Cambridge IGCSE · Mathematics 0580 · Notes

Surface area and volume

Read online at **pastopic.com/cambridge/mathematics-0580/notes/surface-area-and-volume**

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Read first: **Units, area and perimeter**, **Circles, arcs and sectors**

What you need to know

By the end of this topic you should be able to:

1. **Work out the volume and the surface area of a cuboid** (including a cube), and solve problems with them, such as finding a missing length.
2. **Work out the volume and the surface area of a prism**: any solid with the same cross-section all the way along, such as a triangular prism or a prism with a trapezium cross-section.
3. **Work out the volume, the curved surface area and the total surface area of a cylinder.**
4. **Work out the volume and the surface area of a pyramid.**
5. **Work out the volume, the curved surface area and the total surface area of a cone.**
6. **Work out the volume and the surface area of a sphere**, and of a hemisphere (half a sphere).
7. **Work out the surface area and volume of compound solids and parts of solids**: solids joined together (a cone on a hemisphere, a cylinder with a hemisphere on the end) and part of a solid (a hemisphere, a half-cylinder).
8. **Extended only:** work out the surface area and volume of a **frustum**, the part of a cone left when the top is cut off.
9. **Solve problems with these solids**: work backwards from a volume or an area to a length, compare solids with equal volumes or areas, and use volumes for mass, capacity and filling. Answers may be asked for **in terms of** π.
10. **Use nets**: draw the net of a cuboid, prism or pyramid, and use the measurements on a net to work out volumes and surface areas.

Core and Extended share outcomes 1–7, 9 and 10; frustums are Extended only. Extended papers (2 and 4) also combine the solids with algebra; the examples marked **Extended** show this. Changing units such as cm^3 to litres is in Units, area and perimeter, and angles inside solids are in Trigonometry in 3D.

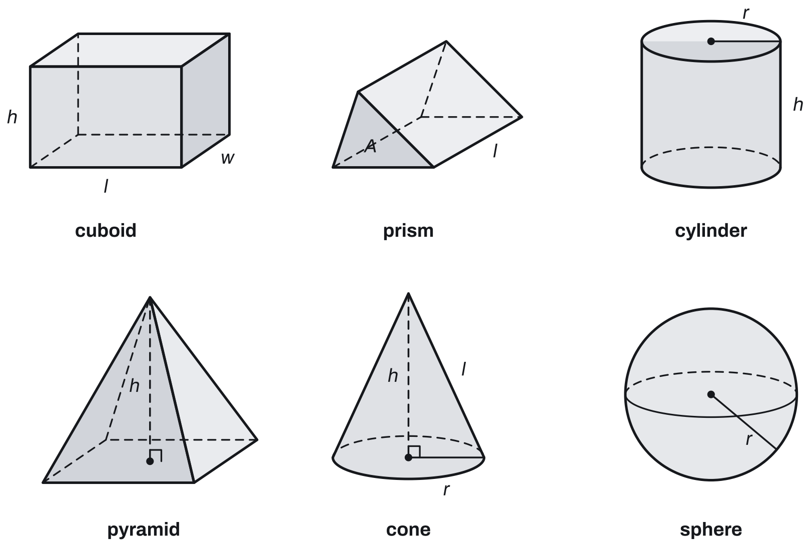
Understand it

Volume and surface area

The **volume** of a solid is the space inside it, measured in cubic units: cm³, m³. A cube with edges 1 cm long has a volume of 1 cm³, so volume counts how many of these little cubes would fill the solid.

The **surface area** is the total area of all its outside faces, measured in square units: cm², m². Imagine wrapping the solid in paper with no overlaps: the area of paper you need is the surface area.

These are the six solids in this topic, with the lengths their formulas use:



Cuboids

A cuboid of length l, width w and height h holds $l \times w$ cubes in each layer and has h layers, so

$$V = lwh.$$

It has six rectangular faces in three matching pairs (top and bottom, front and back, the two ends), so

$$\text{surface area} = 2lw + 2lh + 2wh.$$

A cuboid 5 cm by 4 cm by 3 cm has volume $5 \times 4 \times 3 = 60$ cm³ and surface area $2(20 + 15 + 12) = 94$ cm². A **cube** of side x has $V = x^3$ and surface area $6x^2$.

Prisms and cylinders

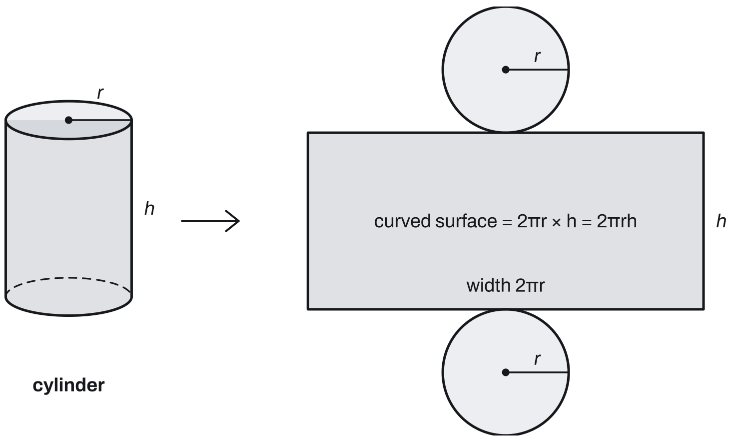
A **prism** has the same cross-section all the way along its length. Every slice is the same shape, so

$$V = \text{area of cross-section} \times \text{length} = Al.$$

A triangular prism whose cross-section has base 6 cm and height 4 cm, and whose length is 10 cm, has $A = \frac{1}{2} \times 6 \times 4 = 12$ cm² and $V = 12 \times 10 = 120$ cm³. The cross-section can be any shape: a trapezium (a water trough), a semicircle (a half-cylinder), a sector (a cake with a slice cut out). Find its area first, using the area formulas you already know, then multiply by the length.

A **cylinder** is a prism with a circle as its cross-section, so $V = \pi r^2 h$.

Its surface is two circles and a curved part. Cut the curved part straight down and unroll it: it becomes a rectangle. Its height is h and its width is the distance round the circle, $2\pi r$.



$$\text{curved surface area} = 2\pi rh \qquad \text{total surface area} = 2\pi rh + 2\pi r^2.$$

A cylinder with $r = 4$ cm and $h = 10$ cm has $V = \pi \times 16 \times 10 = 160\pi = 503$ cm³, curved surface 80π cm², and total surface $80\pi + 32\pi = 112\pi = 352$ cm² (3 s.f.). An **open** cylinder, such as a cup, has only one end: $2\pi rh + \pi r^2$. A pipe or a label has no ends: just $2\pi rh$.

The surface area of any prism is the two ends plus the rectangles round the sides. A net (the solid opened out flat) shows you every face, so none gets missed.

Pyramids and cones

A **pyramid** rises from a base to a single point. It holds exactly one third of the prism with the same base and the same height, so

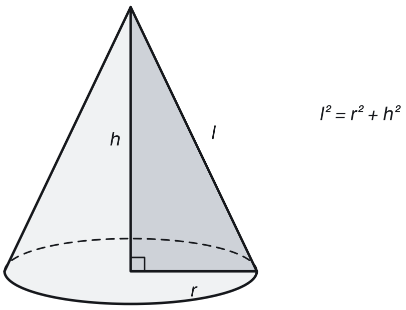
$$V = \tfrac{1}{3} \times \text{base area} \times h,$$

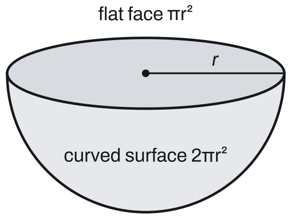
where h is the **perpendicular** height, measured straight up from the base to the top, not along a sloping edge. Its surface area is the base plus the triangles. For a square-based pyramid with base 10 cm and height 12 cm: $V = \frac{1}{3} \times 100 \times 12 = 400$ cm³. Each triangular face has height $\sqrt{12^2 + 5^2} = 13$ cm (Pythagoras, using half the base), so the surface area is $100 + 4 \times \frac{1}{2} \times 10 \times 13 = 360$ cm².

A **cone** is a pyramid with a circular base, so $V = \frac{1}{3}\pi r^2 h$. Its curved surface uses the **slant height** l, the length down the sloping side:

$$\text{curved surface area} = \pi rl \qquad \text{total surface area} = \pi rl + \pi r^2.$$

The radius, the perpendicular height and the slant height make a right-angled triangle, so $l^2 = r^2 + h^2$. You often need this to get l from h, or h from l.





A cone with $r = 5$ cm and $h = 12$ cm has $l = \sqrt{25+144} = 13$ cm, $V = \frac{1}{3} \times \pi \times 25 \times 12 = 100\pi = 314$ cm³, curved surface $\pi \times 5 \times 13 = 65\pi$ cm² and total surface $65\pi + 25\pi = 90\pi = 283$ cm² (3 s.f.).

Spheres and hemispheres

A sphere of radius r has

$$V = \tfrac{4}{3}\pi r^3 \qquad \text{surface area} = 4\pi r^2.$$

With $r = 6$ cm: $V = \frac{4}{3} \times \pi \times 216 = 288\pi = 905$ cm³ and surface area $4 \times \pi \times 36 = 144\pi = 452$ cm².

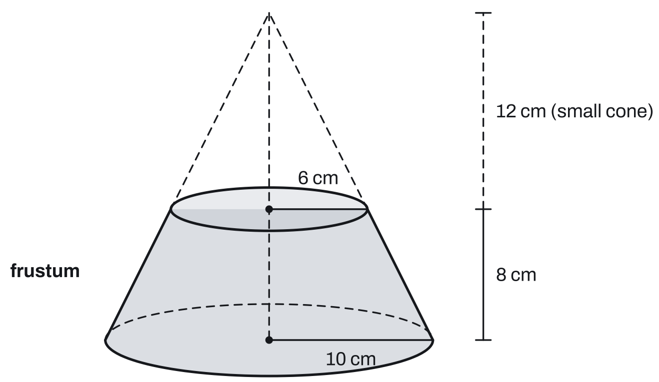
A **hemisphere** is half a sphere. Its volume is half: $\frac{2}{3}\pi r^3$. Its surface is **half the sphere's surface plus a flat circle**: curved $2\pi r^2$, flat face πr^2, total $3\pi r^2$. For $r = 6$: volume 144π, curved surface 72π and total 108π.

Solids made from other solids

A toy, a silo or a capsule is often a cylinder, cone and hemisphere joined together.

- **Volume**: add the volumes of the parts.
- **Surface area**: add **only the faces on the outside**. Where two parts are joined, the faces touching each other are hidden. A cone on top of a hemisphere (same radius) has surface $\pi r l + 2\pi r^2$, with no circles at all. A cylinder with a hemisphere on one end has one flat circle (the other end), the curved surface of the cylinder, and $2\pi r^2$ for the hemisphere.

Extended only: cut the top off a cone with a cut parallel to the base and the bottom part is a **frustum**. Its volume is the large cone minus the small cone. The small cone is a smaller copy of the large one (they are similar), so their radii are in the same ratio as their heights, and so are their slant heights. Its **curved surface** is the large cone's curved surface minus the small cone's, $\pi R l_{\text{large}} - \pi r l_{\text{small}}$; for the **total surface** of a solid frustum, add the two flat circles, $\pi R^2 + \pi r^2$.



Working backwards

If you know a volume or an area, put it into the formula and solve for the missing length. Rearrange first, then use the calculator once.

- A cylinder with volume 500 cm³ and height 8 cm: $\pi r^2 \times 8 = 500$, so $r^2 = \dfrac{500}{8\pi}$ and $r = 4.46$ cm. You **square root** at the end because the formula has r^2.
- A sphere with volume 1000 cm³: $\frac{4}{3}\pi r^3 = 1000$, so $r^3 = \dfrac{3000}{4\pi}$ and $r = 6.20$ cm. You **cube root** at the end because the formula has r^3.
- A cube with surface area 150 cm²: $6x^2 = 150$, so $x^2 = 25$ and $x = 5$ cm.

Equal volumes, melting and filling

When a solid is **melted and made into** another shape, the volume stays the same. Set the two volume expressions equal and solve. If the question says the two solids have equal surface areas, set the areas equal instead.

When water is poured from one container into another, the **volume of water** is what stays the same. When a solid is dropped into water and sinks completely, the water rises by the solid's volume: the rise in level is $\dfrac{\text{volume of the solid}}{\text{area of the water surface}}$.

For "what percentage is empty" or "left over", work out the unused volume, divide by the **whole** starting volume and multiply by 100.

Mass, density and capacity

Density is the mass of each cm³ (or m³) of a material: $\text{density} = \dfrac{\text{mass}}{\text{volume}}$, so $\text{mass} = \text{density} \times \text{volume}$. A cylinder of volume 200 cm³ made of a metal with density 8 g/cm³ has mass 1600 g $= 1.6$ kg. The question prints the density formula, but it expects you to use volume, never surface area.

Capacity is the volume a container holds. Learn: 1 cm³ $= 1$ ml, 1000 cm³ $= 1$ litre, 1 m³ $= 1000$ litres. So a tank holding 2.5 litres has capacity 2500 cm³.

Key facts and formulas

"Given" means it is on the List of formulas printed on page 2 of every 0580 paper (Core and Extended). "Learn it" means you must remember it.

Formula	Status
Volume of a prism $V = Al$ (A = cross-section area, l = length)	Given
Volume of a pyramid $V = \frac{1}{3}Ah$ (A = base area)	Given
Volume of a cylinder $V = \pi r^2 h$	Given
Curved surface area of a cylinder $A = 2\pi rh$	Given
Volume of a cone $V = \frac{1}{3}\pi r^2 h$	Given
Curved surface area of a cone $A = \pi rl$ (l = slant height)	Given
Volume of a sphere $V = \frac{4}{3}\pi r^3$	Given
Surface area of a sphere $A = 4\pi r^2$	Given
Area of a circle πr^2; area of a triangle $\frac{1}{2}bh$	Given
Cuboid: $V = lwh$, surface area $= 2lw + 2lh + 2wh$; cube: $V = x^3$, surface area $6x^2$	Learn it
Total surface area of a cylinder $= 2\pi rh + 2\pi r^2$	Learn it
Total surface area of a cone $= \pi rl + \pi r^2$, with $l^2 = r^2 + h^2$	Learn it
Hemisphere: $V = \frac{2}{3}\pi r^3$, curved surface $2\pi r^2$, total surface $3\pi r^2$	Learn it
Extended: frustum volume $=$ large cone $-$ small cone	Learn it
Extended: frustum curved surface $=$ large cone's $\pi Rl -$ small cone's πrl; total surface adds $\pi R^2 + \pi r^2$	Learn it
Density $= \dfrac{\text{mass}}{\text{volume}}$ (printed in the question when needed)	Learn it
1 cm³ $= 1$ ml, 1000 cm³ $= 1$ litre, 1 m³ $= 1000$ litres $= 1\,000\,000$ cm³	Learn it

Use π from your calculator or 3.142. On Papers 1 and 2 (no calculator) leave π in the answer.

How to do it

In the questions we have tagged for this topic (87 questions, 84 different, 2021–2025), the types came up this often. 55 of the 84 different questions mix two or more types, so the counts add up to more than 84.

Question type	Questions
Volume of cuboids, prisms, cylinders and pyramids	36
Finding a length from a volume or surface area	30
Cones, spheres, hemispheres and solids made from them	29
Surface area of cuboids, prisms, cylinders and pyramids	26
Mass, density and capacity	18
Nets, faces and edges	16
Equal volumes or areas, melting, filling and packing	15
Frustums (Extended only)	3

1. Volume of cuboids, prisms, cylinders and pyramids (36 questions)

1. **Name the solid** and write its formula: cuboid lwh, prism Al, cylinder $\pi r^2 h$, pyramid $\frac{1}{3}Ah$.
2. **For a prism, find the cross-section area first** (triangle $\frac{1}{2}bh$, trapezium $\frac{1}{2}(a+b)h$, semicircle $\frac{1}{2}\pi r^2$ ...), then multiply by the length.
3. **Substitute and calculate.** Give the units (cm³, m³) and 3 significant figures unless the answer is exact.

For example, a trough whose cross-section is a trapezium with parallel sides 30 cm and 50 cm and height 20 cm, and whose length is 150 cm: $A = \frac{1}{2}(30 + 50) \times 20 = 800$ cm², so $V = 800 \times 150 = 120\,000$ cm³ $= 120$ litres.

Variations you will meet:

- **All lengths in the same unit.** A 1.2 m length with a cross-section in cm: change it to 120 cm before multiplying.
- **A cuboid built from 1 cm cubes**: the volume is the number of cubes. A different cuboid with the same volume has three whole-number sides with the same product ($24 = 2 \times 3 \times 4 = 1 \times 4 \times 6$ ...).
- **"In terms of π"**: a cylinder with $r = 8$, $h = 12$ has $V = 768\pi$ cm³.
- **An unusual cross-section**: a regular polygon, a sector or a segment of a circle. Work out its area with the methods from Circles, arcs and sectors or trigonometry, then multiply by the length.
- **Volume of empty space**: the container's volume minus what is inside it.
- **A volume written with algebra**, such as $V = x^2(9 - x)$ for a cuboid with a square base. Substitute values for a table or graph, or form and solve an equation.

2. Finding a length from a volume or surface area (30 questions)

1. Write the formula with the numbers you know, set it equal to the given value.
2. **Rearrange for the unknown**: divide by everything else first.
3. Take the **square root** if the unknown is squared (r^2, x^2) and the **cube root** if it is cubed (r^3, x^3).

For example, a cuboid with volume 357 cm³, length 8.5 cm and width 6 cm has height $357 \div (8.5 \times 6) = 7$ cm.

Variations you will meet:

- **Cylinder radius from volume and height**: $r = \sqrt{\frac{V}{\pi h}}$. **Height**: $h = \frac{V}{\pi r^2}$.
- **Cube side from surface area**: $x = \sqrt{\text{area} \div 6}$. From volume: $x = \sqrt[3]{V}$. A cube with volume 125 m³ has side 5 m and surface area 150 m².
- **A square base from volume and height**: side $= \sqrt{V \div h}$.
- **Sphere or hemisphere radius from volume**: $r = \sqrt[3]{\frac{3V}{4\pi}}$ for a sphere, $r = \sqrt[3]{\frac{3V}{2\pi}}$ for a hemisphere.
- **Surface area given in terms of** π: a cylinder with radius 5 has total surface area 170π, so $2\pi \times 5 \times h + 2\pi \times 25 = 170\pi$. Divide by π: $10h + 50 = 170$, so $h = 12$.
- **Slant height from a surface area**, then Pythagoras if the perpendicular height is wanted.
- **Height of water in a container**: height $=$ volume of water $\div$ base area.

3. Cones, spheres, hemispheres and solids made from them (29 questions)

1. **Split the solid into parts** and list each part's radius, height and slant height.
2. **Volume**: add the parts' volumes. Halve the sphere formula for a hemisphere.
3. **Surface area**: add only the outside faces: the cone's curved surface $\pi r l$, the cylinder's curved surface $2\pi r h$, a hemisphere's curved surface $2\pi r^2$, plus any flat circle that is on the outside.
4. **Missing slant or perpendicular height**: $l^2 = r^2 + h^2$.

For example, a cylinder of radius 3 cm and length 10 cm with a hemisphere on one end: surface area $= \pi \times 9 + 2\pi \times 3 \times 10 + 2\pi \times 9 = 9\pi + 60\pi + 18\pi = 87\pi$ cm².

Variations you will meet:

- **"Show that the total surface area is 1131 cm², correct to the nearest cm²"**: write each part with the numbers in, give a value with more figures (such as $1130.97\ldots$), then round.
- **Cone from its perpendicular height**: find l by Pythagoras before the curved surface area.
- **A diameter in the question**: halve it. A sphere with diameter 4.8 cm has $r = 2.4$ cm.
- **A container made from a hemisphere, cylinder and cone**: add the three volumes; if the total is given, solve for the missing height.
- **A percentage of the surface**: a label on the curved surface as a percentage of the total surface is $\frac{\pi r l}{\pi r l + \pi r^2} \times 100$.

- **Painting**: cost per cm² $=$ total cost $\div$ surface area. Number of solids one tin covers $=$ tin's area $\div$ one solid's area (same units; round **down** for complete solids).
- **A sphere that just fits inside a cube** (it touches all six faces): its diameter equals the cube's side, so a cube of side 10 cm holds a sphere of radius 5 cm. A cylinder that just fits in a cube has the same radius and a height equal to the side.
- **A cone made from a sector**: the sector's radius is the slant height and its arc is the base circumference (see Circles, arcs and sectors).

4. Surface area of cuboids, prisms, cylinders and pyramids (26 questions)

1. **List every face** (a sketch or net helps): a cuboid has three pairs; a prism has two ends and a rectangle for each side of the cross-section; a pyramid has the base and triangles.
2. **Find each area**, using Pythagoras for any missing length (the sloping side of a triangular prism, the height of a pyramid's triangles).
3. **Add them.** For a cuboid, $2(lw + lh + wh)$: don't forget to double.

For example, a cuboid 8 cm by 6 cm by 3 cm: $2(48 + 24 + 18) = 180$ cm².

Variations you will meet:

- **Open box (no lid)**: leave out one face. A box $5 \times 4 \times 2$ with an open top has area $20 + 2(10 + 8) = 56$ cm².
- **Total surface of a cylinder** $= 2\pi rh + 2\pi r^2$; the curved surface alone is $2\pi rh$. Read which one is asked.
- **A surface area as an algebraic expression**, such as a cuboid x by x by $(9 - x)$: $2x^2 + 4x(9 - x) = 36x - 2x^2$.
- **Surface area from a net** with the lengths marked: add the areas of the faces you see.

5. Mass, density and capacity (18 questions)

1. **Find the volume** of the solid.
2. **Mass** $=$ **density** $\times$ **volume**, with matching units (g/cm³ with cm³; kg/m³ with m³).
3. **Convert** at the end if asked: 1000 g $= 1$ kg; 1000 cm³ $= 1$ litre.

Variations you will meet:

- **Density from mass and volume**: density $=$ mass $\div$ volume, with its units (g/cm³ or kg/m³). Change the lengths to metres **before** working out the volume if the answer must be in kg/m³.
- **Capacity in litres**: work out the volume in cm³ and divide by 1000.
- **Filling at a rate**: time $=$ volume $\div$ rate. 120 litres at 500 ml per second takes $120\,000 \div 500 = 240$ s $= 4$ minutes.
- **Flow through a pipe**: the volume each second is the pipe's cross-section area $\times$ the distance the liquid moves each second.

- **Emptying by mass** (grain removed at so many kg per hour): mass $=$ density $\times$ volume, then time $=$ mass $\div$ rate.

6. Nets, faces and edges (16 questions)

1. **A net is the surface opened out flat.** A cuboid's net has six rectangles; a triangular prism's has two triangles and three rectangles; a square-based pyramid's has a square and four triangles.
2. **Drawing a net on a grid**: draw one face, then attach each other face along the edge it shares, using the real lengths. Check that the edges which will meet when folded are the same length.
3. **Reading a net**: its face sizes give the solid's dimensions, so you can find the volume and surface area.

Variations you will meet:

- **Faces, edges and vertices**: a cuboid has 6 faces, 12 edges, 8 vertices; a triangular prism $5, 9, 6$; a square-based pyramid $5, 8, 5$.
- **Which corners meet** when the net is folded: fold it in your head one face at a time, or count along the edges.
- **Name the solid** from its net: rectangles and two triangles give a triangular prism.
- **An open box**: its net has only five faces.

7. Equal volumes or areas, melting, filling and packing (15 questions)

1. **Write each solid's volume** (or surface area) as an expression.
2. **Set them equal** and solve for the unknown. Cancel π from both sides when it is in every term.
3. For several copies, multiply one volume by the number of copies.

For example, a sphere of radius 3 cm is melted into a cylinder of radius 2 cm: $\pi \times 4 \times h = \frac{4}{3}\pi \times 27 = 36\pi$, so $h = 9$ cm.

Variations you will meet:

- **Water level rise or fall**: rise $=$ solid's volume $\div$ area of the water surface. Pouring into another container: the volume of water is the same, so height $=$ volume $\div$ new base area.
- **Percentage empty or left over**: $\dfrac{\text{whole} - \text{used}}{\text{whole}} \times 100$. Divide by the **whole**, not by the part used.
- **Algebra (Extended)**: radii written as $2R$ or x. Collect the terms, cancel π, and give the answer asked for, such as R in terms of x or $r = \sqrt{\frac{7}{2}}\,x$.
- **Packing boxes**: count how many fit along each edge (round **down**), then multiply the three counts. Check the lengths are in the same units.

8. Frustums (Extended) (3 questions)

Extended only.

1. **Complete the cone**: find the height of the whole cone and of the small cone that was removed.

2. **Use similar triangles**: $\frac{\text{small radius}}{\text{large radius}} = \frac{\text{small height}}{\text{large height}}$.
3. **Volume** $=$ **large cone** $-$ **small cone**, both with $\frac{1}{3}\pi r^2 h$.
4. **Surface area**: find each cone's slant height by Pythagoras, then curved surface $=$ large $\pi R l$ $-$ small $\pi r l$. Add the two circles for a solid frustum (only the bottom one for an open container such as a bucket).

If you are given the frustum's height and the small cone's height, add them for the large cone's height. If you are given a diameter, halve it.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

Core, calculator allowed.

A box is a cuboid 8 cm long, 5 cm wide and 3.5 cm high.

(a) Work out the volume of the box. **[2]**

(b) Work out the total surface area of the box. **[3]**

(c) A cube has a total surface area of 150 cm². Work out the volume of the cube. **[3]**

(a) $8 \times 5 \times 3.5$ **M1** $= 140$ cm³. **A1**

(b) The three different faces are $8 \times 5 = 40$, $8 \times 3.5 = 28$ and $5 \times 3.5 = 17.5$. **M1** for at least one pair's area.

$$2 \times 40 + 2 \times 28 + 2 \times 17.5 \quad \mathbf{M1}$$

$= 171$ cm². **A1**

(c) A cube has six equal square faces, so each face is $150 \div 6 = 25$ cm². **M1**

The side is $\sqrt{25} = 5$ cm **M1**, so the volume is $5^3 = 125$ cm³. **A1**

Example 2

A solid metal cylinder has radius 3.5 cm and height 12 cm.

(a) Calculate the volume of the cylinder. **[2]**

(b) Calculate the total surface area of the cylinder. **[3]**

(c) The density of the metal is 8.9 g/cm³. Calculate the mass of the cylinder. Give your answer in kilograms. [Density = mass ÷ volume] **[3]**

(a) $\pi \times 3.5^2 \times 12$ **M1** $= 147\pi = 461.8\ldots = 462$ cm³ (3 s.f.). **A1**

(b) Two circles: $2 \times \pi \times 3.5^2 = 76.96\ldots$ **M1**. Curved surface: $2 \times \pi \times 3.5 \times 12 = 263.89\ldots$ **M1**

Total $= 340.8\ldots = 341$ cm² (3 s.f.). **A1**

(c) Mass $= 8.9 \times 461.8\ldots$ **M1** $= 4110.1\ldots$ g **A1**, which is 4.11 kg (3 s.f.). **B1** for changing g to kg (divide by 1000).

Keep the full volume from (a) in your calculator; using 462 gives 4111.8 g, which still rounds to 4.11 kg here but can push other answers out of range.

Example 3

Extended.

A solid is made from a cone and a hemisphere joined at their circular faces. Both have radius 6 cm. The perpendicular height of the cone is 8 cm.

(a) Show that the slant height of the cone is 10 cm. **[1]**

(b) Find the total surface area of the solid. Give your answer in terms of π. **[3]**

(c) Find the volume of the solid. Give your answer in terms of π. **[3]**

(d) The solid is melted down and made into a sphere. Calculate the radius of the sphere. **[3]**

(a) $l = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ cm. **B1**

(b) The joined circles are inside the solid, so only the two curved surfaces count.

Cone: $\pi \times 6 \times 10 = 60\pi$. **M1** Hemisphere: $\frac{1}{2} \times 4 \times \pi \times 6^2 = 72\pi$. **M1**

Total $= 132\pi$ cm². **A1**

(c) Cone: $\frac{1}{3} \times \pi \times 6^2 \times 8 = 96\pi$. **M1** Hemisphere: $\frac{1}{2} \times \frac{4}{3} \times \pi \times 6^3 = 144\pi$. **M1**

Total $= 240\pi$ cm³. **A1**

(d) The volume stays the same: $\frac{4}{3}\pi r^3 = 240\pi$. **M1**

$r^3 = 240 \times \frac{3}{4} = 180$ **M1**, so $r = \sqrt[3]{180} = 5.646\ldots = 5.65$ cm (3 s.f.). **A1**

Example 4

Extended.

A solid cone has base radius 10 cm and height 20 cm. A cut is made parallel to the base, 8 cm above it, and the small cone above the cut is removed. The solid that remains is a frustum (the fourth diagram in "Understand it").

(a) Show that the radius of the top of the frustum is 6 cm. **[2]**

(b) Calculate the volume of the frustum. **[3]**

(c) Calculate the total surface area of the frustum. **[4]**

(a) The small cone has height $20 - 8 = 12$ cm. **M1** It is similar to the large cone, so

$$\frac{r}{10} = \frac{12}{20} \;\Rightarrow\; r = 10 \times \frac{12}{20} = 6 \text{ cm.} \quad \textbf{A1}$$

(b) Large cone: $\frac{1}{3} \times \pi \times 10^2 \times 20 = \frac{2000}{3}\pi$. **M1** Small cone: $\frac{1}{3} \times \pi \times 6^2 \times 12 = 144\pi$. **M1**

Frustum $= \frac{2000}{3}\pi - 144\pi = \frac{1568}{3}\pi = 1642.0\ldots = 1640$ cm³ (3 s.f.). **A1**

(c) Slant heights: large cone $\sqrt{10^2 + 20^2} = \sqrt{500} = 22.36\ldots$, small cone $\sqrt{6^2 + 12^2} = \sqrt{180} = 13.41\ldots$ **M1**

Curved surface $= \pi \times 10 \times 22.36\ldots - \pi \times 6 \times 13.41\ldots = 702.4\ldots - 252.8\ldots = 449.5\ldots$ **M1**

The two circles: $\pi \times 10^2 + \pi \times 6^2 = 136\pi = 427.2\ldots$ **M1**

Total $= 449.5\ldots + 427.2\ldots = 876.8\ldots = 877$ cm² (3 s.f.). **A1**

Common mistakes

- **Mixing up volume and surface area.** Examiners often see $8 \times 6 \times 3$ given as a surface area. Volume multiplies three lengths (cm³); surface area adds face areas (cm²).
- **Not doubling a cuboid's faces**, or using four of one face and two of another. There are three pairs: find three areas, add, then double.
- **Using only the curved surface when the total is asked**, or leaving out the flat base of a cone or the ends of a cylinder. Reports mention this every year. "Total surface area" means every outside face.
- **Adding hidden faces in a joined solid**, or using the full sphere's $4\pi r^2$ for a hemisphere. A hemisphere's curved surface is $2\pi r^2$; add πr^2 only if its flat face is on the outside.
- **Using the full sphere for a hemisphere's volume**, then, when working backwards, halving the radius instead of the volume.
- **Square root instead of cube root.** If the formula has r^3, the last step is a cube root.
- **Slant height and perpendicular height confused.** $\pi r l$ needs the slant height; $\frac{1}{3}\pi r^2 h$ needs the perpendicular height. Use Pythagoras to get one from the other.

- **Diameter used as radius.** Halve the diameter before squaring or cubing.
- **Units.** Examiners saw cm³ divided by 100 to get m³ (it is $1\ 000\ 000$). Change the lengths to metres first if the answer is in m³ or kg/m³. 1 litre $= 1000$ cm³.
- **Percentage of the wrong total.** "Percentage left over" divides the leftover by the whole starting amount.
- **Rounding too early.** In long questions keep full values in the calculator (or keep π as a letter) until the last line; premature rounding often pushes the final answer outside the accepted range.

How to get the marks

- **Write the calculation with the numbers in**, such as $\pi \times 3.1^2 \times h = \frac{256}{3}\pi$. That earns the method mark even if the arithmetic slips.
- **"Show that"** means the answer is printed. Show every substitution and step, including a value with more figures than the rounded answer, such as $268.08\ldots$ before "268". Examiners withhold marks when the substitution or the division step is missing.
- **"In terms of π"**: keep π as a letter and simplify the number, 132π, not 414.7. On Papers 1 and 2 this is how you give an exact answer.
- **Accuracy**: give non-exact answers to 3 significant figures unless told otherwise. Use the calculator's π or 3.142.
- **Units**: when a question says "give the units of your answer", the unit earns a mark (cm³ for a volume, cm² for an area, g/cm³ for a density).
- **Use the List of formulas** on page 2: it has every curved surface area and volume formula except the cuboid's. Copy the formula, then substitute.
- **Method marks follow through**: a wrong volume from part (a), used correctly for the mass in part (b), still earns the method mark, so finish the question.

Quick check

1. A triangular prism has length 15 cm. Its cross-section is a right-angled triangle with the two shorter sides 6 cm and 8 cm. Work out its volume.
2. A cylinder has volume 1000 cm³ and height 10 cm. Calculate its radius.
3. A sphere has radius 3 cm. Find its volume and its surface area, in terms of π.
4. A square-based pyramid has base side 6 cm and perpendicular height 4 cm. Work out its volume and its total surface area.
5. A cylinder of radius 8 cm contains water. A solid metal cube of side 5 cm is placed in the water and is completely covered. Calculate how far the water level rises.

Answers

1. Cross-section $= \frac{1}{2} \times 6 \times 8 = 24$ cm². Volume $= 24 \times 15 = 360$ cm³.
2. $\pi r^2 \times 10 = 1000$, so $r^2 = \dfrac{1000}{10\pi} = 31.83\ldots$ and $r = 5.64$ cm (3 s.f.).
3. Volume $= \frac{4}{3} \times \pi \times 27 = 36\pi$ cm³. Surface area $= 4 \times \pi \times 9 = 36\pi$ cm².
4. Volume $= \frac{1}{3} \times 36 \times 4 = 48$ cm³. Each triangle has height $\sqrt{4^2 + 3^2} = 5$ cm, so the surface area $= 36 + 4 \times \frac{1}{2} \times 6 \times 5 = 96$ cm².
5. The cube's volume is 125 cm³. Rise $= \dfrac{125}{\pi \times 8^2} = 0.622$ cm (3 s.f.).

Past questions to try

- **0580/22/M/J/25 Q16**: the surface area of a cylinder with a hemisphere, in terms of π.
- **0580/32/M/J/25 Q23**: a cone with the same volume as a sphere.
- **0580/43/O/N/22 Q5**: a trapezium prism, litres, emptying time and a cylindrical tank.
- **0580/22/O/N/22 Q21**: the volume of a frustum.