Cambridge IGCSE · Mathematics 0580 · Notes

Probability and expected frequency

Read online at **pastopic.com/cambridge/mathematics-0580/notes/probability-basics**

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Read first: **Fractions, decimals, ordering and the four operations**, **Ratio, proportion and rates**

What you need to know

By the end of this topic you should be able to:

1. **Use the probability scale from 0 to 1.** Place a probability on the scale, read one from it, and describe it in words (impossible, unlikely, even chance, likely, certain).
2. **Work out the probability of a single event**, such as taking a green ball from a bag or a spinner landing on 3, using information given in words, tables, charts or Venn diagrams. Give it as a fraction, a decimal or a percentage.
3. **Use the fact that the probability of an event not happening is 1 minus the probability that it happens.** This also means that the probabilities of all the possible outcomes add up to 1.
4. **Understand relative frequency** (how often something happened in an experiment, as a fraction of the number of trials) as an estimate of a probability, and know what **fair** and **biased** mean.
5. **Calculate expected frequencies**: how many times you expect an event to happen in a number of trials, or how many people in a population you expect to have some feature.
6. **Extended only:** read and write probability notation: $P(A)$ is the probability of event A, and $P(A')$ is the probability of A **not** happening.

Two or more events together (tree diagrams, "both", picking twice, with or without replacement) and conditional probability are in the note on combined events and conditional probability.

Understand it

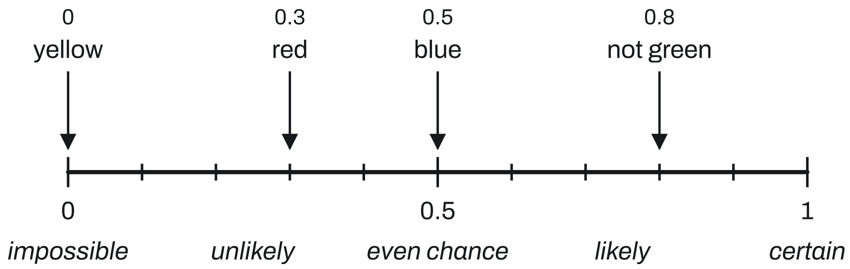
What a probability is

A **probability** is a number that measures how likely something is to happen. The "something" is an **event**, such as "the dice shows a 6"; each possible result of the experiment is an **outcome**.

Every probability is between 0 and 1:

- 0 means the event is **impossible** (a normal dice can't show 7);
- 1 means it is **certain** (the dice shows a number less than 7);
- $\frac{1}{2} = 0.5$ is an **even chance**, like a fair coin landing on heads.

A probability can never be negative or bigger than 1. If you get an answer like 1.3 or $\frac{12}{7}$, something has gone wrong.



On a **probability scale**, work out the probability first, then find it on the line. Count the marks carefully: if the line from 0 to 1 is split into 6 equal parts, each mark is $\frac{1}{6}$; if it is split into 10, each mark is 0.1; if into 8, each is $\frac{1}{8} = 0.125$.

Equally likely outcomes

When every outcome is **equally likely** (a fair dice, a fair spinner with equal sections, a ball taken "at random" from a bag),

$$\text{probability of an event} = \frac{\text{number of outcomes where the event happens}}{\text{total number of outcomes}}.$$

For the bag in the diagram, with 3 red, 5 blue and 2 green counters (10 in total):

- probability of red $= \frac{3}{10} = 0.3$;
- probability of blue $= \frac{5}{10} = \frac{1}{2}$;
- probability of yellow $= \frac{0}{10} = 0$, because there are no yellow counters.

The same idea works with any list of equally likely outcomes. On a fair 12-sided spinner numbered 1 to 12, "a prime number" happens for 2, 3, 5, 7 and 11, so its probability is $\frac{5}{12}$. If the numbers on a fair spinner repeat (say 1, 2, 2, 2, 5), count every section: the probability of 2 is $\frac{3}{5}$.

Ratios. If a box has blue and yellow pencils in the ratio $4 : 5$, then out of every $4 + 5 = 9$ pencils, 4 are blue, so the probability of blue is $\frac{4}{9}$. A probability is **never written as a ratio**: $4 : 9$ or $4 : 5$ earns nothing.

Tables, charts and Venn diagrams. Read off the count for the event and the right total. The words after "picked at random from..." tell you which total to divide by. In this table,

	Walk	Bus	Total
Boys	18	22	40
Girls	27	13	40
Total	45	35	80

"a student picked at random walks" is $\frac{45}{80}$, but "a **girl** picked at random walks" is $\frac{27}{40}$, because only the 40 girls are being picked from. The same goes for a Venn diagram: the total is every number inside the rectangle, including the one outside the circles.

"Not happening" and the total of 1

Something either happens or it doesn't, so the two probabilities add up to 1:

$$\text{probability that an event does not happen} = 1 - \text{probability that it happens}.$$

If the probability of rain tomorrow is 0.38, the probability of no rain is $1 - 0.38 = 0.62$. With the counters above, "not green" is $1 - \frac{2}{10} = \frac{8}{10} = 0.8$, the same as counting the 8 counters that are red or blue.

More generally, if a list of outcomes covers **everything** that can happen and no two can happen together, their probabilities **add up to** 1. That is how you fill a gap in a probability table: for a spinner with colours red 0.15, blue 0.45 and green, green must be $1 - (0.15 + 0.45) = 0.4$.

The probability of "red **or** blue" on one spin is the sum of the two: $0.15 + 0.45 = 0.6$, which is also $1 - 0.4$ (not green).

Fair and biased

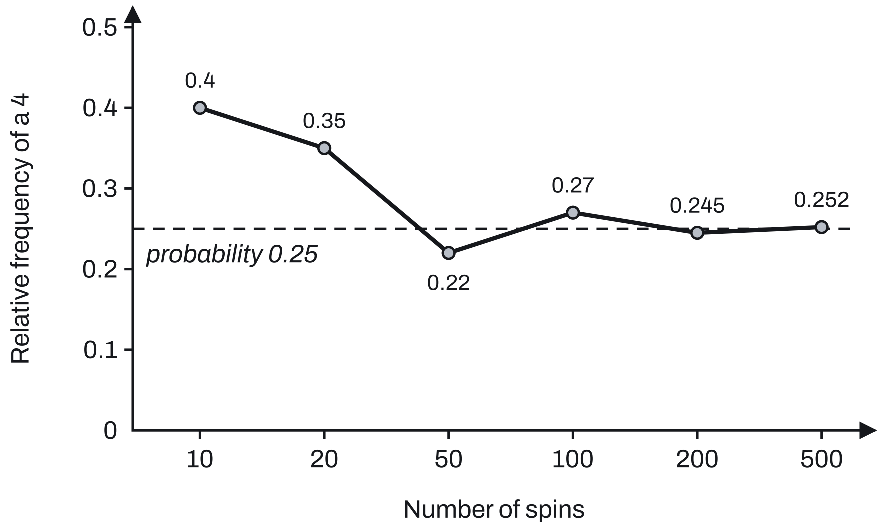
A dice, coin or spinner is **fair** if every outcome is equally likely. It is **biased** if some outcomes are more likely than others (for example a weighted dice). For a biased spinner you can't count sections; you have to be told the probabilities or estimate them from an experiment.

Relative frequency

If you can't work a probability out by counting, do an experiment. Repeat it many times (each go is a **trial**) and count how often the event happens:

$$\text{relative frequency} = \frac{\text{number of times the event happens}}{\text{number of trials}}.$$

A relative frequency is an **estimate** of the probability. The more trials, the better the estimate: with only a few trials it jumps about, but with many trials it settles down close to the true probability.



This is how you decide whether something might be biased. A fair four-sided spinner should give each number about $\frac{1}{4}$ of the time. If, after 500 spins, a 4 came up 250 times (relative frequency 0.5), the spinner is very likely biased towards 4. A small difference after only 20 spins proves nothing.

A survey works the same way. If 45 out of 120 people asked walk to work, the relative frequency $\frac{45}{120} = 0.375$ estimates the probability that a person picked at random from the same population walks to work.

Expected frequency

If the probability of an event is p and you do n trials, you **expect** it to happen

$$\text{expected frequency} = p \times n \text{ times.}$$

A fair dice rolled 300 times is expected to show a 6 about $\frac{1}{6} \times 300 = 50$ times. It won't be exactly 50 every time; "expected" means the number you would predict.

The same rule estimates a number in a population. If 45 out of 120 people surveyed walk to work, then in a town of 2000 working people you would expect about $\frac{45}{120} \times 2000 = 750$ to walk to work.

An expected frequency is a **number of times** (or of people, cars, games...), not a probability. It is often a whole number, but it doesn't have to be.

Probability notation (Extended only)

Extended only: events are named with capital letters and $P(\ldots)$ means "the probability of". If R is the event "the counter is red", then $P(R) = 0.3$, and $P(R')$, the probability that the counter is **not** red, is $1 - P(R) = 0.7$. Core candidates can write everything in words instead.

Key facts and formulas

None of these is on the List of formulas printed in the exam paper, so you must know them all.

Formula	Status
$0 \leqslant \text{probability} \leqslant 1$; 0 is impossible, 1 is certain	Learn it
Equally likely outcomes: $\text{probability} = \dfrac{\text{number of ways the event happens}}{\text{total number of outcomes}}$	Learn it
Probability of not happening $= 1 - \text{probability of happening}$; Extended: $P(A') = 1 - P(A)$	Learn it
The probabilities of all the possible outcomes add up to 1	Learn it
One of several outcomes on a single trial ("red or blue"): add their probabilities	Learn it
$\text{relative frequency} = \dfrac{\text{number of times the event happens}}{\text{number of trials}}$	Learn it
$\text{expected frequency} = \text{probability} \times \text{number of trials}$	Learn it
Ratio $a : b$ of two kinds: probability of the first kind $= \dfrac{a}{a+b}$	Learn it

How to do it

In the questions we have tagged for this topic (79 questions from Papers 1–4, 2021–2025, six of them printed on two papers), the types came up this often. Three of the 79 are Venn diagram or dice-results parts that ask no probability question, so the table counts the other 76. 18 of those 76 mix two or three types, so the counts add up to more than 76.

Question type	Questions
Probability of a single event from counts, tables, charts or Venn diagrams	23
Missing probability in a table	22
Expected frequency	21
Probability that an event does not happen	18
The probability scale	5
Working backwards from a probability	5
Relative frequency from results	1

1. Probability of a single event from counts, tables, charts or Venn diagrams (23 questions)

1. **Find the total** of equally likely outcomes: all the balls in the bag, all the sections of the spinner, or the right row, column or set total in a table or Venn diagram.
2. **Count the outcomes where the event happens.** List them if it helps ("multiples of 4 from 1 to 30: 4, 8, 12, 16, 20, 24, 28", so 7 out of 30).
3. **Write count $\div$ total** as a fraction (or decimal or percentage). Simplify if you like; you only have to when the question says "in its simplest form".

Variations you will meet:

- **"Or" on one pick**, such as "red or blue": add the counts (or the probabilities).
- **Most likely / least likely outcome**: the one with the most (or fewest) equally likely sections or items. On a fair spinner with sections 2, 4, 4, 4, 7, 9, the most likely number is 4.
- **Frequency tables** (for example words and their numbers of letters): "more than 3 letters" means add the frequencies for 4, 5, 6, ... and divide by the total frequency.
- **Two-way tables**: "a girl, picked at random" divides by the number of girls; "a person, picked at random from those who ..." divides by that group's total.
- **Venn diagrams**: add the numbers in the regions you want and divide by the total of **every** number in the diagram, including the one outside the circles.
- **Pie charts and bar charts**: find the frequency first (from the angle: $\frac{\text{angle}}{360} \times \text{total}$, or from the bar's height).
- **"Explain why the probability is $\frac{1}{5}$"**: list the outcomes that work (for example the square numbers 1, 4, 9, 16 on a fair spinner numbered 1 to 20) and say that there are 20 equally likely outcomes, and $\frac{4}{20} = \frac{1}{5}$.

2. Missing probability in a table (22 questions)

1. **Add the probabilities you are given.**
2. **Take the sum from 1.** Show this line: $1 - (0.22 + 0.14 + 0.31 + 0.08) = 0.25$ earns a method mark even if you slip later.
3. **If two gaps share what is left**, split it in the ratio given. For example, if 0.42 is left and blue : green $= 4 : 3$, then blue $= \frac{4}{7} \times 0.42 = 0.24$ and green $= \frac{3}{7} \times 0.42 = 0.18$. Check the two add up to 0.42, and put each in the right box.

Variations you will meet:

- **"A is half as likely as B"**: if $A = x$ and $B = 2x$, then $3x$ is what is left, so $x = \frac{\text{what is left}}{3}$.
- **"There are four times as many blue beads as white beads"** is the ratio blue : white $= 4 : 1$, so blue gets $\frac{4}{5}$ of what is left.
- **The missing value is a letter**, such as "find x": it is the same calculation.
- **Relative frequency tables** work the same way: relative frequencies of all the outcomes also add up to 1.
- **"Red or blue" from the table**: add the two probabilities.
- **Given "not" probabilities**: if P(not orange) $= 0.8$ and P(not purple) $= 0.7$, then P(orange) $= 0.2$ and P(purple) $= 0.3$, so the only other colour has $1 - (0.2 + 0.3) = 0.5$.
- **Fractions in the table**: use a common denominator, for example $1 - \left(\frac{1}{3} + \frac{1}{4}\right) = 1 - \frac{7}{12} = \frac{5}{12}$.

3. Expected frequency (21 questions)

1. **Find the probability** of the event: given, counted, read from a table, or a relative frequency from a survey or experiment.
2. **Multiply by the number of trials** (spins, games, days, cars, people): expected frequency $= p \times n$.
3. **Give a number of times** that makes sense in the context; don't leave it as a probability.

For example, if the probability that a bus is late is 0.08, over 350 journeys you would expect $0.08 \times 350 = 28$ late buses.

Variations you will meet:

- **From a sample to a population**: "84 out of 600 people surveyed wear glasses; how many in a town of 9000?" is $\frac{84}{600} \times 9000 = 1260$. Use the fraction as it is; if it isn't an exact decimal (such as $\frac{7}{30}$), rounding it early gives a wrong answer.
- **"4 of the 50 bulbs tested are faulty"** means $\frac{4}{50}$, not $\frac{4}{54}$: the 4 faulty ones are part of the 50.
- **A probability you work out in an earlier part** (often from a table): use that value, even if it has been rounded in the question.
- **Read which event is asked for**: expected number who **pass**, not fail; expected **wins**, not losses.

4. Probability that an event does not happen (18 questions)

1. Find the probability that the event **does** happen.
2. Take it from 1: $1 - 0.58 = 0.42$; $1 - \frac{3}{11} = \frac{8}{11}$.

Variations you will meet:

- **From counts**: 7 of 25 bikes are red, so "not red" $= \frac{18}{25}$ (either $1 - \frac{7}{25}$ or count the 18 bikes that are not red).
- **The other way round**: if P(does not score) $= 0.91$, then P(scores) $= 0.09$.
- **Extended only:** in notation, $P(C) = 0.61$ means $P(C') = 0.39$.

5. The probability scale (5 questions)

1. **Work out the probability** as a fraction or decimal.
2. **Find it on the scale.** Count how many equal parts the line from 0 to 1 is split into; each mark is $1 \div$ that number. Draw the arrow exactly on a mark (or exactly halfway if the value is there), and label it if asked.
3. For "which arrow shows...", work out each probability, then match it to the nearest letter in the right order.

Impossible events go at 0 and certain events at 1. A "not" probability such as "not blue" is usually near the right-hand end.

Variation: the arrow is given and you find a number. If the arrow shows 0.4 and there are 14 red beads, then 14 is 0.4 of the total, so the total is $14 \div 0.4 = 35$.

6. Working backwards from a probability (5 questions)

Here you know a probability and have to find how many items there are.

1. **Write the probability as count $\div$ total**, using a letter for what you don't know.
2. **Solve** the equation, or use the fraction to find the total directly.

For example, a box has 6 red and x blue pens, and the probability of red is $\frac{2}{9}$. Then the total is $6 \div \frac{2}{9} = 27$ pens, so $x = 27 - 6 = 21$. With letters in both places, cross-multiply: $\frac{3}{9+x} = \frac{1}{5}$ gives $15 = 9 + x$, so $x = 6$.

Variations you will meet:

- **Find another probability afterwards**: once you know every count, the probability of yellow (say) is its count over the new total.
- **The count of one colour comes from a probability**: 48 counters and P(green) $= \frac{5}{12}$ means 20 green counters.
- **Comparing two bags**: compare the **probabilities**, not the numbers of items. A bag with 5 black out of 12 (0.417) is **less** likely to give black than one with 3 black out of 7 (0.429), even though it has more black marbles.
- **Adding items to make two probabilities equal**: write the new probability with the added items and set it equal to the target fraction.

7. Relative frequency from results (1 question)

1. Find the number of times the event happened and the total number of trials (add the frequencies if needed).
2. Relative frequency $= \frac{\text{number of times}}{\text{number of trials}}$, as a fraction, decimal or percentage. The answer is **not** the frequency itself.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

A spinner can land on red, blue, green or yellow. The table shows some of the probabilities.

Colour	Red	Blue	Green	Yellow
Probability	0.31	0.14		

The probabilities of green and yellow are in the ratio green : yellow $= 4 : 7$.

(a) Complete the table. **[3]**

(b) Anya spins the spinner 80 times. Find the number of times she expects it to land on yellow. **[1]**

(a) Green and yellow together: $1 - (0.31 + 0.14) = 1 - 0.45 = 0.55$. **M1**

Split 0.55 in the ratio $4 : 7$ (11 parts, so each part is 0.05):

green $= 4 \times 0.05 = 0.2$, yellow $= 7 \times 0.05 = 0.35$. **A1 A1** (one for each value, in the right box)

Check: $0.31 + 0.14 + 0.2 + 0.35 = 1$.

(b) $0.35 \times 80 = 28$ times. **B1**

Example 2

A bag contains 4 red counters, 6 blue counters and some white counters. A counter is taken from the bag at random. The probability that it is white is $\frac{3}{8}$.

(a) Find the number of white counters in the bag. **[3]**

(b) Find the probability that the counter is **not** red. **[1]**

(a) Let there be w white counters, so there are $10 + w$ counters in total.

$$\frac{w}{10 + w} = \frac{3}{8}$$

M1 for a correct equation.

$8w = 3(10 + w) \Rightarrow 8w = 30 + 3w \Rightarrow 5w = 30$ **M1**, so $w = 6$. **A1**

(Check: $\frac{6}{16} = \frac{3}{8}$.)

(b) There are 16 counters, 4 of them red, so the probability of not red is $1 - \frac{4}{16} = \frac{12}{16} = \frac{3}{4}$. **B1**

Example 3

Leon thinks his six-sided dice may be biased. He rolls it 250 times. The table shows his results.

Score	1	2	3	4	5	6
Frequency	38	41	36	44	39	52

(a) Find the relative frequency of scoring a 6. **[1]**

(b) Leon says his dice is biased towards 6. Give a reason why he might be right. **[1]**

(c) Leon rolls the same dice another 1000 times. Use your answer to part (a) to estimate the number of times it shows a 6. **[1]**

(a) $\dfrac{52}{250} = 0.208$ **B1**

(b) For a fair dice the probability of a 6 is $\frac{1}{6} = 0.167$ (3 s.f.). After a large number of rolls the relative frequency, 0.208, is noticeably higher than this. **B1**

(c) $0.208 \times 1000 = 208$ times. **B1**

Example 4

In a survey, 45 out of 120 students at a school said they walk to school.

(a) The school has 1480 students. Estimate how many of them walk to school. **[2]**

(b) **Extended only:** W is the event "a student picked at random walks to school". Use the survey to write down an estimate of $P(W')$. **[1]**

(a) Relative frequency $= \frac{45}{120} = 0.375$. **M1** for $\frac{45}{120} \times 1480$

$0.375 \times 1480 = 555$ students. **A1**

(b) $P(W') = 1 - 0.375 = 0.625$. **B1**

Common mistakes

- **Writing a probability as a ratio or in words.** $3 : 8$, "3 in 8" and "3 out of 8" are not accepted. Write $\frac{3}{8}$, 0.375 or 37.5%.
- **Dividing by the wrong total.** "A **girl** picked at random" uses the number of girls, not everyone in the table. In a Venn diagram, don't forget the number outside the circles.
- **Guessing a missing table value from a pattern.** The gap is never "between" its neighbours or the next term of a sequence. The probabilities must add up to 1, so work it out as $1 - (\text{the others})$.
- **Getting the ratio split right but not adding to what is left**, or putting the two values in the wrong boxes. Check both conditions: the right ratio **and** the right total.
- **Place-value slips when adding decimals.** $0.4 + 0.35 = 0.75$, not 0.39. Line the decimal points up.
- **Giving a probability when the question asks for an expected number**, or dividing instead of multiplying. Expected frequency is probability $\times$ number of trials, and it is a count of things.
- **Answering for the wrong event.** "Expected number who pass" is not the number who fail. Underline the event in the question.
- **Miscounting the marks on a probability scale.** Count the equal parts between 0 and 1 first. $\frac{1}{8}$ on a line split into 8 is the first mark, not the second or the eighth; don't put the arrow between marks.
- **Writing the frequency instead of the relative frequency.** A relative frequency is a fraction of the trials: $\frac{9}{40}$, not 9.
- **Adding the sample to its part.** "4 of the 50 tested are faulty" is $\frac{4}{50}$; it is not $\frac{4}{54}$ and not $\frac{4}{46}$.
- **Comparing numbers instead of probabilities.** More black marbles doesn't mean a higher chance of black; work out and compare both probabilities.

- **Rounding too early.** Keep fractions like $\frac{7}{30}$ exact (or use the calculator's full value) until the final multiplication.

How to get the marks

- **Show the "1 minus" line.** For a missing probability, writing $1-(0.27+0.4+0.12)$ usually earns a method mark, and so does the total of the given ones (such as 0.79) even if the last step goes wrong.
- **Form of the answer.** A fraction, decimal or percentage is fine; a fraction doesn't have to be simplified unless the question says "simplest form". Never give a ratio.
- **"Write down"** means you can answer without working, but a mark scheme still needs the exact value, so don't round a fraction to a decimal that isn't exact.
- **Expected frequency marks follow through.** If part (a) asks for a probability and part (b) for an expected number, $(\text{your }(a))\times n$ still earns the mark in (b), as long as your probability is between 0 and 1. So always attempt (b).
- **Survey questions** give a method mark for $\frac{\text{count}}{\text{sample}}\times\text{population}$ written down, so show it before you calculate.
- **"Explain"** or **"give a reason"** needs numbers: list the outcomes that work and the total, or compare two probabilities with their values ("$\frac{5}{12}<\frac{3}{7}$, so the second bag").
- **Probability scale:** the arrow must be on the exact point, with the label asked for.
- **Accuracy:** give exact answers where you can. Otherwise use 3 significant figures, as for every non-exact answer on these papers.

Quick check

1. The probability that it snows on a given day is 0.07. Find the probability that it does not snow on that day.
2. A spinner can land on A, B, C or D. P(A) $=0.15$, P(B) $=0.4$ and P(C) $=0.3$. Find the probability that it lands on D.
3. A fair 10-sided spinner is numbered 1 to 10. (a) Find the probability that it lands on a multiple of 3. (b) It is spun 250 times. Find the expected number of times it lands on a multiple of 3.
4. A bag contains 5 red balls and x blue balls. The probability of taking a red ball at random is $\frac{1}{4}$. Find x.
5. In a game you can win, lose or draw. The probability of losing is 0.23, and winning is six times as likely as drawing. (a) Find the probability of winning. (b) You play 50 games. Find the expected number of wins.

Answers

1. $1 - 0.07 = 0.93$.
2. $1 - (0.15 + 0.4 + 0.3) = 1 - 0.85 = 0.15$.
3. (a) The multiples of 3 are 3, 6 and 9, so the probability is $\frac{3}{10} = 0.3$. (b) $0.3 \times 250 = 75$ times.
4. The total is $5 \div \frac{1}{4} = 20$ balls, so $x = 20 - 5 = 15$.
5. (a) Win and draw together: $1 - 0.23 = 0.77$. Win : draw $= 6 : 1$, so win $= \frac{6}{7} \times 0.77 = 0.66$ (and draw $= 0.11$). (b) $0.66 \times 50 = 33$ wins.

Past questions to try

- **0580/31/M/J/21 Q2**: comparing the probabilities from two bags.
- **0580/32/M/J/25 Q9**: a "not" probability, a table split in a ratio, then an expected number.
- **0580/22/O/N/21 Q7**: working backwards from a probability with algebra.
- **0580/12/O/N/22 Q19**: scaling a survey up to a whole town.