Cambridge IGCSE · Mathematics 0580 · Notes

Sequences

Read online at **pastopic.com/cambridge/mathematics-0580/notes/sequences**

Last checked 29 September 2026

Read first: **Equations and changing the subject**

What you need to know

Core and Extended share the same three outcomes, but Extended goes further with the kinds of sequence. The parts marked **Extended only** are for Papers 2 and 4. By the end of this topic you should be able to:

1. **Continue a number sequence or a pattern.** Write down the next terms of a list of numbers, fill in missing terms, and draw the next pattern in a sequence of shapes.
2. **Recognise patterns in sequences.** Describe the **term-to-term rule** (how to get from one term to the next, such as "add 7" or "multiply by 3"), and see how two sequences are linked, for example when one is the other plus 1, or when a fraction is made from two sequences.
3. **Find and use the nth term** (a formula that gives any term from its position n) of **linear** sequences, such as $4n + 3$, and of **simple quadratic and cubic** sequences, such as $n^2 + 1$ or $n^3 - 2$. Use it to work out any term, and to decide whether a number is in the sequence and which term it is.
4. **Extended only:** find and use the nth term of **any** quadratic or cubic sequence, of **exponential** sequences (multiply by the same number each time), and of **simple combinations** of these, such as fractions made from two sequences or one sequence minus another. Know that T_n can stand for the nth term of a sequence T.

The formula $\frac{n}{2}(2a + (n-1)d)$ for adding up the terms of a linear sequence is **not** in the syllabus. If a question needs a total, it gives you the formula.

Understand it

Terms, positions and rules

A **sequence** is a list of numbers (or shapes) in order, following a rule. Each number is a **term**. The **position** says where a term sits: the 1st term, the 2nd term, and so on. Positions are always whole numbers $1, 2, 3, \ldots$; there is no 0th or 2.5th term.

There are two kinds of rule:

- A **term-to-term rule** says how to get the next term from the one before. For $8, 12, 16, 20, \ldots$ it is "add 4". For $3, 6, 12, 24, \ldots$ it is "multiply by 2". To use it you must know a starting term.

- A **position-to-term rule**, the n**th term**, gives a term straight from its position. For $8, 12, 16, 20, \ldots$ it is $4n + 4$: the 10th term is $4 \times 10 + 4 = 44$, without writing out the first nine.

To get terms from an nth term, substitute $n = 1$, then $n = 2$, then $n = 3$. With $n^2 - 2$: $1 - 2 = -1$, then $4 - 2 = 2$, then $9 - 2 = 7$. Square **before** you add or subtract.

Differences

Write the gaps between neighbouring terms underneath. These are the **first differences**, and they show what kind of sequence you have.

Sequence	Terms	First differences	Kind
A	$4, 9, 14, 19, 24$	$5, 5, 5, 5$	linear
B	$3, 6, 11, 18, 27$	$3, 5, 7, 9$	quadratic
C	$2, 6, 18, 54, 162$	$4, 12, 36, 108$	exponential

- The same first difference every time means a **linear** (arithmetic) sequence.
- First differences that go up (or down) by the same amount each time mean a **quadratic** sequence: B's differences go up by 2, so its next difference is 11 and its next term is $27 + 11 = 38$. You can continue it like this even if you can't find its nth term.
- Each term the previous one **times** the same number means an **exponential** (geometric) sequence: C is "multiply by 3". Its differences are not steady, so look at the ratio instead.

Why the nth term of a linear sequence is $dn + c$

In $4, 9, 14, 19, \ldots$ each term is 5 more than the last, just like the five times table $5, 10, 15, 20, \ldots$. Every term is the times-table number minus 1, so the nth term is $5n - 1$.

In general, the **common difference** d goes in front of n, and the number that fixes it is the **zero term**, the term that would come before the first one. Here, stepping back one place from 4 gives $4 - 5 = -1$. So

$$n\text{th term} = dn + (\text{zero term}) = 5n - 1.$$

For a **decreasing** sequence the difference is negative. For $20, 17, 14, 11, \ldots$, $d = -3$ and the zero term is $20 + 3 = 23$, so the nth term is $23 - 3n$. Check: $n = 1$ gives 20.

The same answer comes from the formula $a + (n-1)d$, with first term a: $20 + (n-1)(-3) = 23 - 3n$. Either way, **simplify** and **check** with $n = 1$ and $n = 2$.

Square and cube numbers

Many non-linear sequences are the square numbers or cube numbers with a small change, so know these by heart:

n	**1**	**2**	**3**	**4**	**5**	**6**
n^2	1	4	9	16	25	36
n^3	1	8	27	64	125	216

$3, 6, 11, 18, 27$ is each square number plus 2, so its nth term is $n^2 + 2$. $-1, 6, 25, 62, 123$ is each cube number minus 2, so its nth term is $n^3 - 2$. Always compare the terms with n^2 and n^3 **in the same positions**.

Extended only: any quadratic or cubic

Extended only: for $an^2 + bn + c$, the second differences are always $2a$. So **halve the second difference** to get the number in front of n^2. Then subtract an^2 from each term: what is left is a linear sequence, and its nth term gives $bn + c$.

Extended only: for a cubic $an^3 + \ldots$ the **third** differences are all $6a$. When they are 6, compare with n^3; when they are 12, compare with $2n^3$.

Extended only: exponential sequences

Extended only: in an exponential sequence you multiply by the same number r each time. Starting from the first term a, the 2nd term is ar, the 3rd is ar^2, and the nth is

$$ar^{n-1}.$$

For $2, 6, 18, 54, \ldots$: $a = 2, r = 3$, so the nth term is $2 \times 3^{n-1}$. The power is always one less than the position, because the first term has not been multiplied yet. **Extended only:** if the terms halve, $r = \frac{1}{2}$, and $80, 40, 20, 10, \ldots$ has nth term $80 \times \left(\frac{1}{2}\right)^{n-1}$.

Extended only: when the terms are powers of one number, you can write the nth term as a single power: $\frac{1}{9}, \frac{1}{3}, 1, 3, 9$ are $3^{-2}, 3^{-1}, 3^0, 3^1, 3^2$, so the nth term is 3^{n-3} (check: $n = 1$ gives $3^{-2} = \frac{1}{9}$).

Sequences made from other sequences

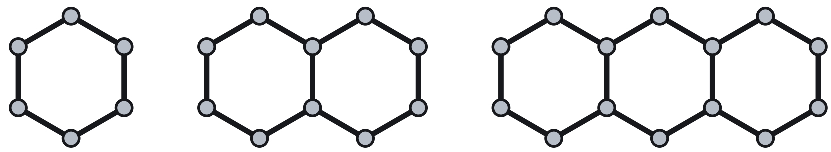
Two sequences can be joined. In a pattern, a fraction such as "the fraction of the tiles that are white" is the white-tile sequence over the total-tile sequence.

Extended only: in a sequence of fractions, the **numerators** make one sequence and the **denominators** make another. For $\frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}$ the numerators $2, 3, 4, 5$ are $n + 1$ and the denominators $5, 7, 9, 11$ are $2n + 3$, so the nth term is $\frac{n+1}{2n+3}$. Don't look at differences between the fractions themselves.

Extended only: a sequence can also be the sum or difference of two known ones. If A is $4n + 1$ and B is 2^n, then the sequence $A - B$ (terms $3, 5, 5, 1, \ldots$) has nth term $4n + 1 - 2^n$.

Patterns of shapes

A sequence can be a list of diagrams. Count something in each (sticks, tiles, dots) and you get a number sequence.



Each new hexagon shares a side with the one before, so it needs only 5 more sticks: $6, 11, 16, \ldots$ with difference 5. The zero term is $6 - 5 = 1$, so Pattern n has $5n + 1$ sticks. You can see the formula in the picture: one starting stick on the left, then 5 sticks for each hexagon.

Key facts and formulas

None of these are on the list of formulas printed in the exam paper, so learn them all.

Formula	Status
Linear sequence with common difference d: nth term $= dn + (\text{zero term})$, where zero term $=$ first term $-d$	Learn it
Same thing another way: nth term $= a + (n-1)d$, with first term a	Learn it
Square numbers n^2: $1, 4, 9, 16, 25, \ldots$; cube numbers n^3: $1, 8, 27, 64, 125, \ldots$	Learn it
Extended only: quadratic $an^2 + bn + c$: second differences $= 2a$	Learn it
Extended only: cubic $an^3 + \ldots$: third differences $= 6a$	Learn it
Extended only: exponential sequence, first term a, multiplier r: nth term $= ar^{n-1}$	Learn it
Extended only: T_n means the nth term of sequence T	Learn it
A number N is a term only if solving "nth term $= N$" gives a **positive whole number** n	Learn it

How to do it

In the questions we have tagged for this topic (66 questions, 2021–2025), the types came up this often. 53 of the 66 questions mix two or more types, so the counts add up to more than 66.

Question type	Questions
nth term of a linear sequence	43
Next terms, missing terms and the term-to-term rule	34
Terms from a given nth term	19
nth term of a quadratic or cubic sequence	14
Exponential, fraction and combined sequences	12
Using the nth term: which term, is it a term?	12
Patterns of shapes	7
Unknown constants in an nth term	2
Totals of terms	2

1. nth term of a linear sequence (43 questions)

1. **Check the differences are all the same.** That number, d, goes in front of n. Take care with signs: a sequence going **down** has a negative d.
2. **Find the zero term**: first term minus d (step one place back).
3. **Write** $dn + \text{zero term}$ and simplify.
4. **Check** with $n = 1$ and $n = 2$.

For example, $9, 15, 21, 27$: $d = 6$, zero term $9 - 6 = 3$, so $6n + 3$. For $14, 10, 6, 2$: $d = -4$, zero term $14 + 4 = 18$, so $18 - 4n$.

Variations you will meet:

- **Negative terms**, as in $-5, -2, 1, 4$: $d = 3$, zero term $-5 - 3 = -8$, so $3n - 8$.
- **The sequence is in a table** with other sequences, or is the number of sticks or tiles in a pattern: it is still the same four steps.
- **A particular term**, such as "find the 50th term": find the nth term first, then substitute $n = 50$. For $6n + 3$ that is 303.

2. Next terms, missing terms and the term-to-term rule (34 questions)

1. **Find how the terms change**: the difference (add or subtract), or a multiplier if the terms are multiplied, or differences that themselves change steadily.
2. **Next terms:** apply the same change again. For $17, 12, 7, 2$ (subtract 5), the next two terms are -3 and -8.
3. **Term-to-term rule:** write it in words with the number, for example "subtract 5" or "multiply by 3". Don't give the nth term here.

Variations you will meet:

- **Changing differences.** $2, 4, 8, 14, 22$ has differences $2, 4, 6, 8$, so the next difference is 10 and the next term 32. The same idea continues a sequence that goes down, such as $40, 38, 34, 28, 20$ (differences $-2, -4, -6, -8$; next $20 - 10 = 10$).
- **Missing terms in the middle** of a linear sequence. If the 1st term is 4 and the 5th is 28, there are 4 equal steps between them: $\frac{28-4}{4} = 6$, so the terms are $4, 10, 16, 22, 28$.
- **Missing terms before the start**: work backwards with the opposite operation.
- **A two-step rule** such as "multiply by 2 then subtract 1". To go **forwards**, do it in that order. To go **backwards** from a known term, undo it in reverse order: add 1, then divide by 2. If the 4th term is 41, the 3rd is $(41 + 1) \div 2 = 21$.
- **"Explain how you worked it out"**: state the rule, for example "each term is 4 less than the one before".

3. Terms from a given nth term (19 questions)

1. Substitute $n = 1, n = 2, n = 3$ in turn (or the position asked for).
2. Follow the order of operations: powers first, then multiply, then add or subtract.
3. Write the terms in order, separated by commas.

For example, $n^2 + 2n$ gives $3, 8, 15$, and $15 - 2n$ gives $13, 11, 9$.

Variations you will meet:

- **"Find the greatest term"**: if the nth term goes down as n goes up (like $15 - 2n$), the first term is the greatest.
- **The difference between two terms**, such as between the 5th and 6th: find both terms, then subtract.
- **Extended only: a fraction nth term** such as $\frac{n}{3n+1}$, giving $\frac{1}{4}, \frac{2}{7}, \frac{3}{10}$. Leave the answers as fractions if the question says so.

4. nth term of a quadratic or cubic sequence (14 questions)

Simple ones (Core and Extended):

1. Work out the differences. If the second differences are all 2, compare with n^2; if you need third differences and they are all 6, compare with n^3.
2. Write n^2 (or n^3) under each term and see what has been added or subtracted.
3. The nth term is n^2 (or n^3) plus or minus that number.

For example, $-2, 5, 24, 61, 122$: the third differences are 6, and each term is 3 less than $1, 8, 27, 64, 125$, so the nth term is $n^3 - 3$. A sequence linked to one you have just found, such as "each term of $n^2 - 2$ plus 3", is $n^2 + 1$.

Extended only: any quadratic.

1. **Extended only:** find the second difference and halve it: that is a, the number in front of n^2.
2. **Extended only:** subtract an^2 from each term.

3. **Extended only:** the numbers left form a linear sequence. Find its nth term, $bn + c$, as in type 1.
4. **Extended only:** the answer is $an^2 + bn + c$. Check it with $n = 3$.

For example, $3, 10, 21, 36, 55$ has first differences $7, 11, 15, 19$ and second differences 4, so $a = 2$. Subtracting $2n^2 = 2, 8, 18, 32, 50$ leaves $1, 2, 3, 4, 5$, which is n. The nth term is $2n^2 + n$.

Variations you will meet:

- **Extended only: a cubic** with third differences 12 (compare with $2n^3$) or a cubic of a shifted bracket, such as $(n + 1)^3$: its terms $8, 27, 64, \ldots$ are the cube numbers starting one place later.
- **Extended only: a multiple of a square**, such as $5, 20, 45, 80$, which is $5n^2$ (second difference 10, halved: 5, and nothing left over).
- **"Find the 10th term"** of a sequence you recognise, such as the cube numbers: $10^3 = 1000$.

5. Exponential, fraction and combined sequences (12 questions)

Continuing these (for example writing the next term of $5, 10, 20, 40$) is on both tiers; finding the nth term is Extended. Core papers have asked for a fraction made from two pattern counts; see type 7.

Exponential:

1. **Extended only:** check the terms are multiplied by the same number r each time (divide a term by the one before).
2. **Extended only:** the nth term is ar^{n-1}, where a is the first term.
3. **Extended only:** if the terms are powers of one number, you can write a single power instead. $9, 27, 81, 243$ are 3^2, 3^3, 3^4, 3^5, so the nth term is 3^{n+1}, the same as $9 \times 3^{n-1}$.

Fractions:

1. **Extended only:** split each term into numerator and denominator (write whole numbers as fractions, such as $1 = \frac{3}{3}$ or $\frac{4}{4}$, to fit the pattern).
2. **Extended only:** find the nth term of the numerators and of the denominators separately, each as in type 1 (or type 4 or 5 if it is not linear).
3. **Extended only:** write the nth term as one fraction, numerator over denominator.

Variations you will meet:

- **Extended only: halving or thirds**: $64, 16, 4, 1$ is $64 \times \left(\frac{1}{4}\right)^{n-1}$.
- **Extended only: a denominator that multiplies**, such as $\frac{3}{2}$, $\frac{5}{4}$, $\frac{7}{8}$, $\frac{9}{16}$, which is $\frac{2n+1}{2^n}$.
- **Extended only: one sequence built from two others** in a table (for example $C = A - B$): check the terms first, then subtract the two nth terms, with brackets.

6. Using the nth term: which term, is it a term? (12 questions)

1. **Set the nth term equal to the number** and solve for n.

2. **If n is a positive whole number**, the number is a term, and n is its position.
3. **If n is a fraction or a decimal**, the number is not a term. Say so and give the reason.

For example, is 162 a term of $4n+3$? $4n+3=162$ gives $4n=159$ and $n=39.75$, which is not a whole number, so no. Which term of $4n+3$ is 203? $4n=200$, so the 50th.

Variations you will meet:

- **A quadratic nth term**, such as n^2+4: $n^2+4=229$ gives $n^2=225$ and $n=15$, so 229 is the 15th term.
- **Undo the operations in the right order**: for $8k-5=115$, add 5 first, then divide by 8: $k=15$.
- **Extended only: a fraction nth term**: $\frac{n}{3n+1}=\frac{3}{10}$ gives $10n=9n+3$, so $n=3$ and it is the 3rd term. With $\frac{n}{3n+1}=\frac{1}{3}$ you get $3n=3n+1$, which has no solution, so $\frac{1}{3}$ is not a term.

7. Patterns of shapes (7 questions)

1. **Draw the next pattern** by seeing what is added each time, and keep the same layout.
2. **Count** the things asked for in each pattern and complete the table.
3. **Find the nth term** from the table, as in type 1.
4. **Use it**: which pattern has a given number, or how many a given pattern needs.

Variations you will meet:

- **"She has 200 sticks; what is the largest pattern she can make, and how many sticks are left?"** For $5n+1$ sticks: $5n+1=200$ gives $n=39.8$, so the largest whole pattern is 39. It uses $5\times 39+1=196$ sticks, leaving 4.
- **Two quantities** (such as white and grey tiles): find each nth term. The total is their sum, and "the fraction that are grey" is grey over total, such as $\frac{2n}{3n+1}$.
- **Repeating patterns** of shapes (for example square, circle, triangle, square, ...): the shape in position p depends on the remainder when p is divided by the length of the repeat. With a repeat of 3, positions $3, 6, 9, \ldots$ are all triangles, so position 60 is a triangle and position 61 a square.

8. Unknown constants in an nth term (2 questions)

1. **Extended only:** substitute each position you are told about into the nth term. "The 1st term is 6" means put $n=1$ and set it equal to 6. Don't put the term's value in for n.
2. **Extended only:** you get simultaneous equations in the unknowns (such as a and b). Solve them by elimination.
3. **Extended only:** check with another known term.

9. Totals of terms (2 questions)

1. The question gives a formula for the **total** of the first k terms (or diagrams). Substitute the value of k asked for.
2. To check a total, add the terms themselves: for sticks $6, 11, 16$ the total of the first 3 patterns is 33.

3. For "how many are left", work out the total needed and subtract it from what is available.

Extended only: if S_n is the total of the first n terms, then the nth term is $S_n - S_{n-1}$ (the total up to n minus the total up to the one before). For $S_n = n^2 + 2n$: $S_2 = 8$ and $S_3 = 15$, so the 3rd term is $15 - 8 = 7$.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

These are the first four terms of a sequence.

$$4, \quad 11, \quad 18, \quad 25$$

(a) Write down the next term. **[1]**

(b) Write down the term-to-term rule. **[1]**

(c) Find the nth term. **[2]**

(d) Is 150 a term in this sequence? Show how you decide. **[2]**

(a) The terms go up by 7, so the next term is 32. **B1**

(b) Add 7. **B1**

(c) $d = 7$ and the zero term is $4 - 7 = -3$, so the nth term is $7n - 3$. **B2** (**B1** for $7n + \ldots$ or for $\ldots - 3$ alone.) Check: $n = 2$ gives 11.

(d) $7n - 3 = 150$ gives $7n = 153$ and $n = 21.857\ldots$ **M1**

n is not a whole number, so 150 is **not** a term. **A1**

Example 2

A sequence of patterns is made from sticks. Pattern 1 is a hexagon, and each new pattern adds a hexagon on the right that shares a side with the one before (see the diagram in "Understand it"). A dot is drawn at every corner, where sticks meet.

(a) Complete the table. **[2]**

Pattern number	1	2	3	4	5
Number of dots	6	10	14		

(b) Find an expression, in terms of n, for the number of dots in Pattern n. **[2]**

(c) One pattern has 130 dots. Work out its pattern number. **[2]**

(d) The number of sticks in Pattern n is $5n+1$. Ravi makes the pattern that has 81 sticks. Work out the number of dots in this pattern. **[3]**

(a) Each pattern has 4 more dots: 18 and 22. **B1 B1**

(b) $d=4$, zero term $6-4=2$, so $4n+2$. **B2** (**B1** for $4n+\dots$ or $\dots+2$.) Check: $n=3$ gives 14.

(c) $4n+2=130$ **M1**, so $4n=128$ and $n=32$. **A1**

(d) $5n+1=81$ **M1**, so $n=16$. **A1**

Dots: $4\times 16+2=66$. **B1** (a follow-through mark for putting your n into your answer to (b)).

Example 3

Extended only: find the nth term of each sequence.

(a) $5,\ 14,\ 27,\ 44,\ 65,\dots$ **[2]**

(b) $162,\ 54,\ 18,\ 6,\ 2,\dots$ **[2]**

(c) $\frac{3}{5},\ \frac{5}{8},\ \frac{7}{11},\ \frac{9}{14},\dots$ **[2]**

(a) First differences $9,13,17,21$; second differences $4,4,4$. **M1** So the nth term starts $2n^2$ (half of 4).

n	1	2	3	4	5
term	5	14	27	44	65
$2n^2$	2	8	18	32	50
term $-2n^2$	3	6	9	12	15

What is left is $3n$, so the nth term is $2n^2+3n$. **A1**

(b) Each term is the one before divided by 3, so $r=\frac{1}{3}$ and $a=162$. **M1** The nth term is $162\times\left(\frac{1}{3}\right)^{n-1}$. **A1** (Equivalent forms such as $486\times\left(\frac{1}{3}\right)^{n}$ are also right.)

(c) Numerators $3,5,7,9$: $2n+1$. Denominators $5,8,11,14$: $3n+2$. **M1** for either. The nth term is $\dfrac{2n+1}{3n+2}$. **A1**

Example 4

Extended only: the nth term of a sequence is an^2+bn+5. The first term is 2 and the third term is 20.

(a) Find the value of a and the value of b. **[4]**

(b) Show that 107 is a term of the sequence and state its position. **[3]**

(a) $n = 1$: $a + b + 5 = 2$, so $a + b = -3$. $n = 3$: $9a + 3b + 5 = 20$, so $9a + 3b = 15$, that is $3a + b = 5$. **M2** for both equations.

Subtracting: $2a = 8$, so $a = 4$ **A1**, and $b = -3 - 4 = -7$. **A1**

Check: the 2nd term is $16 - 14 + 5 = 7$, and the terms $2, 7, 20$ have second difference $8 = 2 \times 4$.

(b) $4n^2 - 7n + 5 = 107$ gives $4n^2 - 7n - 102 = 0$. **M1**

$(n - 6)(4n + 17) = 0$, so $n = 6$ or $n = -\frac{17}{4}$. **M1**

Position must be a positive whole number, so $n = 6$: 107 is the 6th term. **A1** (Check: $4 \times 36 - 42 + 5 = 107$.)

Common mistakes

- **Using the difference as the number added.** For $45, 38, 31, 24$ some write $n - 7$ or $n + 45$. The difference goes **in front of** n: $-7n$, and the zero term is $45 + 7 = 52$, so the answer is $52 - 7n$.
- **Using the first term instead of the zero term.** $38 - 7n$ gives 31 for the first term, not 38. Always check your answer with $n = 1$.
- **Losing the minus sign** for a decreasing sequence, which gives answers such as $52 + 7n$. If the terms go down, the n term must be negative.
- **Mixing up the two rules.** The term-to-term rule is "subtract 7", not $-7n$ or $52 - 7n$. The nth term must contain n.
- **Starting at** $n = 0$ when writing the first terms, or reading "nth term" as the 9th term.
- **Undoing in the wrong order.** For $5k - 3 = 97$, add 3 **then** divide by 5 ($k = 20$), not the other way round. If k comes out as a decimal, check your working before you conclude it isn't a term.
- **Extended only: putting the terms in for** n**.** In "the first term of $an^2 + bn - 1$ is 4", use $n = 1$: $a + b - 1 = 4$. Don't put $n = 4$.
- **Arithmetic slips in differences**, especially with negative numbers, so the second or third differences never come out equal. Do the subtractions carefully and write each row of differences out.
- **Extended only: simplifying powers wrongly.** $5 \times 2^{n-1}$ is **not** 10^{n-1}, and $36 \times \left(\frac{1}{2}\right)^{n-1}$ is not 18^{n-1}. Multiply only numbers that have the same power.
- **Extended only: looking at differences between fractions.** For a sequence of fractions, find the numerators' rule and the denominators' rule separately.
- **Not using an earlier part.** If part (a) was $n^2 + 5$ and part (b)'s terms are each one more, the answer is $n^2 + 6$; starting again from scratch wastes time.
- **Rounding the pattern number the wrong way.** If $5n + 1 = 200$ gives $n = 39.8$, the largest complete pattern is 39, not 40.

How to get the marks

- **Give the nth term as an expression in n**, such as $7n-3$, not "$n=7n-3$" or just "7". Unsimplified forms such as $4+7(n-1)$ usually score, but a correct expression followed by a wrong "simplification" loses the mark: your **final** answer is marked.
- **Partial credit** for a linear nth term usually comes from getting either the n part ($7n$) or the number (-3) right, so write down your difference and zero term.
- **Extended only: for a quadratic or cubic** found by differences, the method mark is for showing the second (or third) differences or for an answer of the right type. Write the rows of differences out.
- **"Write down"** means little or no working is needed. **"Find"** or **"Work out"** means show the method, such as the equation you solve.
- **"Is … a term? Give a reason"** needs the equation, its solution and a sentence: "$n=21.857\ldots$ is not a whole number, so no". "Yes" or "no" alone earns nothing.
- **Term-to-term rules** in words with the operation and the number: "add 7", "multiply by 3", "subtract 4". A two-step rule needs both steps in order.
- **"Give your answers as fractions"** means fractions, not decimals.
- **Draw patterns neatly on the grid**, with the same layout as the ones given; the mark is for a correct drawing.
- **Method marks follow through**: if your nth term is wrong but you use it correctly in the next part, you still get the method mark, so keep going.

Quick check

1. Write down the first three terms of the sequence with nth term (a) $20-3n$, (b) $2n^2-1$.
2. These are the first four terms of a sequence: $31, 26, 21, 16$. Find the nth term, and find which term is -49.
3. The term-to-term rule for a sequence is "multiply by 2 then add 3". The 5th term is 93. Find the 4th term and the 3rd term.
4. (a) Write down the next term of $2, 3, 5, 8, 12, \ldots$ (b) Find the nth term of $-4, -1, 4, 11, 20, \ldots$
5. **Extended only:** find the nth term of (a) $3, 12, 48, 192, \ldots$ (b) $4, 12, 26, 46, 72, \ldots$

Answers

1. (a) $17, 14, 11$. (b) $2 \times 1 - 1 = 1, 2 \times 4 - 1 = 7, 2 \times 9 - 1 = 17$: $1, 7, 17$.
2. $d = -5$, zero term $31 + 5 = 36$, so $36 - 5n$. Then $36 - 5n = -49$ gives $5n = 85$ and $n = 17$: the 17th term.
3. Undo in reverse: subtract 3, then divide by 2. 4th term $= (93 - 3) \div 2 = 45$; 3rd term $= (45 - 3) \div 2 = 21$. (Check: $21 \times 2 + 3 = 45$ and $45 \times 2 + 3 = 93$.)
4. (a) The differences are $1, 2, 3, 4$, so the next difference is 5 and the next term is 17. (b) The second differences are 2; each term is 5 less than $1, 4, 9, 16, 25$, so the nth term is $n^2 - 5$.
5. (a) Multiply by 4 each time, first term 3: $3 \times 4^{n-1}$. (b) First differences $8, 14, 20, 26$; second differences 6, so $3n^2$. Subtracting $3, 12, 27, 48, 75$ leaves $1, 0, -1, -2, -3$, which is $2 - n$. The nth term is $3n^2 - n + 2$.

Past questions to try

- **0580/12/O/N/21 Q17**: the next term, the term-to-term rule and the nth term of a linear sequence, then its 40th term.
- **0580/33/M/J/23 Q5**: a pattern of stick diagrams: draw the next one, complete the table, find the nth term and which diagram has 73 sticks, then use a given formula for the total number of sticks.
- **0580/42/M/J/23 Q6**: **Extended only:** terms of a fraction nth term and the position of a given term, then the nth terms of a cubic sequence and a halving sequence.
- **0580/22/M/J/22 Q21**: **Extended only:** find a and b in an nth term $an^2 + bn - 4$ from the first two terms.