}{10} = \frac{3\sqrt{10}}{5}$. Cancel 6 and 10 by 2, but never cancel a number with the number inside a root.

4. Simplifying, multiplying and expanding surds (6 questions)

1. **Simplify each surd**: find the largest square factor of the number under the root $(4, 9, 16, 25, 36, 49, \ldots)$ and take its root outside. $\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$.
2. **Add or subtract like surds** by adding the numbers in front: $\sqrt{48} + \sqrt{75} = 4\sqrt{3} + 5\sqrt{3} = 9\sqrt{3}$.
3. **Multiply** numbers with numbers and roots with roots, then simplify the root.

Variations you will meet:

- **Expand** $(a + b\sqrt{n})(c + d\sqrt{n}) = p + q\sqrt{n}$ **and find** p **and** q: expand all four terms, remember $\sqrt{n} \times \sqrt{n} = n$, then collect. For $(3 - \sqrt{2})(1 + 2\sqrt{2})$ the four terms are 3, $6\sqrt{2}$, $-\sqrt{2}$ and -4, so $p = -1$ and $q = 5$. Write both values on the answer lines.
- **Area of a rectangle or square with surd sides**: multiply the sides. A rectangle $2\sqrt{3}$ by $5\sqrt{12}$ has area $10\sqrt{36} = 60$. A square of side $\sqrt{5} + 1$ has area $(\sqrt{5} + 1)^2 = 5 + 2\sqrt{5} + 1 = 6 + 2\sqrt{5}$.
- **"Simplify" with one term**: $\sqrt{98} = 7\sqrt{2}$. Give the smallest possible number under the root.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

This question is from a calculator paper.

(a) Write down the value of $\sqrt{121}$. **[1]**

(b) Work out $-2^4 + (-2)^3$. **[1]**

(c) Calculate 2.3^4. Give your answer correct to 3 significant figures. **[2]**

(d) Calculate $\sqrt[5]{7776}$. **[1]**

(e) Find the value of $9^0 + 4^{-2}$. **[2]**

(a) $11^2 = 121$, so $\sqrt{121} = 11$. **B1**

(b) $-2^4 = -(2^4) = -16$ and $(-2)^3 = -8$, so the total is $-16 + (-8) = -24$. **B1**

(c) $2.3^4 = 27.9841$. To 3 significant figures: 28.0 (the zero shows the third significant figure). **B2**, or **B1** for $27.98\ldots$ seen.

(d) $\sqrt[5]{7776} = 6$. **B1** Check: $6^5 = 7776$.

(e) $9^0 = 1$ and $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$. **M1** for either one correct.

$1 + \frac{1}{16} = \frac{17}{16}$ (or $1\frac{1}{16}$, or 1.0625). **A1**

Example 2

Calculators must not be used in this question. Parts (c) to (e) are Extended.

(a) Find the value of $5^7 \div 5^4$. **[1]**

(b) Write $8^3 \times 16 \div 2^5$ in the form 2^n. **[2]**

(c) Find the value of $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$. **[2]**

(d) Find the value of x when $9^x = \frac{1}{27}$. **[2]**

(e) $\sqrt{\frac{10^{360}}{10^{100} \times 10^{60}}} = 10^k$. Find the value of k. **[3]**

(a) $5^{7-4} = 5^3 = 125$. **B1**

(b) $8^3 = (2^3)^3 = 2^9$ and $16 = 2^4$. **M1** for writing every number as a power of 2.

$2^9 \times 2^4 \div 2^5 = 2^{9+4-5} = 2^8$. **A1**

(c) The minus sign turns the fraction upside down: $\left(\frac{8}{27}\right)^{\frac{2}{3}}$. The cube root of $\frac{8}{27}$ is $\frac{2}{3}$. **M1** for either the reciprocal or the cube root.

$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$. **A1**

(d) $9 = 3^2$ and $\frac{1}{27} = 3^{-3}$, so $3^{2x} = 3^{-3}$. **M1**

$2x = -3$, so $x = -\frac{3}{2}$. **A1**

(e) Bottom: $10^{100} \times 10^{60} = 10^{160}$. **M1** for any correct first step.

Inside the root: $10^{360} \div 10^{160} = 10^{200}$. **M1**

$\sqrt{10^{200}} = 10^{100}$, so $k = 100$. **A1** (A square root halves the index, because $10^{100} \times 10^{100} = 10^{200}$.)

Example 3

This question is Extended. Calculators must not be used.

(a) Simplify $\sqrt{72} - \sqrt{18}$. **[2]**

(b) $(4+\sqrt{3})(2-\sqrt{3}) = a + b\sqrt{3}$. Find the value of a and the value of b. **[2]**

(c) Rationalise the denominator of $\frac{15}{\sqrt{5}}$. Give your answer in its simplest form. **[2]**

(d) Rationalise the denominator and simplify $\frac{11}{5-\sqrt{3}}$. **[3]**

(a) $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$ and $\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$. **B1** for either.

$6\sqrt{2} - 3\sqrt{2} = 3\sqrt{2}$. **B1**

(b) $4 \times 2 = 8$, $4 \times (-\sqrt{3}) = -4\sqrt{3}$, $\sqrt{3} \times 2 = 2\sqrt{3}$ and $\sqrt{3} \times (-\sqrt{3}) = -3$.

$8 - 3 - 4\sqrt{3} + 2\sqrt{3} = 5 - 2\sqrt{3}$, so $a = 5$ **B1** and $b = -2$ **B1**. (If one value is wrong, three correct terms from the expansion still earn **B1**.)

(c) $\frac{15}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{15\sqrt{5}}{5}$. **M1**

$= 3\sqrt{5}$. **A1**

(d) Multiply top and bottom by $5 + \sqrt{3}$. **M1**

Bottom: $(5-\sqrt{3})(5+\sqrt{3}) = 25 - 3 = 22$. **M1**

$\frac{11(5+\sqrt{3})}{22} = \frac{5+\sqrt{3}}{2}$. **A1**

Common mistakes

- **Halving or doubling instead of rooting.** $\sqrt{36}$ is 6, not 18 or 72, and $\sqrt[3]{27}$ is 3, not 9. A root asks "what number multiplied by itself gives this?"
- **Multiplying the base by the index.** 2^5 is 32, not 10; $125^{\frac{2}{3}}$ is not $125 \times \frac{2}{3}$. A power is repeated multiplication.
- **Answering 0 or the base for a zero index.** $8^0 = 1$, not 0 or 8.
- **Making a negative index give a negative answer.** $2^{-3} = \frac{1}{8}$, not -8 or -6. The minus means "one over".
- **Stopping part-way with a negative fractional index.** Examiners have seen negative fractional indices worked as far as the power and then left there, or given a minus sign. For example, $36^{-\frac{3}{2}}$ is not 216 or -216: finish with the reciprocal, $\frac{1}{216}$.
- **Mixing up the rules.** When you multiply powers of the same base, add the indices ($y^4 \times y^6 = y^{10}$, not y^{24}); for a power of a power, multiply them. Subtract in the right order when dividing: $p^5 \div p^8 = p^{-3}$, not p^3.
- **Combining different bases.** $3^5 \times 9^2$ is not 27^7 or 3^7. Write every number as a power of the same prime first: $3^5 \times (3^2)^2 = 3^5 \times 3^4 = 3^9$.
- **Misreading $\sqrt[4]{\ }$.** $\sqrt[4]{0.0081}$ is the fourth root (0.3), not 4×0.0081 and not the square root (0.09).
- **Forgetting the order of operations.** $-3^2 = -9$, but $(-3)^2 = 9$.

- **Rounding a calculator answer too far, or to the wrong size.** $13^5 = 371\,293$ to 3 significant figures is $371\,000$, not 371 or $370\,000$.
- **Surd arithmetic slips.** $\sqrt{2} \times \sqrt{2} = 2$, not 4 and not left as $\sqrt{4}$; $-\sqrt{7} \times 2\sqrt{7} = -14$; $\sqrt{20}$ is $2\sqrt{5}$, not $4\sqrt{5}$ (the 4 comes out as its root, 2).
- **Adding roots as if they were one root.** $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$.
- **Rationalising with the wrong multiplier.** For $\frac{15}{6+\sqrt{11}}$, multiply by $\frac{6-\sqrt{11}}{6-\sqrt{11}}$, not by $\frac{\sqrt{11}}{\sqrt{11}}$. And never cancel a number outside a root with the number inside it.
- **Not comparing both parts.** In "$= a + b\sqrt{5}$", once you have expanded to, say, $-4 + 11\sqrt{5}$, read off $a = -4$ and $b = 11$; don't try to rearrange an equation for a.

How to get the marks

- **"Write down"** means no working is needed: recall the square, cube or root. **"Work out"**, **"Find the value of"** and **"Calculate"** want a single number.
- **Exact answers.** On a non-calculator paper, give fractions and surds, such as $\frac{1}{64}$ or $3\sqrt{5}$, not rounded decimals. An exact decimal such as 0.0625 is also fine.
- **Show the steps on the non-calculator paper.** With an index question worth 2 or 3 marks, the method mark is often for a correct middle step, such as $16^{\frac{3}{2}} = 64$, $125^{\frac{1}{3}} = 5$ or $27 = 3^3$. Write it down.
- **Accuracy.** "Correct to 3 significant figures" means write the full calculator value first (it usually earns a mark), then round. If the answer is exact, give the exact value.
- **"In the form 3^n"** wants the power 3^{11}, not its value.
- **"Simplify"** a surd means the smallest possible number under the root, with like surds collected, for example $3\sqrt{5}$.
- **"Rationalise the denominator"** means no root left on the bottom. **"Give your answer in its simplest form"** or **"and simplify"** means cancel any common factor too; the last mark is often for this.
- **"Find the value of a and the value of b"** needs both numbers on the answer lines. Each correct value usually scores, so give both even if you are unsure of one.
- **"Show that"** (for example, that an expression equals a given surd) needs every step, ending with exactly the printed answer.

Quick check

1. Without a calculator, work out $\sqrt{144} - \sqrt[3]{64}$.
2. Find the value of (a) $6^{-2} \times 6^3$, (b) $\dfrac{3}{2^{-2}}$.
3. (Extended) Find the value of $81^{-\frac{3}{4}}$.
4. (Extended) Simplify $\sqrt{50} + \sqrt{32}$.
5. (Extended) Rationalise the denominator and simplify $\dfrac{6}{3+\sqrt{7}}$.

Answers

1. $\sqrt{144} = 12$ and $\sqrt[3]{64} = 4$, so $12 - 4 = 8$.
2. (a) $6^{-2} \times 6^3 = 6^{-2+3} = 6^1 = 6$. (b) $\frac{3}{2^{-2}} = 3 \times 2^2 = 12$.
3. $81^{-\frac{3}{4}} = \frac{1}{(\sqrt[4]{81})^3} = \frac{1}{3^3} = \frac{1}{27}$.
4. $\sqrt{50} = 5\sqrt{2}$ and $\sqrt{32} = 4\sqrt{2}$, so the sum is $9\sqrt{2}$.
5. $\frac{6(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})} = \frac{6(3-\sqrt{7})}{9-7} = 3(3-\sqrt{7}) = 9 - 3\sqrt{7}$.

Past questions to try

- **0580/31/O/N/21 Q1**: squares, cubes, roots and a zero index in a first Core question.
- **0580/22/M/J/25 Q17**: fractional and negative fractional indices without a calculator.
- **0580/23/O/N/25 Q12**: zero and negative indices, and writing a product as a power of 3.
- **0580/21/O/N/25 Q21**: expanding a bracket of surds and rationalising a denominator.