Cambridge IGCSE · Mathematics 0580 · Notes

Vectors

Read online at **pastopic.com/cambridge/mathematics-0580/notes/vectors**

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Read first: **Transformations**, **Pythagoras' theorem and right-angled triangles**

What you need to know

Extended only: this whole topic is Extended content (syllabus E7.2 to E7.4). Since 2025, Core candidates only meet vectors as the column vector of a translation (see the Transformations note); Core papers before 2025 also asked for simple column vector sums, so older Core questions still make good practice.

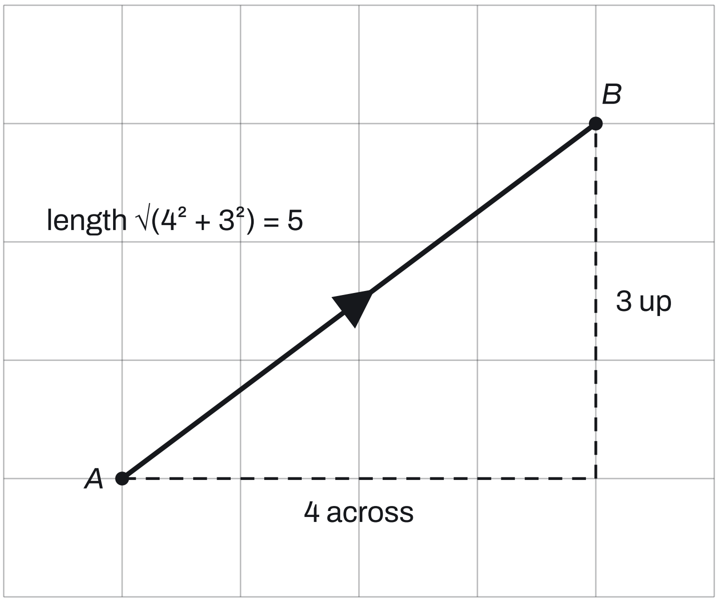
By the end of this topic you should be able to:

1. **Read and write vectors** in each form the exam uses: a column vector $\begin{pmatrix} x \\ y \end{pmatrix}$, a line between two points $\overrightarrow{AB}$, or a bold letter $\mathbf{a}$. Describing a translation with a column vector is taught in the Transformations note.
2. **Add and subtract vectors**, and **multiply a vector by a number** (a scalar).
3. **Find the magnitude** (length) of a column vector: $\left|\begin{pmatrix} x \\ y \end{pmatrix}\right| = \sqrt{x^2 + y^2}$. The exam writes magnitudes with modulus signs, $|\mathbf{a}|$ or $|\overrightarrow{AB}|$.
4. **Draw a vector as an arrow** (a directed line segment) and read one from a diagram or grid.
5. **Use position vectors**: the vector from the origin O to a point.
6. **Write a vector in terms of two given vectors**, such as $\mathbf{a}$ and $\mathbf{b}$, by adding and subtracting along a route through the diagram.
7. **Use vectors to prove and solve geometry problems**: show that two lines are parallel, show that three points are in a straight line (collinear), and solve problems about ratios along lines and similar shapes.

Understand it

What a vector is

A **vector** is a move: it has a size and a direction. A **scalar** is just a size, such as 5 or -2. The column vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ means "4 across and 3 up", exactly as for a translation: the top number is the move across (negative is left) and the bottom number is the move up (negative is down).



In the diagram the arrow from $A(1,1)$ to $B(5,4)$ is the vector $\overrightarrow{AB} = \begin{pmatrix}4\\3\end{pmatrix}$. The arrow shows the direction: $\overrightarrow{BA} = \begin{pmatrix}-4\\-3\end{pmatrix}$ is the same line travelled the other way, so $\overrightarrow{BA} = -\overrightarrow{AB}$.

A vector says nothing about **where** the move starts. Any arrow 4 across and 3 up is the same vector, wherever it is drawn. So opposite sides of a parallelogram, which are the same length and point the same way, are **equal vectors**.

In print, a vector named by one letter is **bold**: $\mathbf{a}$. You can't write bold by hand, so underline it: $\underline{a}$. Without the bold or the underline it is just a number.

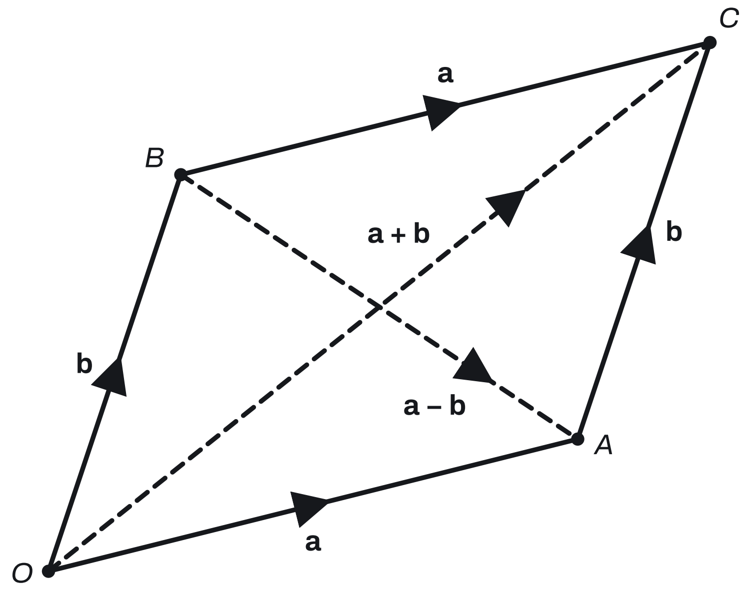
Column vectors: working with the numbers

Work on the top numbers and the bottom numbers **separately**. A column vector is not a fraction: never draw a line between the numbers.

- **Add:** $\begin{pmatrix}4\\1\end{pmatrix} + \begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}5\\4\end{pmatrix}$.
- **Subtract:** $\begin{pmatrix}4\\1\end{pmatrix} - \begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}3\\-2\end{pmatrix}$. Take care with negatives: $5-(-2)=7$.
- **Multiply by a scalar:** multiply both numbers: $3\begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}12\\3\end{pmatrix}$ and $-2\begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}-8\\-2\end{pmatrix}$.

Adding and subtracting as journeys

Adding vectors means doing one move and then the other. In the diagram, $\mathbf{a} = \begin{pmatrix}4\\1\end{pmatrix}$ takes you from O to A and then $\mathbf{b} = \begin{pmatrix}1\\3\end{pmatrix}$ takes you from A to C. The single move from O to C is $\mathbf{a}+\mathbf{b} = \begin{pmatrix}5\\4\end{pmatrix}$, the diagonal of the parallelogram.



To go from B to A, go back down $\mathbf{b}$ to O (that is $-\mathbf{b}$) and then along $\mathbf{a}$: $\overrightarrow{BA} = -\mathbf{b} + \mathbf{a} = \mathbf{a} - \mathbf{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

This is the key idea of the topic: **any vector can be found by a route along lines you know**. Going along an arrow the right way adds it; going against the arrow subtracts it.

Multiplying by a scalar: parallel vectors

$2\mathbf{a}$ is the move $\mathbf{a}$ done twice: the same direction, twice as long. $\frac{1}{2}\mathbf{a}$ is half as long, and $-\mathbf{a}$ is the same length in the opposite direction.

So **two vectors are parallel when one is a number times the other**. $\begin{pmatrix} 6 \\ -9 \end{pmatrix} = 3\begin{pmatrix} 2 \\ -3 \end{pmatrix}$, so these are parallel and the first is 3 times as long. In letters, $6\mathbf{a} - 4\mathbf{b} = 2(3\mathbf{a} - 2\mathbf{b})$, so a line with vector $6\mathbf{a} - 4\mathbf{b}$ is parallel to a line with vector $3\mathbf{a} - 2\mathbf{b}$ and twice as long.

Magnitude

The **magnitude** of a vector is the length of its arrow. The across and up moves are the two short sides of a right-angled triangle, so by Pythagoras

$$\left|\begin{pmatrix} x \\ y \end{pmatrix}\right| = \sqrt{x^2 + y^2}.$$

In the first diagram, $|\overrightarrow{AB}| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$. A negative number squared is positive: $\left|\begin{pmatrix} -5 \\ 12 \end{pmatrix}\right| = \sqrt{25 + 144} = 13$.

Position vectors

The **position vector** of a point A is $\overrightarrow{OA}$, the move from the origin to A. If A is $(2, 5)$, its position vector is $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$: the same numbers, written as a column. In letters, the position vector of A is usually called $\mathbf{a}$.

Going from A to B means going back from A to O, then out to B:

$$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}.$$

"End minus start." With numbers, if A is $(2, 5)$ and B is $(7, 1)$, then $\overrightarrow{AB} = \begin{pmatrix} 7-2 \\ 1-5 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$. The other way round, a point plus a vector gives a new point: $A(2, 5)$ moved by $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$ lands on $(7, 1)$.

Points part of the way along a line

If P is on AB with $AP : PB = 2 : 3$, then AB splits into $2 + 3 = 5$ equal parts and P is 2 of them from A. So $\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$, and

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \mathbf{a} + \tfrac{2}{5}(\mathbf{b} - \mathbf{a}) = \tfrac{3}{5}\mathbf{a} + \tfrac{2}{5}\mathbf{b}.$$

The **midpoint** is the case $1 : 1$: $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}(\mathbf{a} + \mathbf{b})$.

Proving things with vectors

- **Parallel lines.** Find both vectors in terms of the same two letters. If one is a number times the other, the lines are parallel, and the number is the ratio of their lengths.
- **Three points in a straight line (collinear).** Show that $\overrightarrow{PQ} = k\overrightarrow{PR}$ for some number k. That makes PQ and PR parallel, and because they **share the point** P they must be the same straight line. You need to say both facts.
- **Similar shapes.** A line parallel to one side of a triangle cuts the other two sides in the same ratio. So if Y is on UT with $UY : YT = 3 : 1$ and YZ is parallel to UO, then Z is on OT with $OZ : ZT = 3 : 1$ too, so $\overrightarrow{OZ} = \frac{3}{4}\overrightarrow{OT}$.

Key facts and formulas

None of these are in the 0580 list of formulas. "Learn it" means you must remember it.

Formula	Status
$\begin{pmatrix} a \\ b \end{pmatrix} \pm \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a \pm c \\ b \pm d \end{pmatrix}$ and $k\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ka \\ kb \end{pmatrix}$	Learn it
Magnitude: $\left\lvert \begin{pmatrix} x \\ y \end{pmatrix} \right\rvert = \sqrt{x^2 + y^2}$	Learn it
$\overrightarrow{BA} = -\overrightarrow{AB}$	Learn it

Formula	Status
$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \mathbf{b} - \mathbf{a}$ (end minus start)	Learn it
Point + vector = point: $A(p, q)$ moved by $\begin{pmatrix} x \\ y \end{pmatrix}$ gives $(p + x,\ q + y)$	Learn it
P on AB with $AP : PB = m : n$: $\overrightarrow{AP} = \dfrac{m}{m+n}\overrightarrow{AB}$	Learn it
Midpoint M of AB: $\overrightarrow{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$	Learn it
Parallel: $\mathbf{u} = k\mathbf{v}$; collinear: $\overrightarrow{PQ} = k\overrightarrow{PR}$ with P a shared point	Learn it

How to do it

In the questions we have tagged for this topic (57 questions, 2021–2025, Papers 1 to 4), the types came up this often. Most questions have several parts, so 28 of the 57 mix two or more types and the counts add up to more than 57.

Question type	Questions
Column vector arithmetic	30
Column vectors, points and position vectors	24
A vector in terms of two given vectors	23
Magnitude of a vector	17
Parallel, collinear, ratio and meeting-point problems	11

1. Column vector arithmetic (30 questions)

1. **Multiply first.** Work out any scalar multiples, such as $2\mathbf{a}$ and $3\mathbf{b}$, one number at a time.
2. **Then add or subtract** the top numbers together and the bottom numbers together.
3. **Write the answer as a column vector**, in brackets, with no fraction line.

For example, with $\mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$: $\mathbf{a} - 2\mathbf{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 5 \\ -10 \end{pmatrix}$.

Variations you will meet:

- **Unknown numbers**, such as "$\mathbf{a} + k\mathbf{b} = \begin{pmatrix} m \\ 10 \end{pmatrix}$". The top row and the bottom row each give an equation. With the $\mathbf{a}$ and $\mathbf{b}$ above, the bottom row gives $-2 + 4k = 10$, so $k = 3$; then the top row gives $m = 3 + 3 \times (-1) = 0$.
- **Drawing a vector on a grid** (such as $2\mathbf{a}$ or $\mathbf{a} - \mathbf{b}$): work out the column vector, then draw an arrow that many squares across and up, from any starting point, with an arrowhead and a label.

2. Column vectors, points and position vectors (24 questions)

1. **Vector between two points**: end minus start. From $A(3, -1)$ to $B(-2, 4)$: $\overrightarrow{AB} = \begin{pmatrix} -2-3 \\ 4-(-1) \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$.
2. **A point from a point and a vector**: add. If A is $(3, -1)$ and $\overrightarrow{AC} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$, then C is $(9, 1)$.
3. **A position vector** is the point's coordinates written as a column vector, and the other way round.

Variations you will meet:

- **The vector is given backwards**, for example you know A and $\overrightarrow{BA}$ and want B. Then $\overrightarrow{AB} = -\overrightarrow{BA}$, so **subtract**: $B = A - \overrightarrow{BA}$. With $A(2, 7)$ and $\overrightarrow{BA} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$, B is $(-3, 10)$.
- **A multiple of a vector from a point**: if H is on FG extended with $\overrightarrow{FH} = 2\overrightarrow{FG}$, then $\overrightarrow{OH}$ is $\overrightarrow{OF}$ **plus** $2\overrightarrow{FG}$; $2\overrightarrow{FG}$ on its own is not the answer.
- **A chain of vectors**: $\overrightarrow{FG} + \overrightarrow{GH} = \overrightarrow{FH}$. To reach K from H via J, first find J, then find the vector from J to K.
- **A point dividing a line** given in coordinates: with $AP : PB = 2 : 5$, $\overrightarrow{OP} = \overrightarrow{OA} + \frac{2}{7}\overrightarrow{AB}$.
- **On a grid**: read a vector by counting across then up; mark a point by counting the vector's moves from the given point.

3. A vector in terms of two given vectors (23 questions)

These are geometry diagrams, usually "not to scale", with two vectors such as $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$ given.

1. **Mark what you know on the diagram**, including equal vectors: opposite sides of a parallelogram are equal, and "OA is parallel to CB with $OA : CB = 5 : 2$" means $\overrightarrow{CB} = \frac{2}{5}\mathbf{a}$.
2. **Write a route** from the start to the end of the vector you want, along lines you know, for example $\overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{OA} + \overrightarrow{AP}$. Write this route down: it earns a method mark on its own.
3. **Replace each piece** by its vector in $\mathbf{a}$ and $\mathbf{b}$. Going against an arrow changes its sign. Use the ratio for part of a line: if $AP : PB = 2 : 3$ then $\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$.
4. **Simplify**: expand brackets and collect the $\mathbf{a}$ terms and the $\mathbf{b}$ terms, giving something like $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$.

Variations you will meet:

- **"The position vector of M"** means $\overrightarrow{OM}$. Find it with a route that starts at O.
- **Ratios the other way round.** $AE : EB = 3 : 4$ puts E three sevenths of the way from A; $DF = 2FK$ means $DF : FK = 2 : 1$, so $\overrightarrow{DF} = \frac{2}{3}\overrightarrow{DK}$.
- **Diagonals of a rhombus or parallelogram** cut each other in half, so the point where they cross is the midpoint of both.
- **A point on a line extended beyond the shape**: if OBD is a straight line with $OB : BD = 2 : 1$, then $\overrightarrow{OD} = \frac{3}{2}\overrightarrow{OB}$.

4. Magnitude of a vector (17 questions)

1. Square the top number, square the bottom number, add, and square-root: $\left|\begin{pmatrix}-6\\4\end{pmatrix}\right| = \sqrt{36+16} = \sqrt{52} = 7.21$ (3 s.f.).
2. For the magnitude of a combination such as $|\mathbf{p}-\mathbf{q}|$, **work out the column vector first**, then find its magnitude.

Variations you will meet:

- **An unknown from a magnitude**, such as "$\begin{pmatrix}k\\12\end{pmatrix}$ has magnitude 13". Square both sides: $k^2+144=169$, so $k^2=25$ and $k=5$ or $k=-5$. Give both values unless the question rules one out. An unknown inside, such as $\begin{pmatrix}9\\m+3\end{pmatrix}$, works the same way: solve $(m+3)^2=\ldots$ and keep both square roots.
- **An exact or surd answer**, such as "the length of AB is $k\sqrt{7}$": $\sqrt{63}=\sqrt{9\times 7}=3\sqrt{7}$, so $k=3$.
- **"Show that $|\overrightarrow{OB}|=7.3$, correct to 1 decimal place"**: find $\overrightarrow{OB}$ (say $\begin{pmatrix}7\\2\end{pmatrix}$), write the Pythagoras calculation $\sqrt{49+4}=\sqrt{53}$ and the unrounded value $7.280\ldots$ before rounding.
- **Magnitude with a direction**: a vector $\begin{pmatrix}a\\b\end{pmatrix}$ with gradient 2 has $b=2a$; with magnitude 10 that gives $a^2+4a^2=100$, so $a^2=20$ and (taking both positive) $a=2\sqrt{5}$, $b=4\sqrt{5}$.

5. Parallel, collinear, ratio and meeting-point problems (11 questions)

These come after type 3, using the vectors you have just found.

1. **Show two lines are parallel**: write both vectors in terms of the same two letters, and show one is a multiple of the other, for example $\overrightarrow{QP}=\frac{2}{3}\overrightarrow{CB}$. Then say it: "so QP is parallel to CB, and QP is $\frac{2}{3}$ of its length".
2. **Show a shape is a trapezium**: show one pair of sides is parallel (one vector is a multiple of the other).
3. **Show three points are collinear**: find two vectors that share a point, such as $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, and show $\overrightarrow{OP}=k\overrightarrow{OQ}$. Write the conclusion: "they are parallel and share the point O, so O, P and Q lie on a straight line". The ratio $OP:OQ$ is then $k:1$.
4. **Find a ratio from vectors**: if $\overrightarrow{OT}=\frac{3}{7}\overrightarrow{OP}$, then $OT:OP=3:7$ and $OT:TP=3:4$.
5. **Two lines extended until they meet at G**: G is on both lines, so write $\overrightarrow{OG}$ in two ways. One way uses a multiple of one line's vector, and the other a multiple of the other's. Match the amounts of each letter to find the multiples. Often one line is along $\mathbf{a}$, so $\overrightarrow{OG}=t\mathbf{a}$: set the amount of the other letter to 0.

Variations you will meet:

- **Given a vector expression, find the ratio in which a point divides a line**, for example $\overrightarrow{OD}=\frac{1}{3}\mathbf{x}+\frac{2}{3}\mathbf{y}$ on AB where $\overrightarrow{OA}=\mathbf{x}$, $\overrightarrow{OB}=\mathbf{y}$: then $\overrightarrow{AD}=\frac{2}{3}(\mathbf{y}-\mathbf{x})=\frac{2}{3}\overrightarrow{AB}$, so $AD:DB=2:1$.
- **A line parallel to a side of a triangle** (similar triangles): it cuts the other two sides in the same ratio, so a vector along one side is that fraction of the whole side.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

$\mathbf{p} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

(a) Find $2\mathbf{p} - \mathbf{q}$. **[2]**

(b) Find $|\mathbf{q}|$. **[2]**

(c) A is the point $(-1, 6)$ and $\overrightarrow{AB} = 2\mathbf{p} - \mathbf{q}$. Find the coordinates of B. **[2]**

(d) The vector $\begin{pmatrix} k \\ 15 \end{pmatrix}$ has magnitude 17. Find the two possible values of k. **[2]**

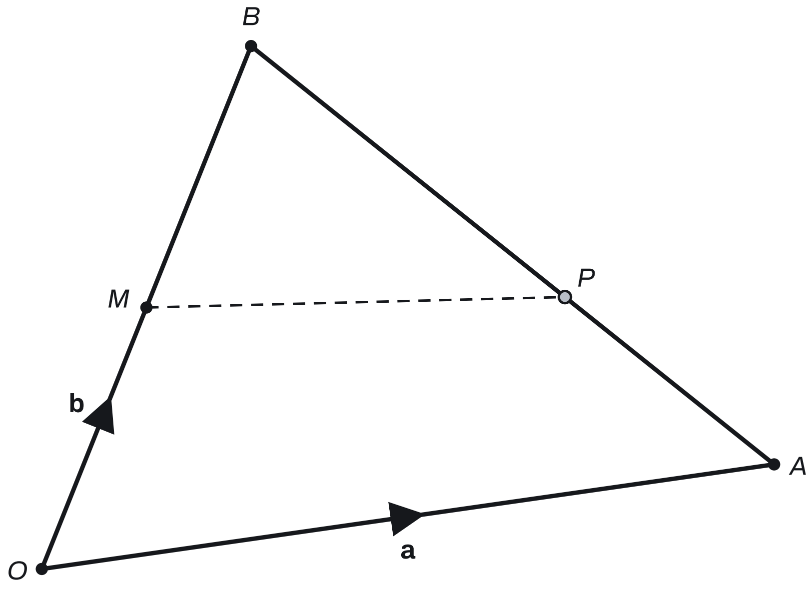
(a) $2\mathbf{p} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$, so $2\mathbf{p} - \mathbf{q} = \begin{pmatrix} 10 - (-3) \\ -4 - 4 \end{pmatrix} = \begin{pmatrix} 13 \\ -8 \end{pmatrix}$. **B2** (**B1** for one number right)

(b) $|\mathbf{q}| = \sqrt{(-3)^2 + 4^2}$ **M1** $= \sqrt{25} = 5$. **A1**

(c) $B = (-1 + 13,\ 6 + (-8)) = (12, -2)$. **B1** for each coordinate.

(d) $k^2 + 15^2 = 17^2$ **M1**, so $k^2 = 289 - 225 = 64$ and $k = 8$ or $k = -8$. **A1**

Example 2



In the diagram, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. P is the point on AB with $AP : PB = 2 : 3$, and M is the midpoint of OB.

(a) Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. **[1]**

(b) Find the position vector of P, in terms of $\mathbf{a}$ and $\mathbf{b}$, in its simplest form. **[2]**

(c) Find $\overrightarrow{MP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, in its simplest form. **[2]**

(a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b}$. **B1**

(b) AB is in $2 + 3 = 5$ parts and P is 2 of them from A:

$$\overrightarrow{OP} = \overrightarrow{OA} + \tfrac{2}{5}\overrightarrow{AB} = \mathbf{a} + \tfrac{2}{5}(\mathbf{b} - \mathbf{a})$$

M1 for a correct route with $\frac{2}{5}$,

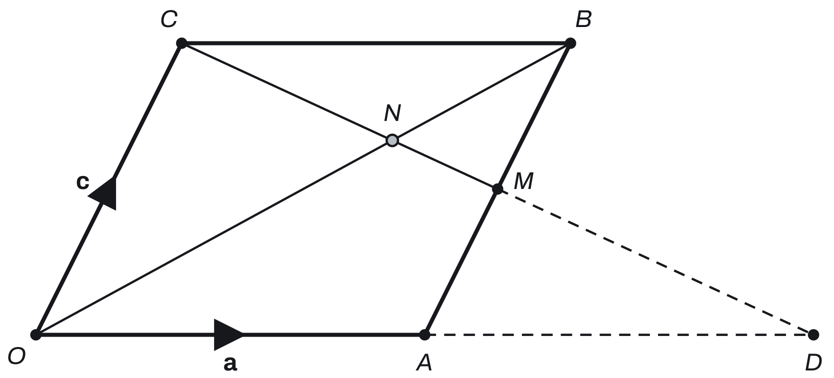
$$= \tfrac{3}{5}\mathbf{a} + \tfrac{2}{5}\mathbf{b}.$$

A1

(c) $\overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{OP} = -\frac{1}{2}\mathbf{b} + \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ **M1**

$= \frac{3}{5}\mathbf{a} - \frac{1}{10}\mathbf{b}$. **A1**

Example 3



$OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of AB, and N is the point on OB with $ON : NB = 2 : 1$.

(a) Find, in terms of $\mathbf{a}$ and $\mathbf{c}$, in their simplest form, (i) $\overrightarrow{CM}$, (ii) $\overrightarrow{CN}$. **[4]**

(b) Show that C, N and M lie on a straight line, and write down the ratio $CN : NM$. **[3]**

(c) CM and OA are extended to meet at D. Find the position vector of D in terms of $\mathbf{a}$. **[2]**

(a)(i) $\overrightarrow{CB} = \overrightarrow{OA} = \mathbf{a}$ (opposite sides of a parallelogram) and $\overrightarrow{BA} = -\mathbf{c}$, so $\overrightarrow{BM} = -\frac{1}{2}\mathbf{c}$.

$\overrightarrow{CM} = \overrightarrow{CB} + \overrightarrow{BM} = \mathbf{a} - \frac{1}{2}\mathbf{c}$. **M1 A1**

(ii) $\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$, so $\overrightarrow{ON} = \frac{2}{3}(\mathbf{a} + \mathbf{c})$. **M1**

$\overrightarrow{CN} = \overrightarrow{CO} + \overrightarrow{ON} = -\mathbf{c} + \frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{c} = \frac{2}{3}\mathbf{a} - \frac{1}{3}\mathbf{c}$. **A1**

(b) $\frac{2}{3}\mathbf{a} - \frac{1}{3}\mathbf{c} = \frac{2}{3}\left(\mathbf{a} - \frac{1}{2}\mathbf{c}\right)$, so $\overrightarrow{CN} = \frac{2}{3}\overrightarrow{CM}$. **M1**

So CN is parallel to CM, and they share the point C, so C, N and M lie on a straight line. **A1**

CN is $\frac{2}{3}$ of CM, so $CN : NM = 2 : 1$. **B1**

(c) D is on the line CM, so $\overrightarrow{OD} = \overrightarrow{OC} + s\overrightarrow{CM} = \mathbf{c} + s\mathbf{a} - \frac{1}{2}s\mathbf{c}$ for some number s. D is also on the line OA, so it has no $\mathbf{c}$ part: $1 - \frac{1}{2}s = 0$, giving $s = 2$. **M1**

$\overrightarrow{OD} = 2\mathbf{a}$. **A1**

Common mistakes

- **Treating a column vector as a fraction.** Drawing a line between the numbers, or "adding" $\begin{pmatrix}3\\5\end{pmatrix}$ and $\begin{pmatrix}2\\7\end{pmatrix}$ as fractions, loses the marks. Examiners see this every year.

- **Slips with negative numbers.** $5-(-2)=7$, not 3; $(-5)+(-1)=-6$, not -4. Write each row out on its own line if it helps.
- **Multiplying only one number.** $3\begin{pmatrix}6\\-4\end{pmatrix}=\begin{pmatrix}18\\-12\end{pmatrix}$: both numbers are multiplied.
- **Getting the direction backwards.** $\overrightarrow{AB}=\mathbf{b}-\mathbf{a}$, not $\mathbf{a}-\mathbf{b}$. Many wrong answers to position-vector questions start with $\overrightarrow{AB}$ the wrong way round. Say "end minus start" as you write it.
- **Using $\overrightarrow{BA}$ as if it were $\overrightarrow{AB}$.** Given A and $\overrightarrow{BA}$, you must subtract to find B.
- **Giving coordinates when a vector is asked for, or the other way round.** A vector goes in a column; a point goes in round brackets with a comma.
- **Stopping early.** If $\overrightarrow{FH}=2\overrightarrow{FG}$, then $\overrightarrow{OH}$ is $\overrightarrow{OF}+2\overrightarrow{FG}$, not just $2\overrightarrow{FG}$. Adding the two position vectors together is also wrong.
- **Magnitude errors.** Squaring -5 as -25, forgetting the square root, or forgetting the $\pm$ when finding an unknown from a magnitude. Some candidates didn't know that $|\ |$ means magnitude.
- **Using the wrong fraction of a line.** $AE : EB = 3 : 4$ means $\overrightarrow{AE}=\frac{3}{7}\overrightarrow{AB}$, not $\frac{3}{4}\overrightarrow{AB}$.
- **Not simplifying.** An answer such as $\mathbf{p}+\mathbf{q}+\frac{3}{7}(\mathbf{p}-\mathbf{q})$ left as it is, or a slip when collecting terms, loses the last mark when the question says "simplest form".
- **No route written down.** Many who can't finish a vector geometry question score nothing because they never wrote a route such as $\overrightarrow{OP}+\overrightarrow{PS}$, which earns a method mark by itself.

How to get the marks

- **"In its simplest form"**: expand the brackets and collect terms, giving something like $\frac{3}{5}\mathbf{a}+\frac{2}{5}\mathbf{b}$ or $\frac{2}{5}(3\mathbf{a}+2\mathbf{b})$. A correct but unsimplified answer usually scores all but the last mark.
- **Write the route.** For vector geometry, mark schemes give a method mark for a correct route along the lines of the diagram, even before any letters go in, such as $\overrightarrow{OM}=\overrightarrow{OA}+\overrightarrow{AM}$.
- **Show the Pythagoras step** for a magnitude, such as $\sqrt{(-6)^2+4^2}$. It earns the method mark if the arithmetic slips. Give a non-exact answer to 3 significant figures, such as 7.21.
- **"Show that" (parallel, trapezium, collinear)**: write both vectors, show one is a multiple of the other (write the multiple), and finish with a sentence. For collinear points also say they share a point.
- **"Make two statements about the relationship between two lines"**: one is that they are **parallel**, the other is how many times longer one is (from the multiple).
- **Answer in the form asked**: a column vector in brackets, coordinates in round brackets, a ratio as $2 : 3$ (or $1 : n$ if asked).
- **Follow-through**: later parts usually allow your earlier answers, so keep going even if you aren't sure about part (a).

Quick check

1. $\mathbf{a} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$. Find (a) $3\mathbf{a} - 2\mathbf{b}$ and (b) $|\mathbf{b}|$, correct to 3 significant figures.
2. P is the point $(3, -4)$ and $\overrightarrow{PQ} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$. (a) Find the coordinates of Q. (b) The line PQ is extended beyond Q to R, so that PR is four times as long as PQ. Write $\overrightarrow{OR}$ as a column vector.
3. The vector $\begin{pmatrix} 7 \\ t \end{pmatrix}$ has magnitude 25. Find the two possible values of t.
4. In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. X is the point on AB with $AX : XB = 3 : 1$. Find the position vector of X in its simplest form.
5. $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = 3\mathbf{b} - 2\mathbf{a}$. Show that A, B and C lie on a straight line, and find the ratio $AB : BC$.

Answers

1. (a) $\begin{pmatrix} -6 \\ 15 \end{pmatrix} - \begin{pmatrix} 8 \\ -2 \end{pmatrix} = \begin{pmatrix} -14 \\ 17 \end{pmatrix}$. (b) $\sqrt{4^2 + (-1)^2} = \sqrt{17} = 4.12$.
2. (a) $(3 - 5,\ -4 + 2) = (-2, -2)$. (b) $\overrightarrow{PR} = 4\overrightarrow{PQ} = \begin{pmatrix} -20 \\ 8 \end{pmatrix}$, so $\overrightarrow{OR} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \begin{pmatrix} -20 \\ 8 \end{pmatrix} = \begin{pmatrix} -17 \\ 4 \end{pmatrix}$.
3. $49 + t^2 = 625$, so $t^2 = 576$ and $t = 24$ or $t = -24$.
4. $\overrightarrow{OX} = \mathbf{a} + \frac{3}{4}(\mathbf{b} - \mathbf{a}) = \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$.
5. $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ and $\overrightarrow{AC} = 3\mathbf{b} - 2\mathbf{a} - \mathbf{a} = 3\mathbf{b} - 3\mathbf{a} = 3\overrightarrow{AB}$. So AC is parallel to AB and they share the point A: A, B and C are collinear. $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = 2\overrightarrow{AB}$, so $AB : BC = 1 : 2$.

Past questions to try

- **0580/42/M/J/21 Q5**: column vector sums, a magnitude and two unknowns, then a parallelogram with two lines extended to meet.
- **0580/42/O/N/21 Q9**: points and magnitudes, then showing three points are collinear and a ratio.
- **0580/42/O/N/24 Q6**: vectors in a trapezium, then two statements about parallel lines.
- **0580/43/M/J/25 Q26**: a vector across a quadrilateral, then the ratio in which the diagonals cut.