Cambridge IGCSE · Mathematics 0580 · Notes

Pythagoras' theorem and right-angled triangles

Read online at **pastopic.com/cambridge/mathematics-0580/notes/pythagoras-and-right-angled-triangles**

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Read first: **Powers, roots, indices and surds**, **Units, area and perimeter**

What you need to know

By the end of this topic you should be able to:

1. **Know and use Pythagoras' theorem**: in a right-angled triangle, find the third side from the other two, whether the missing side is the longest one or one of the shorter ones.
2. **Know and use sine, cosine and tangent** for the acute angles of a right-angled triangle, to find a missing side or a missing angle. Angles are in degrees, and an angle that isn't exact is given to one decimal place.
3. **Solve problems in two dimensions with Pythagoras and trigonometry together**: right-angled triangles hidden inside other shapes (isosceles triangles, rectangles, trapeziums, circles, the faces of solids), two right-angled triangles joined along a side, and problems that use bearings.
4. **Extended only:** know that the **shortest distance** from a point to a line is the **perpendicular** distance, and use it.
5. **Extended only:** work out **angles of elevation and depression**.

Core and Extended share outcomes 1–3. Exact values such as $\sin 30^\circ = \frac{1}{2}$ are in Exact values and trigonometric functions, the sine and cosine rules are in Non-right-angled triangles, and three-dimensional problems are in Trigonometry in 3D.

Understand it

The sides of a right-angled triangle

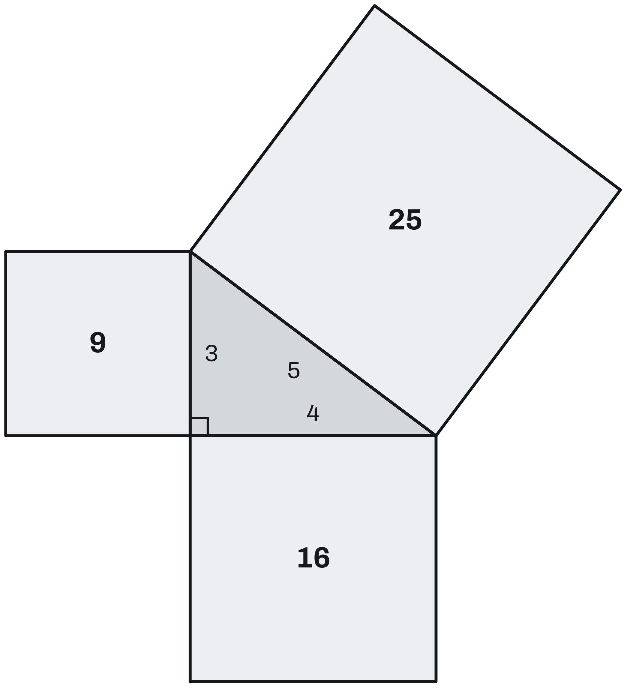
The side opposite the right angle is the **hypotenuse**. It is always the longest side. The other two sides meet at the right angle and are called the **shorter sides** (or legs).

Pythagoras' theorem

In any right-angled triangle with hypotenuse c and shorter sides a and b,

$$a^2 + b^2 = c^2.$$

Draw a square on each side. The two smaller squares together have exactly the same area as the square on the hypotenuse.



9 + 16 = 25

You use it two ways round:

- **Finding the hypotenuse: add the squares.** Shorter sides 6 and 8: $c = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$.
- **Finding a shorter side: subtract.** Hypotenuse 13 and one side 5: $b = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$.

A quick check: the hypotenuse must come out **longer** than both other sides. If your "shorter side" is longer than the hypotenuse, you added when you should have subtracted.

Exact answers. If the question says "exact", leave the square root: shorter sides 2 and 3 give a hypotenuse of $\sqrt{13}$, not 3.61.

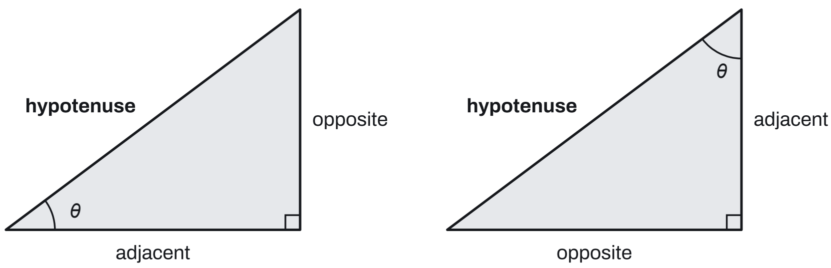
Is it right-angled? The theorem works backwards too. If the square of the longest side equals the sum of the squares of the other two, the angle opposite the longest side is 90°. For sides 20, 21 and 29: $20^2 + 21^2 = 400 + 441 = 841 = 29^2$, so the triangle is right-angled. For sides 7, 9 and 12: $49 + 81 = 130$, but $12^2 = 144$, so it is not.

Naming the sides for trigonometry

Trigonometry links the **angles** of a right-angled triangle to the **ratios** of its sides. First, name the sides **from the angle you are using**, called θ here:

- **hypotenuse**: the longest side, opposite the right angle;
- **opposite**: the side across the triangle from θ;

- **adjacent**: the side next to θ that is not the hypotenuse.



The opposite and adjacent swap when you use the other acute angle; the hypotenuse never changes.

Sine, cosine and tangent

For an acute angle θ in a right-angled triangle,

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}.$$

Many students remember these as **SOH CAH TOA**. Every right-angled triangle with the same angle θ is the same shape (they are similar), so these ratios depend only on the angle, and your calculator knows them.

To choose which one: mark the side you know and the side you want. The two sides involved pick the ratio.

- **Finding a side on top of the fraction: multiply.** Hypotenuse 12, angle 40°. The opposite is $12\sin 40^\circ = 7.71$ and the adjacent is $12\cos 40^\circ = 9.19$ (3 s.f.).
- **Finding a side on the bottom of the fraction: divide.** Adjacent 5, angle 62°: $\cos 62^\circ = \dfrac{5}{h}$, so $h = \dfrac{5}{\cos 62^\circ} = 10.7$ (3 s.f.).
- **Finding an angle: use the inverse.** Opposite 3, adjacent 7: $\tan\theta = \dfrac{3}{7}$, so $\theta = \tan^{-1}\left(\dfrac{3}{7}\right) = 23.2^\circ$ (1 d.p.). On a calculator $\tan^{-1}$ is usually **shift** then **tan**.

Your calculator must be in **degrees** (a small D or DEG on the screen). In radians, $\sin 30^\circ$ comes out as -0.988 instead of 0.5.

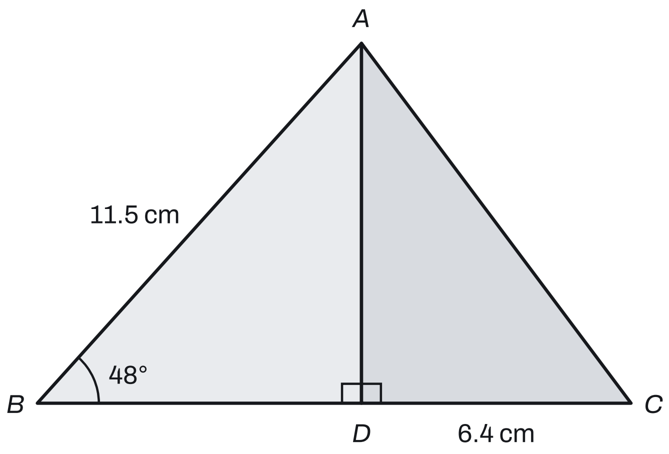
Right-angled triangles inside other shapes

Many questions hide the right angle. Look for it:

- **An isosceles triangle** splits into two identical right-angled triangles by the line from the top to the middle of the base. With sides 13, 13 and base 10: half the base is 5, so the height is $\sqrt{13^2 - 5^2} = 12$ and the area is $\frac{1}{2} \times 10 \times 12 = 60$.
- **A rectangle's diagonal** makes a right-angled triangle with two sides of the rectangle.
- **A trapezium** with one sloping side: drop a perpendicular to make a right-angled triangle. Its base is the difference between the parallel sides.

- **A circle**: the angle in a semicircle is 90°, and a tangent meets a radius at 90°.
- **The face of a pyramid** or the cross-section of a cone is often an isosceles or right-angled triangle.

When a question shows **two right-angled triangles joined along a side**, that shared side links them: work it out in the first triangle, then use it in the second.



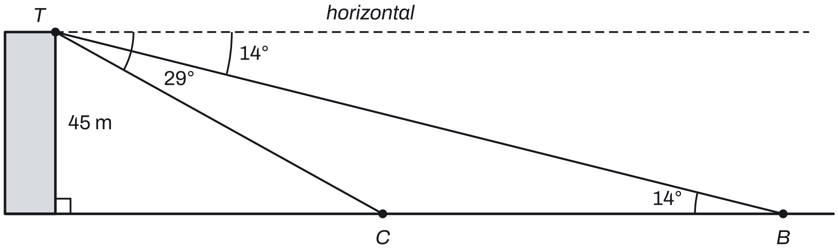
Bearings

Bearings are measured clockwise from North. If a ship sails on a bearing of 040° and then turns onto 130°, the two paths meet at $130^\circ - 40^\circ = 90^\circ$, so they are the shorter sides of a right-angled triangle. Sailing 12 km then 5 km puts the ship $\sqrt{12^2 + 5^2} = 13$ km from its start. The angle at the start is $\tan^{-1}\left(\frac{5}{12}\right) = 22.6^\circ$, so the ship's bearing from its start is $040^\circ + 22.6^\circ = 062.6^\circ$.

Angles of elevation and depression

Extended only. Both are measured **from the horizontal**.

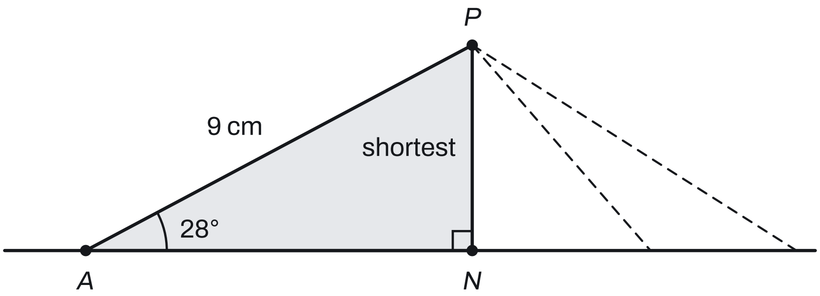
- The **angle of elevation** is the angle you look **up** through, from the horizontal, to see something above you.
- The **angle of depression** is the angle you look **down** through, from the horizontal, to see something below you.



The horizontal at the top and the ground are parallel, so the angle of depression from T to the boat **equals** the angle of elevation from the boat to T (they are alternate angles). Either way, the right-angled triangle is the cliff, the sea and the line of sight, and the angle sits at the boat's end of the horizontal side.

The shortest distance from a point to a line

Extended only. Of all the lines from a point P to a straight line, the shortest is the one that meets it at **right angles**. The perpendicular is the shorter side of a right-angled triangle, and every other line from P is a hypotenuse, so it is longer.



If $AP = 9$ cm makes an angle of 28° with the line, the shortest distance from P to the line is $PN = 9\sin 28^\circ = 4.23$ cm (3 s.f.). "The distance from a point to a line" always means this perpendicular distance.

Key facts and formulas

"Given" means it is on the List of formulas printed on page 2 of every 0580 paper. "Learn it" means you must remember it. Nothing in this topic is on the list except the area of a triangle.

Formula	Status
Pythagoras' theorem: $a^2 + b^2 = c^2$, where c is the hypotenuse	Learn it
Hypotenuse $c = \sqrt{a^2 + b^2}$; shorter side $b = \sqrt{c^2 - a^2}$	Learn it
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	Learn it
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	Learn it
$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	Learn it
Angles in a triangle add up to 180°, so the two acute angles of a right-angled triangle add up to 90°	Learn it
Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$ (for a right-angled triangle, the two shorter sides)	Given
Extended: angles of elevation and depression are measured from the horizontal	Learn it
Extended: the shortest distance from a point to a line is the perpendicular distance	Learn it

How to do it

In the questions we have tagged for this topic (60 questions, 59 different, 2021–2025), the types came up this often. 29 of the 59 different questions mix two or more types, so the counts add up to more than 59.

Question type	Questions
Area, perimeter and right-angled triangles inside other shapes	27
Pythagoras: find a missing side	23
Trigonometry: find a missing side	19
Trigonometry: find a missing angle	14
Two right-angled triangles joined together	8
Shortest distance from a point to a line (Extended only)	3
Angles of elevation and depression (Extended only)	1

1. Area, perimeter and right-angled triangles inside other shapes (27 questions)

1. **Find the right angle.** It may be marked, or it may come from a fact: an isosceles triangle split down the middle, a rectangle, the angle in a semicircle, a tangent and radius, two bearings 90° apart.
2. **Sketch just that triangle** and label the two sides (or side and angle) you know.
3. **Find the missing length** with Pythagoras or trigonometry.
4. **Finish the question**: an area ($\frac{1}{2}\times$ base $\times$ height), a perimeter (add every outside edge), or the next step in a longer problem.

For example, a rectangle has one side 24 cm and a diagonal 26 cm. The other side is $\sqrt{26^2 - 24^2} = 10$ cm, so the area is $24 \times 10 = 240$ cm².

Variations you will meet:

- **Perimeter of a trapezium with a sloping side.** Parallel sides 9 cm and 21 cm, perpendicular height 8 cm and one side at right angles to the parallel sides: the sloping side is the hypotenuse of a triangle with sides $21 - 9 = 12$ and 8, so it is $\sqrt{12^2 + 8^2} = 14.42\ldots$ cm, and the perimeter is $9 + 21 + 8 + 14.42\ldots = 52.4$ cm.
- **Isosceles triangle**: halve the base, then Pythagoras (or $\tan$ with a base angle) gives the height. Use it for the area, or as a face of a pyramid on a net.
- **Area of a right-angled triangle**: the two shorter sides are the base and height. If you know the area and one side, $\frac{1}{2} \times b \times h = \text{area}$ gives the other. If the sides are algebra, such as x and $x + 7$, the area gives an equation to solve.
- **"Show that the angle is 90°"**: check that the longest side squared equals the sum of the other two squared, and write the numbers out. With bearings, subtract the bearings.
- **A triangle drawn in a circle** on a diameter: say "the angle in a semicircle is 90°", then use Pythagoras.

- **The slant height of a pyramid's face** or of a cone: a right-angled triangle made by the height and half the base (or the radius).

2. Pythagoras: find a missing side (23 questions)

1. **Label the hypotenuse** (opposite the right angle) first.
2. **Hypotenuse missing: add.** $c^2 = a^2 + b^2$, then square root.
3. **Shorter side missing: subtract.** $b^2 = c^2 - a^2$, then square root.
4. **Check** the hypotenuse is the longest side, and round to 3 significant figures (or leave a surd if "exact").

For example, hypotenuse 17.5 cm and one side 9.8 cm: $b^2 = 17.5^2 - 9.8^2 = 210.21$, so $b = 14.5$ cm (3 s.f.).

Variations you will meet:

- **"Show that $BC = 20$ m"** (hypotenuse 29 m, other side 21 m): write $BC^2 = 29^2 - 21^2 = 400$, then $BC = \sqrt{400} = 20$. Show both lines.
- **"Work out the exact length"**: leave the answer as a surd, such as $\sqrt{41}$ from sides 4 and 5; don't round.
- **Perimeter after finding the side**: add all three sides.
- **A path, a ladder, a diagonal**: the real-life object is the hypotenuse.

3. Trigonometry: find a missing side (19 questions)

1. **Label** the sides hypotenuse, opposite and adjacent **from the given angle**.
2. **Choose the ratio** that uses the side you know and the side you want: $\sin$ (O and H), $\cos$ (A and H) or $\tan$ (O and A).
3. **Write the equation**, for example $\sin 24^\circ = \dfrac{15}{x}$.
4. **Rearrange**: if the unknown is on top, multiply ($x = 20\cos 41^\circ$); if it is underneath, divide ($x = \dfrac{15}{\sin 24^\circ}$).

For example, the angle is 33° and the side opposite it is 6.8 cm: $\sin 33^\circ = \dfrac{6.8}{h}$, so $h = \dfrac{6.8}{\sin 33^\circ} = 12.5$ cm (3 s.f.).

Variations you will meet:

- **"Show that $x = 12.5$, correct to 3 significant figures"**: write the calculation, give more figures than asked (such as $12.485\ldots$ before 12.5), then round. The unrounded value earns the last mark.
- **"$\cos x = k$, write down k"**: just the fraction of the two sides, such as $k = \dfrac{5}{13}$ when the adjacent side is 5 and the hypotenuse 13.
- **The other acute angle** is 90° minus the given one; you can use either angle as long as you label the sides from it.
- **Unknown underneath with $\tan$**: $\tan 35^\circ = \dfrac{4.2}{x}$ gives $x = \dfrac{4.2}{\tan 35^\circ} = 6.00$ cm (3 s.f.).

4. Trigonometry: find a missing angle (14 questions)

1. **Label the sides** from the angle you want.
2. **Choose the ratio** using the two sides you know.
3. **Use the inverse**: $\theta = \sin^{-1}(\ldots)$, $\cos^{-1}(\ldots)$ or $\tan^{-1}(\ldots)$.
4. **Give the angle to 1 decimal place.**

For example, the side adjacent to the angle is 8.2 cm and the hypotenuse is 13.5 cm: $\cos\theta = \dfrac{8.2}{13.5}$, so $\theta = \cos^{-1}\left(\dfrac{8.2}{13.5}\right) = 52.6^\circ$.

Variations you will meet:

- **"Show that the angle is 52.6°"**: write $\cos\theta = \dfrac{8.2}{13.5}$, then the value to more decimal places ($52.597\ldots$), then 52.6.
- **Only two sides given and they aren't the right pair**: you can still pick the ratio that uses exactly those two sides; you don't need Pythagoras first.
- **An angle on a grid**: count squares for the opposite and adjacent, then $\tan^{-1}$.
- **A side found earlier** (in part (a)) is used here: use the unrounded value from your calculator.

5. Two right-angled triangles joined together (8 questions)

1. **Find the shared side** in the triangle where you know two things (two sides, or a side and an angle).
2. **Use the shared side in the second triangle** with Pythagoras or trigonometry.
3. **Keep the shared side unrounded** in your calculator.

Variations you will meet:

- **A quadrilateral made of two right-angled triangles**: for its perimeter, add only the outside edges, not the shared side.
- **Lengths along a straight line**: $BC = BD + DC$; take away a known piece to find the part inside one triangle.
- **Exact trigonometric values** (Extended, such as $\cos 60^\circ = \frac{1}{2}$) may be needed on the non-calculator paper: see Exact values and trigonometric functions.

Example 3 below works through one.

6. Shortest distance from a point to a line (3 questions)

Extended only.

1. **Draw the perpendicular** from the point to the line. Mark the right angle.
2. **The perpendicular is a side** of a right-angled triangle whose hypotenuse is a line you already know (such as AP or BC).

3. **Use** $\sin$ (usually): distance $=$ hypotenuse $\times \sin(\text{angle at the line})$.

Only the angle between the known side and the line matters. With $AP = 9$ cm at 28° to the line, the distance is $9 \sin 28^\circ = 4.23$ cm.

Variations you will meet:

- **Given the shortest distance, find a side**: the perpendicular is the opposite side, so $\sin(\text{angle}) = \frac{\text{distance}}{\text{side}}$.
- **Inside a larger problem** (a quadrilateral with the sine and cosine rules): you may need an angle from another part first.

7. Angles of elevation and depression (1 question)

Extended only.

1. **Draw the horizontal** from the observer. The angle is between this horizontal and the line of sight.
2. **Build the right-angled triangle**: the vertical height difference, the horizontal distance and the line of sight.
3. **Use** $\tan$ (usually), since you tend to know or want the height and the horizontal distance.

Variations you will meet:

- **Two poles or buildings of different heights**: the vertical side is the **difference** in heights, not either height. Poles 13 m and 2 m tall have a vertical side of 11 m.
- **Two positions** (a boat moving towards a cliff): find each horizontal distance, then subtract. Example 4 does this.
- **Depression from the top equals elevation from the bottom**: use whichever puts the angle inside your triangle.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.

Example 1

Core, calculator allowed.

(a) Triangle ABC has a right angle at B. $AB = 9.6$ cm and $BC = 7.2$ cm. Calculate AC. **[2]**

(b) Calculate angle BAC. **[2]**

(c) Triangle PQR has a right angle at Q. $PR = 15$ cm and angle $QPR = 34^\circ$. Calculate QR. **[2]**

(a) AC is the hypotenuse. $AC^2 = 9.6^2 + 7.2^2 = 92.16 + 51.84 = 144$ **M1**, so $AC = 12$ cm. **A1**

(b) From angle BAC, BC is opposite and AB is adjacent: $\tan(BAC) = \dfrac{7.2}{9.6}$ **M1**, so angle $BAC = 36.869\ldots = 36.9^\circ$. **A1**

(c) From angle P, QR is opposite and PR is the hypotenuse: $\sin 34^\circ = \dfrac{QR}{15}$ **M1**, so $QR = 15\sin 34^\circ = 8.387\ldots = 8.39$ cm. **A1**

Example 2

Core.

A ladder 6.5 m long leans against a vertical wall. The foot of the ladder is on horizontal ground, 2.1 m from the wall.

(a) Calculate how far up the wall the ladder reaches. **[3]**

(b) Calculate the angle the ladder makes with the ground. **[2]**

(a) The ladder is the hypotenuse, and the wall and ground meet at 90°. Height$^2 + 2.1^2 = 6.5^2$ **M1**

Height $= \sqrt{6.5^2 - 2.1^2}$ **M1** $= \sqrt{37.84} = 6.151\ldots = 6.15$ m. **A1**

(b) From the angle at the ground, 2.1 is adjacent and 6.5 is the hypotenuse: $\cos\theta = \dfrac{2.1}{6.5}$ **M1**, so $\theta = 71.15\ldots = 71.2^\circ$. **A1**

Example 3

Core.

In the third diagram in "Understand it", B, D and C lie on a straight line and AD is perpendicular to BC. $AB = 11.5$ cm, angle $ABD = 48^\circ$ and $DC = 6.4$ cm.

(a) Calculate AD. **[2]**

(b) Calculate AC. **[2]**

(c) Calculate angle ACD. **[2]**

(a) In triangle ABD, AD is opposite 48° and AB is the hypotenuse: $AD = 11.5\sin 48^\circ$ **M1** $= 8.546\ldots = 8.55$ cm. **A1**

(b) In triangle ADC, AC is the hypotenuse: $AC = \sqrt{8.546\ldots^2 + 6.4^2}$ **M1** $= 10.677\ldots = 10.7$ cm. **A1** Keep $8.546\ldots$ in the calculator: using 8.55 still gives 10.7 here, but rounding earlier than that can push the answer out of range.

(c) From angle C, AD is opposite and DC is adjacent: $\tan(ACD) = \dfrac{8.546\ldots}{6.4}$ **M1**, so angle $ACD = 53.17\ldots = 53.2^\circ$. **A1**

Example 4

Extended.

The top of a vertical cliff, T, is 45 m above the sea. A boat at B is seen from T at an angle of depression of 14°.

(a) Calculate the distance from the boat to the foot of the cliff. **[2]**

(b) The boat sails directly towards the cliff to C. From T, the angle of depression of C is 29°. Calculate how far the boat sails. **[3]**

(a) The angle of elevation of T from B is also 14°. In the triangle at B, 45 is opposite and the distance x is adjacent: $\tan 14^\circ = \dfrac{45}{x}$ **M1**, so $x = \dfrac{45}{\tan 14^\circ} = 180.48\ldots = 180$ m (3 s.f.). **A1**

(b) In the same way, C is $\dfrac{45}{\tan 29^\circ} = 81.18\ldots$ m from the cliff. **M1**

Distance sailed $= 180.48\ldots - 81.18\ldots$ **M1** $= 99.30\ldots = 99.3$ m. **A1**

The fourth diagram in "Understand it" shows this cliff.

Common mistakes

- **Adding the squares when a shorter side is missing.** Examiners see this every year. Find the hypotenuse first: if it is the side you know, subtract.
- **Forgetting the square root**, or square-rooting each term separately: $\sqrt{15^2 + 8^2}$ is 17, not $15 + 8 = 23$.
- **Labelling the sides from the wrong angle**, so $\sin$ and $\cos$ swap. Mark the angle you are using, then label opposite and adjacent from it.
- **Multiplying when you should divide.** If the unknown is on the bottom of the fraction ($\cos 62^\circ = \frac{5}{h}$), divide: $h = \frac{5}{\cos 62^\circ}$, not $5\cos 62^\circ$.
- **Calculator in radians** (or gradians), which gives nonsense values. Check for D or DEG on the screen before the exam.
- **Rounding too early.** Reports repeatedly mention answers outside the accepted range because an intermediate length was rounded to 2 or 3 figures. Keep full values in the calculator until the last line.
- **Assuming what isn't given.** A triangle isn't isosceles or right-angled unless the question or diagram says so, and you can't measure angles on a diagram marked "not to scale".
- **Using a single height for elevation between two objects.** For two poles, the vertical side is the difference in heights.
- **Shortest distance drawn to the wrong point.** It must meet the line at right angles; it isn't the line to a vertex or to the middle of a side.

- **Angle answers to 3 significant figures** instead of 1 decimal place, or giving 36.8° when 36.87 rounds to 36.9°.

How to get the marks

- **Write the equation before you work it out**, such as $\tan 48^\circ = \dfrac{h}{7.3}$ or $BC^2 = 11.4^2 - 6.2^2$. That line earns the method mark even if you slip on the calculator.
- **Accuracy**: lengths to 3 significant figures, angles to 1 decimal place, unless the question says otherwise. Exact answers ("exact length") stay as surds.
- **"Show that"**: the answer is printed, so write every step and a value with more figures than the answer ($10.677\ldots$ before 10.7). Without the more accurate value the last mark is lost.
- **"Calculate"** means working is expected; a bare answer that is slightly off earns nothing.
- **Longer questions**: marks for finding an intermediate length are often given on their own, so write it down and label it ($AD = 8.546\ldots$).
- **Method marks follow through**: a wrong length from part (a), used correctly in part (b), still earns the method mark. Keep going.
- **Units**: give cm, m or degrees; if the question says "give the units of your answer", the unit earns a mark.

Quick check

1. A right-angled triangle has shorter sides 5 cm and 7 cm. Find the exact length of its hypotenuse.
2. A right-angled triangle has hypotenuse 9.4 cm and one shorter side 5.1 cm. Calculate the other side.
3. In a right-angled triangle, an angle is 52° and the side adjacent to it is 7 cm. Calculate the side opposite the 52° angle.
4. A right-angled triangle has hypotenuse 11 cm, and the side opposite angle θ is 4.5 cm. Calculate θ.
5. **Extended:** In triangle ABC, $AB = 10$ cm and angle $BAC = 35^\circ$. Calculate the shortest distance from B to the line AC.

Answers

1. $\sqrt{5^2 + 7^2} = \sqrt{74}$ cm.
2. $\sqrt{9.4^2 - 5.1^2} = \sqrt{62.35} = 7.90$ cm (3 s.f.).
3. $\tan 52^\circ = \dfrac{x}{7}$, so $x = 7 \tan 52^\circ = 8.96$ cm (3 s.f.).
4. $\sin \theta = \dfrac{4.5}{11}$, so $\theta = \sin^{-1}\left(\dfrac{4.5}{11}\right) = 24.1^\circ$.
5. The perpendicular from B to AC is opposite the 35° angle, with AB the hypotenuse: $10 \sin 35^\circ = 5.74$ cm (3 s.f.).

Past questions to try

- **0580/33/M/J/25 Q23**: Pythagoras to find the hypotenuse.
- **0580/13/O/N/24 Q23**: a missing side with sine.
- **0580/21/O/N/24 Q20**: two right-angled triangles, then an angle.
- **0580/42/M/J/25 Q20**: Extended; lengths with tangent and cosine, and the shortest distance from a point to a line in parts (a) and (d).