

Types of number and sets

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What you need to know

This topic is the same for Core and Extended, except for the set work marked **Extended**. By the end of it you should be able to:

1. **Recognise and use the main types of number:** natural numbers, integers (positive, zero and negative), prime numbers, square numbers, cube numbers, factors, multiples, common factors and common multiples.
2. **Write a number as a product of its prime factors**, for example $72 = 2^3 \times 3^2$, and use this to find the **highest common factor (HCF)** and the **lowest common multiple (LCM)** of two numbers.
3. **Tell rational and irrational numbers apart**, and write down an irrational number between two given values.
4. **Find the reciprocal** of a whole number, a fraction, a mixed number or a decimal.
5. **Write large numbers in words and in figures**, for example six billion is 6 000 000 000.
6. **Use set language and notation:** $n(A)$ for the number of elements in A , A' for the complement of A , ξ for the universal set, $A \cup B$ for the union and $A \cap B$ for the intersection. Read sets written as lists, such as $\{a, b, c\}$, or with a rule, such as $\{x : x \text{ is a natural number}\}$ or $\{x : 2 \leq x \leq 9\}$.
7. **Draw and use Venn diagrams:** two sets for Core, up to three sets for Extended. Fill them in with elements or with numbers of elements, shade a region described in set notation, and describe a shaded region in set notation.
8. **Extended only:** use $\in$ ("is an element of"), $\notin$ ("is not an element of"), $\emptyset$ (the empty set), $A \subseteq B$ (" A is a subset of B ") and $A \not\subseteq B$ (" A is not a subset of B "), and read sets of points such as $\{(x, y) : y = 2x + 1\}$. Show how sets are related: a subset as a circle inside another circle, and sets with no common elements as separate circles.

Squares, cubes and their roots are worked out in the next topic, Powers, roots, indices and surds; here you only need to recognise square and cube numbers.

Understand it

Kinds of number

- **Natural numbers** are the counting numbers 1, 2, 3, 4, ...

- **Integers** are the whole numbers, including zero and the negatives: $\dots, -2, -1, 0, 1, 2, \dots$. So -7 and 0 are integers, but 2.5 is not.
- **Even** numbers divide exactly by 2; **odd** numbers leave a remainder of 1.
- **Square numbers** come from multiplying a whole number by itself:
 $1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, \dots$
- **Cube numbers** come from using a whole number three times: $1, 8, 27, 64, 125, \dots, 1000 (= 10^3)$. Some numbers, such as 1 and 64, are both square and cube.

Factors and multiples

A **factor** of n divides into n exactly. A **multiple** of n is in the n times table. So 6 is a factor of 30, and 30 is a multiple of 6.

Find **all** the factors of a number in pairs, starting from 1 and stopping when the pairs meet:

$$48 : \quad 1 \times 48, 2 \times 24, 3 \times 16, 4 \times 12, 6 \times 8$$

so the factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48. Working in pairs means you never miss one, and you remember that 1 and the number itself are factors too.

A **common factor** of two numbers is a factor of both; a **common multiple** is a multiple of both. The common factors of 18 and 24 are 1, 2, 3, 6. Some common multiples of 18 and 24 are 72, 144, 216 (any multiple of 72).

Prime numbers

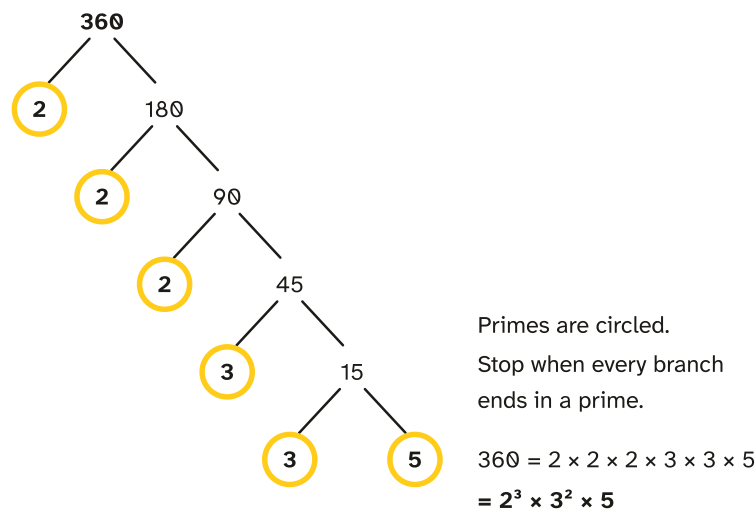
A **prime number** has exactly two factors: 1 and itself. The primes below 50 are

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.$$

- 1 is **not** prime: it has only one factor.
- 2 is the only even prime. Every other even number has 2 as a factor.
- To test a number, try dividing by the primes 2, 3, 5, 7, 11, $\dots$ up to its square root. $91 = 7 \times 13$ and $111 = 3 \times 37$ look prime but are not. A quick test for 3: add the digits; $1 + 1 + 1 = 3$ is a multiple of 3, so 111 is too.

Products of prime factors

Every whole number above 1 can be written as primes multiplied together in exactly one way (apart from the order). To find them, split the number into any two factors, and keep splitting until every branch ends in a prime. This is a **factor tree**. Dividing repeatedly by primes ($360 \div 2 = 180$, $180 \div 2 = 90$, and so on) gives the same result.



Collect equal primes as powers: $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5$. This is called **index form**.

The prime factors show what a number is made of:

- **A square number** has every prime to an even power: $144 = 2^4 \times 3^2$.
- **A cube number** has every power a multiple of 3: $216 = 2^3 \times 3^3$.

HCF and LCM

The **highest common factor** is the biggest number that divides into both. The **lowest common multiple** is the smallest number that both divide into. With the prime factors of both numbers:

- **HCF**: take each prime that is in **both** numbers, with the **lower** power.
- **LCM**: take **every** prime that is in **either** number, with the **higher** power.

For $360 = 2^3 \times 3^2 \times 5$ and $84 = 2^2 \times 3 \times 7$:

$$\text{HCF} = 2^2 \times 3 = 12, \quad \text{LCM} = 2^3 \times 3^2 \times 5 \times 7 = 2520.$$

A check: $\text{HCF} \times \text{LCM}$ equals the two numbers multiplied together, $12 \times 2520 = 30\,240 = 360 \times 84$.

For small numbers, lists work too. Multiples of 6: 6, 12, 18, 24, 30, ...; multiples of 8: 8, 16, 24, ... The first one in both lists, 24, is the LCM.

The same rules work with letters: the HCF of $12a^3b$ and $18a^2b^2$ is $6a^2b$ (lower power of each letter), and the LCM is $36a^3b^2$ (higher power of each).

LCM in real life. Whenever two things repeat on different cycles and you want the next time they happen together, you need the LCM. Buses every 12 minutes and every 20 minutes leave together again after $\text{LCM}(12, 20) = 60$ minutes.

Rational and irrational numbers

A **rational** number can be written as a fraction $\frac{p}{q}$ of two integers: $5 = \frac{5}{1}$, $-0.25 = -\frac{1}{4}$, $0.333 \dots = \frac{1}{3}$ (a recurring decimal), $\sqrt{49} = 7$ and $\sqrt[3]{8} = 2$ are all rational.

An **irrational** number cannot. Its decimal goes on forever with no repeating pattern. The ones you meet are:

- square roots (or cube roots) of numbers that are not squares (or cubes): $\sqrt{2}$, $\sqrt{50}$, $\sqrt[3]{10}$;
- π , and simple multiples of these, such as 3π or $2 + \sqrt{3}$.

Be careful: 3.142 and $\frac{22}{7}$ are rational (they are only approximations to π), and $\sqrt{0.25} = 0.5$ is rational.

To write down an irrational number between two values, square them. Between 8 and 9: $8^2 = 64$ and $9^2 = 81$, so $\sqrt{65}$, $\sqrt{70}$ or $\sqrt{80}$ all work.

Reciprocals

The **reciprocal** of a number is 1 divided by it, so a number times its reciprocal is 1. For a fraction, turn it upside down.

- The reciprocal of 8 is $\frac{1}{8}$ (or 0.125).
- The reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$ (or 2.5).
- For a mixed number, make it improper first: $1\frac{3}{4} = \frac{7}{4}$, whose reciprocal is $\frac{4}{7}$.
- For a decimal, write it as a fraction first: $0.4 = \frac{2}{5}$, whose reciprocal is $\frac{5}{2} = 2.5$.

Zero has no reciprocal.

Large numbers in words

Read in groups of three digits: 4 020 015 300 is four billion, twenty million, fifteen thousand, three hundred. One million is 1 000 000 and one billion is 1 000 000 000. A missing group means zeros: ten thousand and seven is 10 007.

Sets

A **set** is a collection of things called **elements**. It can be listed, $A = \{2, 4, 6, 8\}$, or described by a rule, $A = \{x : x \text{ is an even number, } 1 \leq x \leq 9\}$ ("the set of x such that..."). The **universal set**, ξ , holds everything being talked about in the question.

Notation	Means	Example with $\xi = \{1, 2, \dots, 10\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8, 10\}$
$n(A)$	number of elements in A	$n(A) = 4$
$A \cap B$	intersection: in A and in B	$\{2, 4\}$
$A \cup B$	union: in A or B or both	$\{1, 2, 3, 4, 6, 8, 10\}$

Notation	Means	Example with $\xi = \{1, 2, \dots, 10\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8, 10\}$
A'	complement: not in A	$\{5, 6, 7, 8, 9, 10\}$
$x \in A$	x is an element of A (Extended)	$3 \in A$
$x \notin A$	x is not an element of A (Extended)	$6 \notin A$
$\emptyset$	the empty set, with no elements (Extended)	$A \cap \{5, 7\} = \emptyset$
$C \subseteq A$	every element of C is in A (Extended)	$\{1, 3\} \subseteq A$
$B \not\subseteq A$	some element of B is not in A (Extended)	$B \not\subseteq A$

A way to remember: $\cap$ looks like an "n" for **and**, and $\cup$ is a cup that holds everything in either set.

A set can hold points instead of numbers (Extended): $\{(x, y) : y = 2x + 1\}$ is every point on the line $y = 2x + 1$, so $(3, 7)$ is an element and $(1, 4)$ is not.

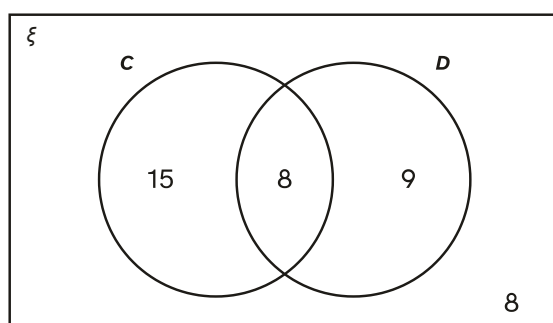
Venn diagrams

A **Venn diagram** draws ξ as a rectangle and each set as a circle inside it. Two overlapping circles split the rectangle into **four regions**: A only, both, B only, and neither. Every element goes in **exactly one** region, so an element that is in both sets goes in the overlap and nowhere else.

A region can hold the elements themselves, or just **how many** elements are there. When you are told totals, remember that the total for a set is **everything inside its circle**, including the overlap:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B),$$

because adding $n(A)$ and $n(B)$ counts the overlap twice.



C only: $23 - 8 = 15$

both: $23 + 17 - 32 = 8$

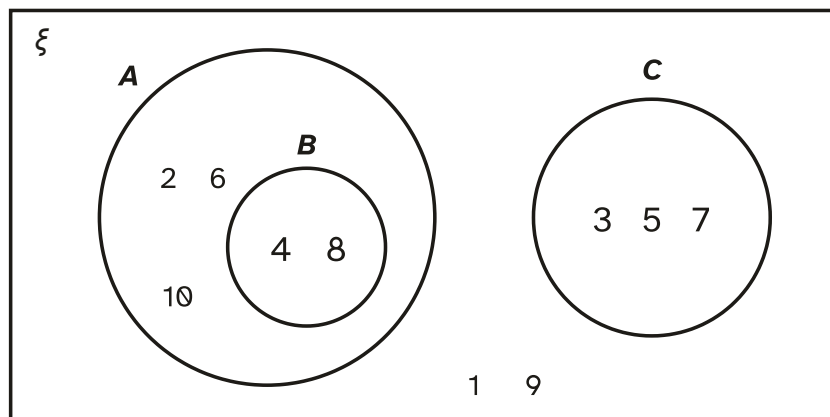
D only: $17 - 8 = 9$

neither: 8 (given)

check: $15 + 8 + 9 + 8 = 40$

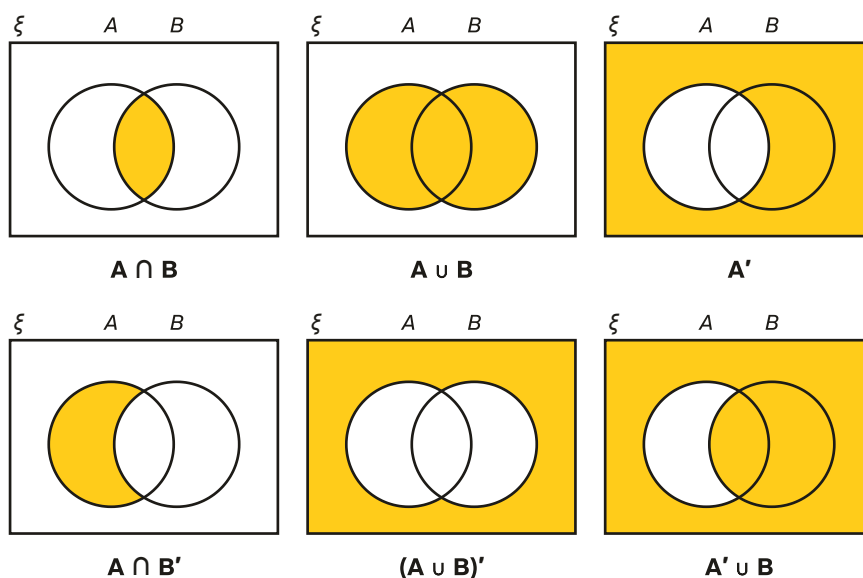
This diagram is Example 3 below: 40 members, 23 in set C , 17 in set D and 8 in neither. The number in at least one set is $40 - 8 = 32$, so the overlap is $23 + 17 - 32 = 8$.

Extended only: circles need not overlap. If $B \subseteq A$, draw B 's circle **inside** A 's. If two sets have no elements in common ($A \cap C = \emptyset$), draw their circles **apart**.



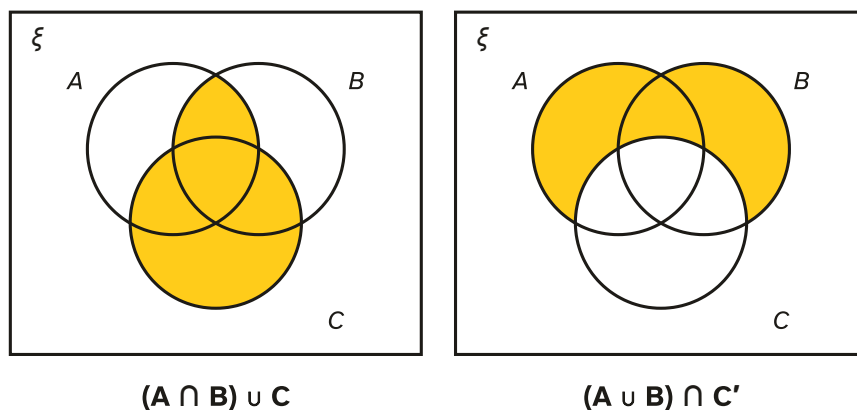
Shading regions

To shade a region, decide for **each separate region** of the diagram whether it fits the description. The diagram below shows the ones that come up most with two sets.



With brackets, work out the bracket first, just as in arithmetic. For $(A \cup B)'$, first find $A \cup B$ (both circles), then take everything else. That is different from $A' \cup B'$, which is everything except the overlap, the same as $(A \cap B)'$.

With three sets (Extended) there are **eight** regions: one in all three, three in exactly two, three in exactly one, and the outside. Test each one against the description.



Key facts and formulas

None of these is on the List of formulas printed in the exam paper, so you must know them all.

Formula	Status
A prime number has exactly two factors, 1 and itself; 1 is not prime; 2 is the only even prime	Learn it
Squares 1^2 to 15^2 (1, 4, 9, ..., 225) and cubes 1, 8, 27, 64, 125 and 1000	Learn it
HCF: common primes, lower powers. LCM: all primes, higher powers	Learn it
$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$	Learn it
Square number: every prime power is even. Cube number: every prime power is a multiple of 3	Learn it
Rational: can be written as $\frac{p}{q}$ with integers p and q , $q \neq 0$	Learn it
Reciprocal of x is $\frac{1}{x}$; reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$; $x \times \frac{1}{x} = 1$	Learn it
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$	Learn it
$n(A') = n(\xi) - n(A)$	Learn it
Probability from a Venn diagram = $\frac{\text{number in the region}}{\text{number in the group chosen from}}$	Learn it

How to do it

In the questions we have tagged for this topic (159 questions from Papers 1–4, 2021–2025, 10 of them printed on two papers), the types came up this often. 42 of the 159 questions mix two or three types, so the counts add up to more than 159.

Question type	Questions
Factors, multiples, primes, squares and cubes	56
Completing and using Venn diagrams	47
Prime factors, HCF and LCM	42

Question type	Questions
Shading and describing regions in set notation	28
Reciprocals	24
Rational and irrational numbers	10
Numbers in words and figures	6

1. Factors, multiples, primes, squares and cubes (56 questions)

Most of these give a list of numbers and ask you to pick one of each kind.

1. **Read which kind is wanted** for each part: factor of, multiple of, prime, square, cube, even, odd, natural.
2. **Test each number in the list**, not just the first that looks right. For a prime, divide by 2, 3, 5, 7, ...; for a factor of 56, check that $56 \div$ the number is whole.
3. **Write one answer** unless the question says "all" or "the prime numbers" (plural). Extra wrong answers cost marks.

Variations you will meet:

- **"Write down all the factors of 42"**: find them in pairs (1×42 , 2×21 , 3×14 , 6×7), then list all eight: 1, 2, 3, 6, 7, 14, 21, 42. Don't list prime factors instead, and don't write the pairs as products.
- **"Find all the common factors of..."**: list the factors of each, then the ones in both lists.
- **"A number is a factor of X and a multiple of Y "**: list the factors of X and keep those in the Y times table.
- **"A prime number between 60 and 70"**: test each odd number that doesn't end in 5: 61 is prime, $63 = 7 \times 9$, 67 is prime, $69 = 3 \times 23$.
- **"Is he correct? Give a reason"**: show a factor. "111 is not prime because $111 = 3 \times 37$." "Not every odd number is prime: $15 = 3 \times 5$."
- **Number puzzles** (a prime that is 2 less than a square, three consecutive numbers that are prime, cube and square, a sum of primes): list the candidates of the rarest kind first (squares or cubes), then test the other conditions.
- **Largest or smallest possible value** of an expression using a common factor, a common multiple and a prime: make each part as large or small as needed. For example, to make a fraction large, put the largest allowed value on top and the smallest on the bottom.
- **"Using your answer to part (a)"**: if a factorised expression such as $(a + 2)(b - 1)$ must equal 18, each bracket is one of a factor pair of 18.
- **A given method** (for example a test for multiples of 11): follow the steps exactly as shown and write each one.

2. Completing and using Venn diagrams (47 questions)

With numbers of elements:

1. **Fill in what you are told directly**, usually the overlap and the "neither" number.
2. **If the overlap is not given**, find it: $n(A \cap B) = n(A) + n(B) - n(A \cup B)$, where $n(A \cup B) = n(\xi) - \text{neither}$.
3. **Find the "only" regions**: $A \text{ only} = n(A) - n(A \cap B)$, and the same for B .
4. **Check** that the four regions add up to $n(\xi)$.
5. **Answer each part from the diagram** by adding the right regions.

With elements:

1. List each set in full first. For $\xi = \{x : 1 \leq x \leq 12\}$ and $M = \{\text{multiples of } 3\}$, $M = \{3, 6, 9, 12\}$.
2. Put the elements in **both** sets in the overlap, then the rest of each set, then everything left over **outside** the circles. Write each element once.
3. Then list or count whatever is asked, such as $A \cap B$ or $n(A' \cup B)$.

Variations you will meet:

- **Finding n of an expression**: shade or mark the regions it covers, then add their numbers. $n(A \cup B)$ is three regions, not two.
- **Probability of one person chosen at random**: the number in the region over the total, for example $\frac{15}{40}$. If the person is chosen from one set only ("a student who studies history"), the total is n of that set. Two people chosen one after another is in the probability topics.
- **An unknown, such as $n(F) = 3 \times n(V)$** : call the overlap x (or the unknown region), write each region in terms of x , and solve an equation using the total.
- **Three sets (Extended)**: start in the middle ($A \cap B \cap C$), then the three "two-set" regions, then the "one set only" regions, then the outside.
- **"Write down a possible value of y " where $y \in (P \cup S)'$** : any element outside both circles. With $\xi = \{1, \dots, 10\}$, $P = \{\text{primes}\}$ and $S = \{\text{squares}\}$, y can be 6, 8 or 10.
- **Describing a set in words**: $F' \cap E$ where $F = \{\text{multiples of } 5\}$ and $E = \{\text{even numbers}\}$ is "even numbers that are not multiples of 5".
- **Subsets and disjoint sets (Extended)**: if every element of B is in A , draw B 's circle inside A 's; if two sets share no elements, draw the circles apart. You may be asked to draw this, or to write the relation, such as $B \subseteq A$ or $A \cap C = \emptyset$.
- **Sets defined by inequalities**, such as $A = \{a : \frac{1}{4} \leq a < 0.6\}$: change every element to a decimal first, then test each one.

3. Prime factors, HCF and LCM (42 questions)

1. **Product of prime factors**: use a factor tree or divide by primes. Write the answer as a product, such as $2^2 \times 3^2 \times 7$, not a list and not a sum.
2. **HCF**: common primes, lower powers. **LCM**: all primes, higher powers. With small numbers, lists of factors or multiples are fine.
3. **Check** with $\text{HCF} \times \text{LCM} = \text{the two numbers multiplied}$.

Variations you will meet:

- **Numbers already in index form**, such as $A = 2^3 \times 3^2 \times 7$ and $B = 2 \times 3^4 \times 5$: read the powers straight off. $\text{HCF} = 2 \times 3^2 = 18$ and $\text{LCM} = 2^3 \times 3^4 \times 5 \times 7$. You can leave an answer in index form unless the question wants a number.
- **HCF or LCM with letters**, such as $15x^2y$ and $18xy^3$: numbers by the usual method, then the lower (HCF) or higher (LCM) power of each letter: $\text{HCF} = 3xy$ and $\text{LCM} = 90x^2y^3$.
- **Timetable problems** (trains, buses, bells, lights): find the LCM in minutes (or seconds), change it to hours and minutes, and add it to the start time. $\text{LCM} = 140$ minutes = 2 hours 20 minutes.
- **"Find two numbers with HCF 4 and LCM 84"**: both numbers are $4 \times$ something, and those two somethings share no factor and multiply to $84 \div 4 = 21$. So they are 1 and 21 or 3 and 7, giving 4 and 84, or 12 and 28. Use any extra condition (for example, both greater than 4) to choose.
- **Multiply or divide a number in index form**: $14M$ adds a 2 and a 7 to M 's primes; $x \div 315$ takes away $3^2 \times 5 \times 7$. Add or subtract the powers of each prime.
- **Smallest R to make a square or a cube**: write the prime factors and look at each power. For a cube, each power must become a multiple of 3. To make $2^7 \times 3^2$ a cube by **multiplying**, raise the powers to 9 and 3: $R = 2^2 \times 3 = 12$. To make it a cube by **dividing**, lower them to 6 and 0: $R = 2 \times 3^2 = 18$.
- **Missing power**: $600 = 2^3 \times 3 \times 5^m$ means $5^m = 600 \div 24 = 25$, so $m = 2$.
- **Largest odd common factor**: find the HCF, then take out every factor of 2.
- **Write a number as a product of two square numbers**: from $3600 = 2^4 \times 3^2 \times 5^2$, group the even powers, such as $2^4 \times (3^2 \times 5^2) = 16 \times 225$.

4. Shading and describing regions in set notation (28 questions)

To shade:

1. Work out any bracket first.
2. Go through the regions one at a time (four for two sets, eight for three) and ask "is this region in the set described?"
3. Shade exactly those regions, and nothing else. The space outside the circles counts as a region too.

To describe a shaded region:

1. Say which circles the shaded region is **inside** and which it is **outside**.
2. Join "inside A " with A , "outside B " with B' , using $\cap$ for "and".
3. If the shading is two separate pieces, join the pieces with $\cup$.

For example, the part of A outside B is $A \cap B'$; the overlap of A and B outside C is $A \cap B \cap C'$; everything outside both circles is $(A \cup B)'$ or $A' \cap B'$.

Variations you will meet:

- **Shading $A' \cup B'$** (everything except the overlap): the most common wrong answer is shading only the outside region.

- **Describing a region in words** from a context: "people who own a cat and a dog but not a rabbit" is $C \cap D \cap R'$.
- **Several correct answers:** $(A \cup B)'$ and $A' \cap B'$ are the same region; either earns the mark.

5. Reciprocals (24 questions)

1. Write the number as a fraction: $5 = \frac{5}{1}$, $0.25 = \frac{1}{4}$, $2\frac{1}{3} = \frac{7}{3}$.
2. Turn it upside down: $\frac{1}{5}$, 4, $\frac{3}{7}$.
3. If the question asks for a decimal, divide: $\frac{1}{16} = 0.0625$.

The reciprocal of a fraction is never the same fraction with a minus sign, and never the decimal of the same fraction.

6. Rational and irrational numbers (10 questions)

1. **Picking the irrational number from a list:** work out any root. If it comes out exact ($\sqrt{144} = 12$, $\sqrt[3]{343} = 7$), it is rational. Fractions, terminating decimals, recurring decimals and percentages are rational. What's left, such as $\sqrt{20}$ or π , is irrational.
2. **Writing an irrational number between two values:** square the two values and write the square root of a non-square number between them. Between 31 and 32: $31^2 = 961$ and $32^2 = 1024$, so $\sqrt{1000}$ works. Write it as $\sqrt{1000}$, not as a decimal with dots.
3. **Natural numbers or integers from a list:** simplify first ($\sqrt{144} = 12$ is a natural number; -2 is an integer but not natural).

7. Numbers in words and figures (6 questions)

These are usually the first part of a long Paper 3 question, worth one mark.

Figures to words:

1. **Split the digits into groups of three from the right:** 12008450 becomes 12 008 450.
2. **Read each group as a number from 1 to 999**, followed by its name: millions, thousands, then nothing for the last group. So 12 008 450 is twelve million, eight thousand, four hundred and fifty.
3. **Skip a group of 000** completely: 5 000 210 is five million, two hundred and ten.
4. **Put "and"** before the tens and units inside a group ("four hundred **and** fifty"), and before a last group below 100 ("eight thousand **and** nine"). Tens and units with a hyphen: twenty-one, ninety-four.

Words to figures:

1. **Write one group of three digits for each word "million" and "thousand"**, plus one group for the rest.
2. **Fill any group the words don't mention with 000**, and pad a small group with zeros in front: "two million, three hundred thousand and seventy" is 2 300 070 (the last group is 070).
3. **Count the digits to check:** a number in the millions has 7 to 9 digits; a billion has 10. "Six and a half million" is 6 500 000.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

(a) Write down all the factors of 60. **[2]**

(b) From this list, write down

39 41 51 57 64 81 91

(i) a prime number, **[1]**

(ii) a number that is both a square number and a cube number, **[1]**

(iii) a multiple of 19. **[1]**

(a) In pairs: 1×60 , 2×30 , 3×20 , 4×15 , 5×12 , 6×10 .

Factors: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60. **B2** (usually **B1** for most of them with no wrong ones)

(b)(i) $39 = 3 \times 13$, $51 = 3 \times 17$, $57 = 3 \times 19$, $91 = 7 \times 13$, 64 is even and $81 = 9 \times 9$. 41 has no factor except 1 and 41. Answer 41. **B1**

(b)(ii) $64 = 8^2 = 4^3$. **B1** ($81 = 9^2$ is square but not a cube.)

(b)(iii) $57 = 3 \times 19$. **B1**

Example 2

(a) Write 252 as a product of its prime factors. Give your answer in index form. **[2]**

(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 252 and 120. **[3]**

(c) A bell rings every 18 minutes and a buzzer sounds every 42 minutes. They both sound at 10 45. Find the next time they sound together. **[3]**

(d) Find the smallest positive integer k such that $252k$ is a square number. **[1]**

(a) $252 = 2 \times 126 = 2 \times 2 \times 63 = 2 \times 2 \times 3 \times 21 = 2 \times 2 \times 3 \times 3 \times 7$. **M1** for a correct tree or division.

$$252 = 2^2 \times 3^2 \times 7$$

A1

(b) $120 = 2^3 \times 3 \times 5$. **M1** for both in prime factors.

HCF: common primes 2 and 3, lower powers: $2^2 \times 3 = 12$. **A1**

LCM: all primes, higher powers: $2^3 \times 3^2 \times 5 \times 7 = 2520$. **A1**

Check: $12 \times 2520 = 30\,240 = 252 \times 120$.

(c) $18 = 2 \times 3^2$ and $42 = 2 \times 3 \times 7$, so $\text{LCM} = 2 \times 3^2 \times 7 = 126$ minutes. **M1 A1**

126 minutes = 2 hours 6 minutes, and $10\,45 + 2\text{ h }6\text{ min} = 12\,51$. **A1**

(d) $252 = 2^2 \times 3^2 \times 7$. The powers of 2 and 3 are already even; 7 needs one more. $k = 7$ (and $252 \times 7 = 1764 = 42^2$). **B1**

Example 3

A games club has 40 members. 23 play chess (C), 17 play draughts (D) and 8 play neither.

(a) Complete the Venn diagram. **[3]**

(b) Find $n(C \cap D')$. **[1]**

(c) Find $n(C' \cup D)$. **[1]**

(d) A member is chosen at random. Find the probability that this member plays exactly one of the two games. **[1]**

(e) Use set notation to describe the members who play neither game. **[1]**

(a) $n(C \cup D) = 40 - 8 = 32$, so $n(C \cap D) = 23 + 17 - 32 = 8$. **M1**

C only = $23 - 8 = 15$ and D only = $17 - 8 = 9$. Put 15, 8, 9 in the circles and 8 outside (the second diagram in "Understand it"). **A1** for the overlap, **B1** for the 8 outside. Check: $15 + 8 + 9 + 8 = 40$.

(b) $C \cap D'$ is "in C , not in D ": 15. **B1**

(c) $C' \cup D$ is everything **except** " C only": $40 - 15 = 25$. **B1**

(d) Exactly one game: $15 + 9 = 24$, so the probability is $\frac{24}{40} = \frac{3}{5}$. **B1**

(e) $(C \cup D)'$, or $C' \cap D'$. **B1**

Example 4

$\xi = \{x : x \text{ is an integer, } 1 \leq x \leq 15\}$

$A = \{\text{prime numbers}\}$, $B = \{\text{odd numbers}\}$, $C = \{\text{multiples of } 3\}$

(a) Complete the Venn diagram to show the elements of these sets. **[3]**

(b) List the elements of $A \cap B'$. **[1]**

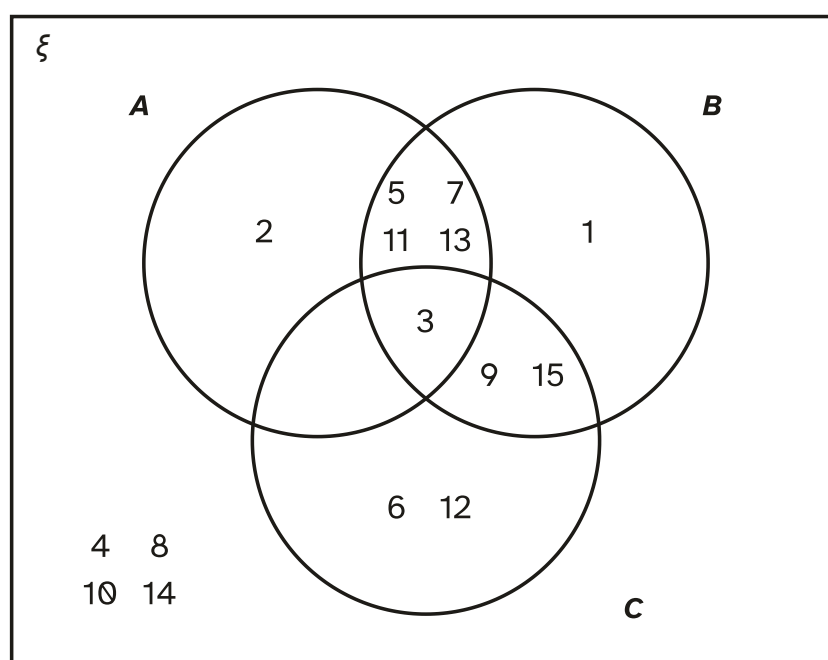
(c) Find $n(A \cup C)$. **[1]**

(d) List the elements of $(A \cup B)'$. **[1]**

(e) Is each statement true or false? (i) $9 \in A$ (ii) $\{3, 5\} \subseteq A \cap B$ **[2]**

(a) List the sets first: $A = \{2, 3, 5, 7, 11, 13\}$, $B = \{1, 3, 5, 7, 9, 11, 13, 15\}$, $C = \{3, 6, 9, 12, 15\}$. **M1** for correct lists or most elements placed.

In all three: 3. In A and B only: 5, 7, 11, 13. In B and C only: 9, 15. In A and C only: none. A only: 2. B only: 1. C only: 6, 12. Outside: 4, 8, 10, 14. **A2 (A1 for most in the right place)**



(b) Prime and not odd: $\{2\}$. **B1**

(c) $A \cup C = \{2, 3, 5, 6, 7, 9, 11, 12, 13, 15\}$, so $n(A \cup C) = 10$. **B1** (Or $6 + 5 - 1 = 10$, since only 3 is in both.)

(d) Not prime and not odd: $\{4, 6, 8, 10, 12, 14\}$. **B1**

(e)(i) False: $9 = 3 \times 3$ is not prime, so $9 \notin A$. **B1 (e)(ii)** True: 3 and 5 are both prime and odd. **B1**

Common mistakes

- **Mixing up factors and multiples.** A factor of 56 is **smaller** than (or equal to) 56; a multiple of 56 is **bigger** (or equal). Say the question to yourself: "goes into" or "times table".

- **Missing factors, or listing prime factors instead.** "All the factors of 42" includes 1 and 42. Work in pairs so none are missed; $2 \times 3 \times 7$ is the wrong kind of answer.
- **Thinking 1 is prime, or that 91, 111, 57 or 51 are.** Each of those four has a factor (7, 3, 3, 3). Use the digit sum to test for 3.
- **Confusing square numbers with cube numbers,** for example giving 9 as a cube. Learn both lists.
- **Mixing up HCF and LCM.** The HCF is no bigger than the smaller number; the LCM is no smaller than the bigger one. Multiplying the two numbers gives a common multiple, but usually not the **lowest**.
- **Adding the prime factors,** or leaving out a prime. $130 = 2 \times 5 \times 13$, not $2 + 5 + 13$. Check by multiplying back.
- **Putting a set's total in its "only" region.** If 45 students play football and 18 play both football and tennis, then $45 - 18 = 27$ go in the "football only" part, not 45. Examiners see this every year.
- **Leaving the overlap empty** when the totals add up to more than the group. 14 play football, 11 play tennis and 9 play neither: $14 + 11 + 9 = 34$, but only 30 are in the group, so $34 - 30 = 4$ must play both.
- **Losing a zero group** when writing words in figures. Two million and fifty is 2 000 050, not 2050 or 200 050: every "million" needs two groups of three digits after it.
- **Writing an element twice,** or forgetting the elements that belong outside all the circles.
- **Reading $\cup$ as $\cap$.** Union ($\cup$) is "or" and covers more; intersection ($\cap$) is "and" and covers less.
- **Shading $(A \cup B)'$ when asked for $A' \cup B'$,** or the other way round. Work out the bracket first and test every region.
- **Giving a number when set notation is asked for,** or labelling a region with a letter instead of describing it.
- **Leaving the reciprocal as a decimal of the same number,** or making it negative. The reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$, not 0.4 or $-\frac{2}{5}$.
- **Choosing $\sqrt{9}$ or a recurring decimal as irrational.** Work out roots first; recurring decimals are rational.

How to get the marks

- **"Write down"** means no working is needed, but only one answer is wanted. A second wrong answer can cancel the right one.
- **"All the factors"** is often worth 2 marks: one for most of them with no wrong extras, both for the complete list. Don't add extra numbers to be safe.
- **Product of prime factors:** give a product such as $2^3 \times 3^2 \times 5$. A factor tree or division on its own, or a list like "2, 3, 5", usually earns only a method mark. "Index form" means use powers.
- **HCF and LCM:** show the prime factors or lists. A wrong answer can still earn a method mark for correct factor trees or lists of multiples.
- **Timetable problems:** the LCM in minutes earns marks even if the final time is wrong, so write it down. Give the time in the same form as the question (for example 11 50).
- **Venn diagrams:** every region needs a number (or element), including the outside. Write 0 in an empty region if you are sure it is empty. Numbers, not tally marks.

- **"Use your Venn diagram"** or a follow-on part: marks follow through from your diagram, so read numbers from what you drew.
- **Probabilities** from a Venn diagram are fractions such as $\frac{55}{100}$; they don't need simplifying unless the question says so.
- **"Give a reason"**: one clear mathematical fact, such as a factor pair (" $111 = 3 \times 37$ ") or a counter-example (" 15 is odd but not prime").
- **Irrational number between two values**: write an exact form such as $\sqrt{150}$ or 4π , and make sure it lies in the range.

Quick check

1. Find the HCF and the LCM of 84 and 90.
2. (a) Find the reciprocal of 0.4 and of $2\frac{1}{3}$. (b) Write 3 040 506 in words. (c) Write seven billion, sixty thousand and five in figures.
3. From the list $\sqrt{49}$, $\sqrt{50}$, $\frac{3}{8}$, π , 0.3, -4 , write down (a) the irrational numbers (b) a natural number (c) the integers.
4. In a group of 50 people, 31 like tea (T), 24 like coffee (C) and 9 like both. Find (a) $n((T \cup C)')$ (b) $n(T' \cap C)$.
5. $M = 2^4 \times 3^5 \times 7$. Find the smallest positive integer k such that Mk is a cube number.

Answers

- $84 = 2^2 \times 3 \times 7$ and $90 = 2 \times 3^2 \times 5$. $\text{HCF} = 2 \times 3 = 6$. $\text{LCM} = 2^2 \times 3^2 \times 5 \times 7 = 1260$. Check: $6 \times 1260 = 7560 = 84 \times 90$.
- (a) $0.4 = \frac{2}{5}$, so its reciprocal is $\frac{5}{2} = 2.5$; $2\frac{1}{3} = \frac{7}{3}$, so its reciprocal is $\frac{3}{7}$. (b) Groups 3 | 040 | 506: three million, forty thousand, five hundred and six. (c) Billions, millions, thousands, units: 7 | 000 | 060 | 005, so 7 000 060 005.
- (a) $\sqrt{50}$ and π . (b) $\sqrt{49} = 7$. (c) $\sqrt{49}$ (that is 7) and -4 .
- T only $= 31 - 9 = 22$, C only $= 24 - 9 = 15$. (a) $n(T \cup C) = 22 + 9 + 15 = 46$, so $n((T \cup C)') = 50 - 46 = 4$. (b) $T' \cap C$ is "coffee but not tea": 15.
- The powers are 4, 5 and 1; the next multiples of 3 are 6, 6 and 3. So $k = 2^2 \times 3 \times 7^2 = 588$, and $Mk = 2^6 \times 3^6 \times 7^3 = (2^2 \times 3^2 \times 7)^3 = 252^3$.

Past questions to try

- 0580/12/F/M/24 Q25**: two numbers from their HCF and LCM.
- 0580/42/M/J/21 Q3**: HCF and LCM from prime factors, and the smallest number that makes a cube (part (c)).
- 0580/22/O/N/22 Q16**: shading regions, then a three-set Venn diagram with numbers.
- 0580/41/M/J/23 Q9**: listing elements, shading, and a three-set Venn diagram with elements.