Cambridge AS & A Level · Mathematics (9709) · Notes

Algebra AS

Read online at pastopic.com/cambridge/mathematics-9709/notes/algebra-p2

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Read first: Quadratics

What you need to know

By the end of this topic you should be able to:

1. **Use the modulus.** Know that $|x|$ means the size of x without its sign, and sketch the V-shaped graph of $y = |ax + b|$. Solve equations and inequalities with moduli in them, such as $|4x + 1| = |x - 5|$ or $3x - 1 < |x + 2|$, using two facts: $|a| = |b|$ exactly when $a^2 = b^2$, and $|x - a| < b$ exactly when $a - b < x < a + b$.
2. **Divide polynomials.** Divide a polynomial of degree 4 or less by a linear expression (like $x - 2$ or $3x + 1$) or a quadratic one (like $x^2 + 3$), and say what the quotient and the remainder are. The remainder may be zero.
3. **Use the factor theorem and the remainder theorem**, including divisors such as $(2x - 1)$ whose x-coefficient is not 1. Use them to find factors and remainders, to solve polynomial equations and to find unknown coefficients.

Only straight-line graphs inside the modulus are in the syllabus: graphs like $y = |x^2 - 4|$ or $y = f(|x|)$ for curves are **not** included.

Understand it

What the modulus means

The **modulus** of a number, $|x|$, is its size with the sign thrown away: $|5| = 5$ and $|-5| = 5$. Think of it as **distance from** 0 on the number line, which is never negative.

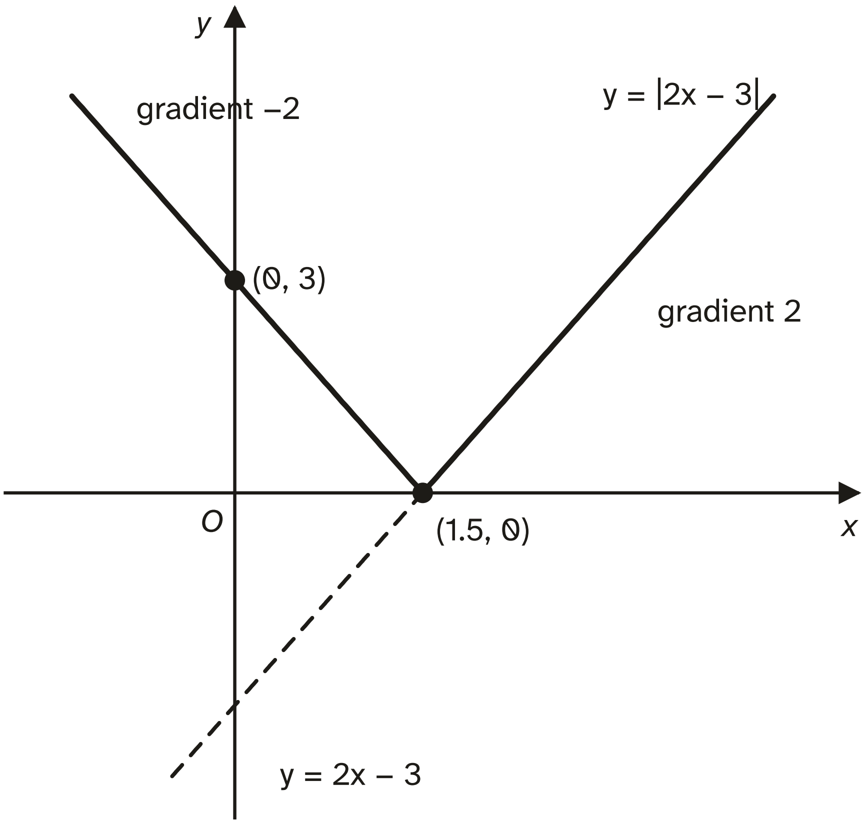
In the same way, $|x - a|$ is the distance between x and a. So $|x - 3| = 2$ means "x is 2 away from 3", giving $x = 1$ or $x = 5$, and $|x - 3| < 2$ means "x is less than 2 away from 3", giving $1 < x < 5$. That is the rule

$$|x - a| < b \iff a - b < x < a + b.$$

For "more than", the answer splits into two pieces: $|x - a| > b \iff x < a - b \text{ or } x > a + b$.

The graph of y = |ax + b|

Draw the straight line $y = ax + b$ first. Wherever it is above the x-axis, the modulus changes nothing. Wherever it is below, the modulus makes every y-value positive, so that part is **reflected in the x-axis**. The result is a V shape.



For $y = |2x - 3|$:

- the **vertex** is on the x-axis where $2x - 3 = 0$, at $(1.5, 0)$;
- the graph meets the y-axis at $y = |-3| = 3$;
- both arms are straight: the right arm has gradient 2 and the left arm has gradient -2, so they are equally steep.

Solving modulus equations

Because $|a| = |b|$ means a and b are the same size, either $a = b$ or $a = -b$. Squaring does the same job in one go, because $a^2 = b^2$ exactly when $a = \pm b$. So there are two methods.

Method 1: two linear equations. For $|x + 5| = |3x - 1|$:

- $x + 5 = 3x - 1$ gives $x = 3$;
- $x + 5 = -(3x - 1)$ gives $4x = -4$, so $x = -1$.

Method 2: square both sides. $(x + 5)^2 = (3x - 1)^2$ gives $x^2 + 10x + 25 = 9x^2 - 6x + 1$, so $8x^2 - 16x - 24 = 0$, that is $x^2 - 2x - 3 = 0$ and $(x - 3)(x + 1) = 0$. Same answers.

When one side has no modulus, as in $|x-4| = 2x+1$, take care. A modulus is never negative, so any answer that makes $2x+1$ negative is false:

- $x-4 = 2x+1$ gives $x=-5$, but then $2x+1=-9<0$, so reject it;
- $-(x-4) = 2x+1$ gives $x=1$, and $2x+1=3$: this works, since $|1-4|=3$.

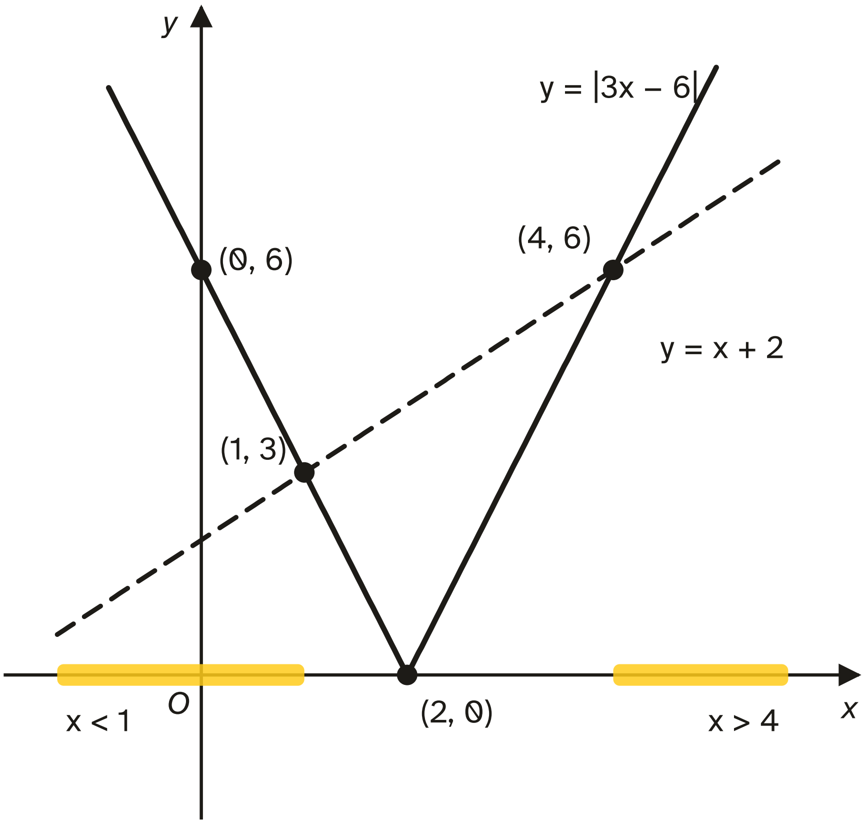
So $x=1$ is the only solution. Squaring would give both 1 and -5, and you would still have to throw -5 away, so a sketch or a check is essential here.

Solving modulus inequalities

Solve an inequality in two stages:

1. **Find the critical values**: solve the matching equation, as above.
2. **Decide which side** of each critical value is wanted: use a sketch, or test one easy number in each region.

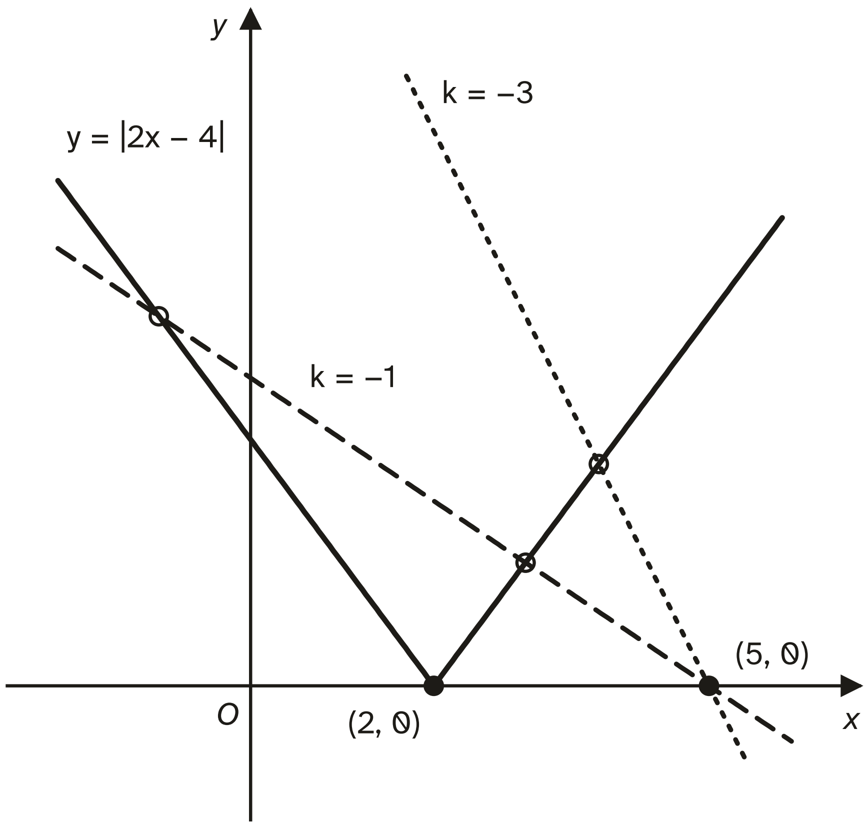
For $|3x-6| > x+2$ (Example 2), the critical values are $x=1$ and $x=4$. The sketch shows the V is above the line outside these values, so $x<1$ or $x>4$.



When both sides are moduli you can also square: $|a|<|b|$ exactly when $a^2<b^2$, so you get a quadratic inequality and solve it as usual. When one side is not a modulus, do **not** square: use two linear equations and a sketch.

Counting roots with a sketch

The number of solutions of an equation is the number of times the two graphs cross. This is how to answer questions such as "for which k does $|2x-4| = k(x-5)$ have exactly two roots?". Every line $y = k(x-5)$ passes through $(5, 0)$, whatever k is, and k is its gradient.



A line through $(5, 0)$ that slopes down from left to right ($k < 0$) always meets the right arm once, between $x = 2$ and $x = 5$. It also meets the left arm only if it is **less steep** than that arm, whose gradient is -2. So there are exactly two roots when $-2 < k < 0$. (At $k = -2$ the line is parallel to the left arm, and at $k = 0$ it is the x-axis, which touches the V only at the vertex.)

Polynomials and division

A **polynomial** is a sum of whole-number powers of x, such as $2x^3 - 3x^2 + 4$. Its **degree** is the highest power. Dividing polynomials works like long division of numbers: $17 \div 5$ is 3 remainder 2 because $17 = 5 \times 3 + 2$. For polynomials,

$$\text{polynomial} = \text{divisor} \times \text{quotient} + \text{remainder},$$

and the remainder always has a **lower degree** than the divisor: a number when you divide by a linear expression, and at most a linear expression $rx + s$ when you divide by a quadratic.

To divide $2x^3 - 3x^2 + 4$ by $x - 2$, first write the missing x term as $0x$, so that every column lines up: $2x^3 - 3x^2 + 0x + 4$. Then repeat "divide the leading terms, multiply back, subtract":

Step	Divide the leading term by x	Multiply $(x-2)$ by it	Subtract to leave
1	$2x^3 \div x = 2x^2$	$2x^3 - 4x^2$	$x^2 + 0x + 4$
2	$x^2 \div x = x$	$x^2 - 2x$	$2x + 4$
3	$2x \div x = 2$	$2x - 4$	8

The quotient is $2x^2 + x + 2$ and the remainder is 8: $2x^3 - 3x^2 + 4 = (x-2)(2x^2 + x + 2) + 8$.

Another way: compare coefficients. Write $2x^3 - 3x^2 + 4 = (x-2)(Ax^2 + Bx + C) + R$ and match the terms: x^3 gives $A = 2$; x^2 gives $B - 2A = -3$, so $B = 1$; x gives $C - 2B = 0$, so $C = 2$; the constant gives $R - 2C = 4$, so $R = 8$.

The remainder theorem

Put $x = 2$ into $2x^3 - 3x^2 + 4 = (x-2)(2x^2 + x + 2) + 8$. The bracket $(x-2)$ becomes 0, so the whole product vanishes and only the remainder is left: $p(2) = 8$. This always works:

- the remainder when $p(x)$ is divided by $(x-a)$ is $p(a)$;
- the remainder when $p(x)$ is divided by $(ax+b)$ is $p\left(-\frac{b}{a}\right)$: use the value of x that makes the divisor zero.

So you can find a remainder without dividing at all. For example, dividing $4x^3 - 2x^2 + x + 3$ by $(2x+1)$ leaves $p\left(-\frac{1}{2}\right) = -\frac{1}{2} - \frac{1}{2} - \frac{1}{2} + 3 = \frac{3}{2}$.

The factor theorem

A **factor** divides exactly, with remainder 0. So:

- $(x-a)$ is a factor of $p(x)$ exactly when $p(a) = 0$;
- $(ax+b)$ is a factor of $p(x)$ exactly when $p\left(-\frac{b}{a}\right) = 0$.

To **factorise a cubic completely**, find one factor (it is usually given, or found by trying small values such as ± 1, ± 2), divide to get a quadratic quotient, then factorise that quadratic if you can. If the quadratic has discriminant $b^2 - 4ac < 0$ it has no real roots and does not factorise, and then the cubic equation has only the one real root that the linear factor gives.

Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. "Learn it" means you must remember it. MF19 has nothing for this topic except the quadratic formula.

Formula	Status
$\lvert x\rvert = x$ if $x \geqslant 0$, and $\lvert x\rvert = -x$ if $x < 0$; so $\lvert x\rvert \geqslant 0$ always	Learn it
$\lvert a\rvert = \lvert b\rvert \iff a^2 = b^2 \iff a = b$ or $a = -b$	Learn it
$\lvert x - a\rvert < b \iff a - b < x < a + b$	Learn it

Formula	Status
$\lvert x-a\rvert > b \iff x < a-b$ or $x > a+b$	Learn it
$y = \lvert ax+b\rvert$: a V with vertex at $\left(-\frac{b}{a}, 0\right)$, y-intercept $\lvert b\rvert$, arm gradients $\pm a$	Learn it
$p(x) = \text{divisor} \times \text{quotient} + \text{remainder}$, with the remainder of lower degree than the divisor	Learn it
Remainder theorem: dividing $p(x)$ by $(x-a)$ leaves $p(a)$; by $(ax+b)$ leaves $p\left(-\frac{b}{a}\right)$	Learn it
Factor theorem: $(x-a)$ is a factor $\iff p(a)=0$; $(ax+b)$ is a factor $\iff p\left(-\frac{b}{a}\right)=0$	Learn it
Quadratic formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$	Given
$b^2-4ac<0$: the quadratic has no real roots	Learn it

How to do it

In the Paper 2 questions we have tagged for this topic (70 questions, 2021–2025; many appear in two or three versions of the same exam, so there are 47 different ones), the types came up this often. 43 of the 70 questions mix two or more types, so the counts add up to more than 70.

Question type	Questions
Modulus equations and inequalities	31
Factorising and solving polynomial equations	29
Finding unknown coefficients with the factor and remainder theorems	26
Sketching modulus graphs	16
Dividing polynomials: quotient and remainder	12

1. Modulus equations and inequalities (31 questions)

1. **Look at the two sides.** Both moduli, like $\lvert x-5\rvert > \lvert 3x+1\rvert$? Then either method works. Only one modulus, like $\lvert 2x-1\rvert > x+4$? Then use two linear equations, not squaring.
2. **Find the critical values.**
   - Two linear equations: $\text{inside} = \text{other side}$ and $\text{inside} = -(\text{other side})$. When both sides are moduli, $\lvert A\rvert = \lvert B\rvert$ gives $A=B$ or $A=-B$.
   - Squaring (both sides moduli only): $A^2 = B^2$, collect into a three-term quadratic and solve it.
3. **Check each value** when one side is not a modulus: the other side must not be negative there. Reject any value that fails, and say so.
4. **For an inequality, pick the regions.** Sketch both graphs or test a number such as $x=0$. The answer is either one interval ($p<x<q$) or two separate pieces ($x<p$ **or** $x>q$). Write two pieces as two separate inequalities; never join them as $q<x<p$.

For example, $|x-5| > |3x+1|$: $x-5=3x+1$ gives $x=-3$ and $x-5=-(3x+1)$ gives $x=1$. At $x=0$, $5>1$ is true, so the answer is the region that contains 0: $-3<x<1$.

Variations you will meet:

- **A simple modulus less than a number**, such as $|3x+2|<11$: use $-11<3x+2<11$, so $-\frac{13}{3}<x<3$.
- **A number times a modulus of x**, such as $4|x|=6-x$: split into $4x=6-x$ and $-4x=6-x$ (giving $x=\frac{6}{5}$ and $x=-2$), and check $6-x>0$ for each. If you square, square **everything**: $16x^2=(6-x)^2$.
- **Use the solutions afterwards**, for example "the solutions are $x=p$ and $x=q$ where $p<q$; find $|2p-1|+|q+3|$". Find p and q exactly first, put them in the right places, then work out each modulus as a positive number. Keep fractions or surds exact.
- **"Hence" with a power of x inside.** After solving $|3x-6|>x+2$, the inequality $|3\times 2^y-6|>2^y+2$ is the same one with $x=2^y$. Use your answer to (a) with x replaced by 2^y, remembering that 2^y is always positive, then solve with logarithms: $2^y<1$ gives $y<0$ and $2^y>4$ gives $y>2$. The same idea works for $e^{0.1N}$, $\ln N$ or 3^y in place of x. If the question asks for the largest or smallest **integer** N, work out the boundary with logarithms first, then choose the integer on the correct side of it.
- **Solving $|4y-5|=a$ where a is a root found earlier**: a modulus can only equal a positive number, so use the positive root only.

2. Factorising and solving polynomial equations (29 questions)

1. **Find a linear factor.** It is usually given, or it comes from the factor theorem.
2. **Divide** by that factor (long division, or compare coefficients) to get the quadratic quotient. Show this working: a factorised answer copied from a calculator earns little.
3. **Factorise the quadratic**, or show it cannot be factorised by working out its discriminant.
4. **Write the full factorisation**, a product of linear factors when that is possible, and read off the roots.

Variations you will meet:

- **"Show that $p(x)=0$ has exactly one real root."** Factorise as (linear)(quadratic), then show the quadratic's discriminant is negative, for example $(x+1)(x^2-2x+5)$, where x^2-2x+5 has discriminant $4-20=-16<0$. Finish with a sentence: "so the only real root is $x=-1$".
- **A substitution**, such as $p(e^y)=0$, $p(3^x)=0$ or $p(\sqrt{x})=0$. Set the new expression equal to each root of $p(x)=0$. Since e^y and 3^x are always positive, only the **positive** roots give answers: $e^y=\frac{3}{2}$ gives $y=\ln\frac{3}{2}$, while $e^y=-2$ has no solution. For $p(3x)=0$, simply divide each root by 3.
- **A trigonometric substitution**, such as $p(\operatorname{cosec}\theta)=0$. Set $\operatorname{cosec}\theta$ equal to each root r, so $\sin\theta=\frac{1}{r}$. Reject any root that gives $|\sin\theta|>1$ (for example $r=\frac{3}{2}$ gives $\sin\theta=\frac{2}{3}$, fine; $r=\frac{1}{2}$ gives $\sin\theta=2$, impossible). Then find the angles in the given range. With $p(\cot 2\theta)=0$, use $\tan 2\theta=\frac{1}{r}$ and halve at the end.
- **Solving $p(x)+c=0$ after a division.** If dividing $p(x)$ by a divisor gave remainder $-c$, then $p(x)+c=\text{divisor}\times\text{quotient}$, so the roots come from divisor $=0$ and quotient $=0$. A divisor such as x^2+4 gives no real roots. Give exact roots with surds when the quotient does not factorise, and throw away any that

are not real.

- **The same idea in another topic**: a cubic in t from parametric equations, with a given factor $(t+1)$. Divide, then use the discriminant of the quadratic to show $t=-1$ is the only solution.

3. Finding unknown coefficients with the factor and remainder theorems (26 questions)

1. **Find the value of x that makes each divisor zero**: $x+2 \to x=-2$; $2x-1 \to x=\frac{1}{2}$.
2. **Substitute it into $p(x)$.** Set the result equal to 0 for a **factor**, or equal to the given **remainder**. Work out every power carefully, especially with negative numbers and fractions: $\left(\frac{1}{2}\right)^3=\frac{1}{8}$ and $(-2)^3=-8$.
3. **Tidy each equation** (multiply out fractions), then solve the pair of simultaneous equations.
4. **Check** by substituting your values back into one condition.

For example, if $(x+2)$ is a factor of $2x^3+ax^2+bx-6$ and the remainder on dividing by $(x-2)$ is 12, then $p(-2)=0$ and $p(2)=12$. Example 3 finishes this.

Variations you will meet:

- **One unknown and one condition**, such as "the remainder when dividing by $(x-3)$ is 20; find a". One substitution, one linear equation.
- **Two given factors** give two equations equal to zero.
- **Two different polynomials**, $f(x)$ and $g(x)$, each with its own condition. Find each constant separately, then (usually) factorise $f(x)-g(x)$: collect like terms first.
- **The factor theorem is quicker than dividing.** Long division with letters in the polynomial is slow and error-prone. Substitution is the expected method; use division only if you prefer and are careful.

4. Sketching modulus graphs (16 questions)

1. **The V**: find the vertex where the inside is zero (on the x-axis), and the y-intercept $|b|$. Draw two straight arms, equally steep.
2. **The other line**: find its y-intercept and x-intercept, and compare its gradient with the arm it crosses. A line steeper than an arm crosses it; one less steep may not.
3. **Check the number of crossings** matches what the question says (for example, "the graphs intersect once") and that each crossing is in the right quadrant.
4. **Label** each graph and the points you worked out.

Variations you will meet:

- **"Hence solve"** after a sketch: the sketch tells you how many solutions there are and which side of each critical value you want. If your algebra gives a value where the graphs do not actually meet, reject it.
- **Lines through a fixed point**, like $y=k(x-5)$: find the gradients of the arms and compare, as in the third diagram. The answer is an inequality in k, often with strict signs.

- **The V with a curve**, such as $y = \ln x$ or $y = e^{-x}$ drawn with $y = |2x - 3|$: draw both carefully, then say in words that the graphs cross twice, so the equation has exactly two roots. To show a root satisfies a rearrangement, use the arm on which it lies: for $x > 1.5$ the V is $y = 2x - 3$; for $x < 1.5$ it is $y = 3 - 2x$.

5. Dividing polynomials: quotient and remainder (12 questions)

1. **Write $p(x)$ in descending powers**, putting 0 in for any missing power, such as $x^4 + 0x^3 + 0x^2 + 7x - 2$.
2. **Divide** step by step (divide the leading terms, multiply back, subtract) until what is left has a lower degree than the divisor. For a divisor like $(3x + 2)$, divide the leading term by $3x$.
3. **State the quotient and the remainder.** If the question says "show that the remainder is 6", your division must end with exactly 6, visibly.

Variations you will meet:

- **Dividing by a quadratic**, such as $x^2 + 3$ or $(x - 2)^2 = x^2 - 4x + 4$: expand the divisor first. Now the remainder can be linear, $rx + s$.
- **A letter in the polynomial**: carry it through, for example quotient $2x^2 + x + (a - 2)$ and a remainder that turns out to be a number.
- **Making it divide exactly**: find the constants that make the remainder zero, as in Example 4.
- **"Hence integrate"**: write $\dfrac{p(x)}{3x+2} = \text{quotient} + \dfrac{\text{remainder}}{3x+2}$. Remember to divide the remainder by the divisor; the integration itself belongs to the integration topic.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

Solve the inequality $|2x + 1| < |x - 4|$. **[4]**

Both sides are moduli, so either method works.

Linear equations. $2x + 1 = x - 4$ gives $x = -5$. **B1**

$2x + 1 = -(x - 4)$ gives $3x = 3$, so $x = 1$. **M1** for an equation with the x terms of opposite sign, **A1** for $x = 1$.

At $x = 0$: $|1| < |-4|$ is true, so the answer is the region containing 0:

$$-5 < x < 1.$$

A1

Or square. $(2x+1)^2 < (x-4)^2$ gives $4x^2 + 4x + 1 < x^2 - 8x + 16$, so $3x^2 + 12x - 15 < 0$, that is $3(x+5)(x-1) < 0$. **B1** for the non-modulus inequality, **M1** for solving the three-term quadratic. The critical values are -5 and 1 (**A1**), and a positive quadratic is negative between its roots: $-5 < x < 1$. **A1**

Example 2

(a) Sketch, on the same diagram, the graphs of $y = |3x - 6|$ and $y = x + 2$. **[2]**

(b) Solve the inequality $|3x - 6| > x + 2$. **[4]**

(c) Hence find the set of values of y for which $|3 \times 2^y - 6| > 2^y + 2$. **[2]**

(a) The V has its vertex at $(2, 0)$ and meets the y-axis at $(0, 6)$. **B1** The line has gradient 1, less steep than the arms (gradient ± 3), and meets the y-axis at $(0, 2)$, below the V, so it crosses the V twice. **B1** The second diagram in "Understand it" shows the sketch.

(b) The right arm: $3x - 6 = x + 2$ gives $x = 4$. **B1**

The left arm: $-(3x - 6) = x + 2$ gives $6 - 3x = x + 2$, so $x = 1$. **M1 A1**

The V is above the line outside these values: $x < 1$ or $x > 4$. **A1**

(Don't square here: $x + 2$ is not a modulus, so squaring can bring in false critical values. The linear method with the sketch is safer.)

(c) This is part (b) with $x = 2^y$, so $2^y < 1$ or $2^y > 4$. **M1** for using the answer to (b).

$2^y < 1 = 2^0$ gives $y < 0$, and $2^y > 4 = 2^2$ gives $y > 2$. So $y < 0$ or $y > 2$. **A1**

Example 3

The polynomial $p(x)$ is defined by $p(x) = 2x^3 + ax^2 + bx - 6$, where a and b are constants. It is given that $(x + 2)$ is a factor of $p(x)$ and that the remainder is 12 when $p(x)$ is divided by $(x - 2)$.

(a) Find the values of a and b. **[4]**

(b) Hence factorise $p(x)$ completely. **[3]**

(c) Solve the equation $p(e^y) = 0$, giving your answer in an exact form. **[2]**

(a) Factor theorem: $p(-2) = -16 + 4a - 2b - 6 = 0$, so $4a - 2b = 22$. **M1** for substituting $x = -2$ and equating to 0.

Remainder theorem: $p(2) = 16 + 4a + 2b - 6 = 12$, so $4a + 2b = 2$. **M1** for substituting $x = 2$ and equating to 12; **A1** for both equations.

Adding: $8a = 24$, so $a = 3$, and then $2b = 2 - 12$, so $b = -5$. **A1**

(b) $p(x) = 2x^3 + 3x^2 - 5x - 6$. Divide by $(x + 2)$:

Step	Divide by x	Multiply $(x+2)$ by it	Subtract to leave
1	$2x^2$	$2x^3 + 4x^2$	$-x^2 - 5x - 6$
2	$-x$	$-x^2 - 2x$	$-3x - 6$
3	-3	$-3x - 6$	0

M1 for dividing at least as far as the x term; **A1** for the quotient $2x^2 - x - 3$.

$$p(x) = (x+2)(2x^2 - x - 3) = (x+2)(2x-3)(x+1).$$

A1

(c) $p(e^y) = 0$ means $e^y = -2$, $e^y = \frac{3}{2}$ or $e^y = -1$. But $e^y > 0$ always, so only $e^y = \frac{3}{2}$ works. **M1**

$$y = \ln\tfrac{3}{2}.$$

A1, with no other values.

Example 4

(a) Find the quotient and the remainder when $2x^4 - 3x^3 + 5x - 1$ is divided by $x^2 - 2x + 3$. **[4]**

(b) Hence find the values of the constants c and d for which $2x^4 - 3x^3 + cx + d$ is exactly divisible by $x^2 - 2x + 3$. **[3]**

(a) Write the missing term: $2x^4 - 3x^3 + 0x^2 + 5x - 1$. Divide the leading term by x^2 each time:

Step	Divide by x^2	Multiply $(x^2 - 2x + 3)$ by it	Subtract to leave
1	$2x^2$	$2x^4 - 4x^3 + 6x^2$	$x^3 - 6x^2 + 5x - 1$
2	x	$x^3 - 2x^2 + 3x$	$-4x^2 + 2x - 1$
3	-4	$-4x^2 + 8x - 12$	$-6x + 11$

M1 for dividing as far as the x term of the quotient, **A1** for $2x^2 + x$, **A1** for the quotient $2x^2 + x - 4$, **A1** for the remainder $-6x + 11$.

The remainder $-6x + 11$ is linear, lower in degree than the divisor, so the division is finished.

(b) The new polynomial is the old one plus $(c - 5)x + (d + 1)$. These extra terms have lower degree than the divisor, so they simply add to the remainder:

$$\text{new remainder} = (-6x + 11) + (c-5)x + (d+1) = (c-11)x + (d+12).$$

M1 For exact division this must be zero for every x, so $c - 11 = 0$ and $d + 12 = 0$. **M1**

$$c = 11, \quad d = -12.$$

A1 Check: $(x^2 - 2x + 3)(2x^2 + x - 4) = 2x^4 - 3x^3 + 11x - 12$.

Common mistakes

- **Squaring only part of a term.** $|5x|$ squared is $25x^2$, not $5x^2$, and $|3 - x|$ squared is $(3 - x)^2$, not $9 - x^2$. Examiners see this in almost every modulus question. Write $(5x)^2$ with the bracket before you expand.
- **Squaring when one side is not a modulus.** It creates false solutions. Use two linear equations, then check each value (or read the sketch) and reject the ones that fail.
- **Giving every critical value as an answer.** For "solve the equation", reject values where the graphs don't meet; for an inequality, a critical value is a boundary, not the answer. Use your sketch from the part before.
- **Joining two regions with one inequality.** $x < -10$ or $x > \frac{4}{3}$ must be written as two separate inequalities; $-10 > x > \frac{4}{3}$ is wrong.
- **Sketching the line with the wrong steepness.** If the question says the graphs meet once, the line must be steeper than the arm it would otherwise run beside. Check gradients and intercepts before drawing.
- **Forgetting to factorise the quadratic.** "Factorise completely" means linear factors if they exist: $(x + 2)(2x^2 - x - 3)$ is not finished.
- **Assuming a quadratic factorises when it doesn't.** Check the discriminant; if it is negative, say so. That is exactly what "exactly one real root" questions want.
- **Missing out placeholder terms.** Dividing $x^4 + 7x - 2$ without writing $0x^3 + 0x^2$ mixes up the columns.
- **Thinking the polynomial is divisor times remainder.** It is divisor $\times$ quotient $+$ remainder.
- **Arithmetic slips with fractions and negatives** when substituting $x = \frac{1}{2}$ or $x = -3$. Write each power out: $a\left(\frac{1}{2}\right)^3 = \frac{a}{8}$.
- **Keeping impossible roots after a substitution.** $e^y = -2$ and $\sin\theta = 2$ have no solutions; say so and leave them out.

How to get the marks

- **"Solve"** a modulus equation: each correct critical value earns a mark, and the last mark needs the final answer with **no** extra invalid values.
- **Inequalities**: the final mark is for the correctly written region. Use $<$ or $\leqslant$ as the question does, and show the critical values you found.
- **"Sketch"**: a V with its vertex on the x-axis and straight arms, and the other graph placed correctly relative to it (steeper or less steep, meeting the axis in the right place). Label the graphs.
- **Factor theorem**: write the substitution with the numbers in and "$= 0$" (or "$=$ the remainder"). The method mark needs the equation, not only the answer.

- **"Hence factorise"**: show a division, comparing coefficients or inspection that gets the quadratic factor. Factors that only come from a calculator's equation solver earn little or nothing.
- **"Show that the remainder is ..."**: the answer is given, so a complete division must reach it. If the question asks for the quotient too, the remainder theorem alone is not enough.
- **"Exactly one real root"**: name the real root, work out the quadratic's discriminant, say it is negative and conclude.
- **"Hence"** means use your answer to the part before, such as your factors or your critical values.
- **"Exact"** means surds, fractions and logarithms like $\ln \frac{3}{2}$, not decimals. Otherwise give non-exact answers to 3 significant figures, or angles to 1 decimal place in degrees.
- **Follow-through**: many marks in later parts are for using your earlier answers correctly, so keep going even if you doubt an earlier value.

Quick check

1. Solve the equation $|2x-7| = |x+1|$.
2. Solve the inequality $|2x-3| < 5$.
3. Find the remainder when $2x^3 + x^2 - 5x + 4$ is divided by $(2x-1)$.
4. Solve the equation $|x-6| = 3x+2$.
5. Show that $(x-2)$ is a factor of $p(x) = x^3 - x^2 - x - 2$, and hence show that the equation $p(x) = 0$ has exactly one real root.

Answers

1. $2x - 7 = x + 1$ gives $x = 8$; $2x - 7 = -(x + 1)$ gives $3x = 6$, so $x = 2$. The solutions are $x = 8$ and $x = 2$.
2. $-5 < 2x - 3 < 5$, so $-2 < 2x < 8$, that is $-1 < x < 4$.
3. $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{8}\right) + \frac{1}{4} - \frac{5}{2} + 4 = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 4 = 2$.
4. $x - 6 = 3x + 2$ gives $x = -4$, but then $3x + 2 = -10 < 0$, so reject it. $-(x - 6) = 3x + 2$ gives $4x = 4$, so $x = 1$, and $|1 - 6| = 5 = 3(1) + 2$. So $x = 1$ only.
5. $p(2) = 8 - 4 - 2 - 2 = 0$, so $(x - 2)$ is a factor. Dividing gives $p(x) = (x - 2)(x^2 + x + 1)$. The quadratic has discriminant $1^2 - 4(1)(1) = -3 < 0$, so it has no real roots, and $x = 2$ is the only real root.

Past questions to try

- **9709/22/F/M/22 Q1**: a modulus equation with moduli on both sides.
- **9709/22/O/N/23 Q4**: sketching a V and a line, then a modulus inequality.
- **9709/22/O/N/24 Q5**: two unknowns from the factor and remainder theorems, then exactly one real root.
- **9709/23/M/J/24 Q5**: dividing by a linear factor and showing the remainder, before integrating.