Cambridge AS & A Level · Mathematics (9709) · Notes

# Circular measure AS

Read online at pastopic.com/cambridge/mathematics-9709/notes/circular-measure

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Read first: Quadratics



## What you need to know

By the end of this topic you should be able to:

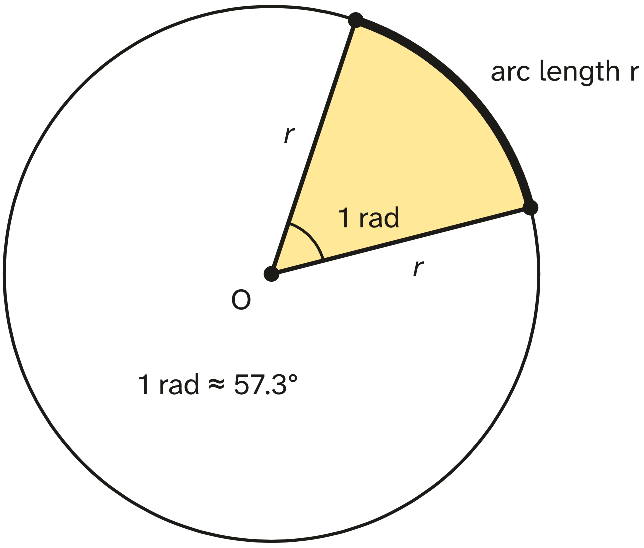
1. **Measure angles in radians.** Know that one radian is the angle at the centre of a circle made by an arc exactly as long as the radius, so a full turn is $2\pi$ radians. Change degrees into radians and radians into degrees.
2. **Find arcs and sectors.** Work out the length of an arc and the area of a sector from the radius and the angle in radians, and run the same formulas backwards to find a radius or an angle from a length or an area you are told.
3. **Bring in triangles.** Use right-angled trigonometry, Pythagoras, the sine and cosine rules and $\frac{1}{2}ab\sin C$ to find the lengths, angles and areas you need, then build the perimeter and area of shapes made from arcs, chords, sectors, segments and triangles.

Graphs and equations of $\sin$, $\cos$ and $\tan$ in radians are in Trigonometry. Solving the quadratic that sometimes appears when you work backwards is in Quadratics.

## Understand it

### What a radian is

Degrees split a full turn into 360 parts, a number chosen long ago for convenience. Radians measure an angle by the circle itself: wrap the radius around the edge, and the angle it covers at the centre is **1 radian**.




The whole circumference is $2\pi r$, so the radius fits around it $2\pi$ times: a full turn is $2\pi$ radians. That gives the one fact you need for converting:

$$\pi \text{ radians} = 180^\circ, \qquad 1 \text{ radian} = \frac{180^\circ}{\pi} \approx 57.3^\circ.$$

- **Degrees to radians:** multiply by $\dfrac{\pi}{180}$. So $40^\circ = \dfrac{40\pi}{180} = \dfrac{2\pi}{9}$.
- **Radians to degrees:** multiply by $\dfrac{180}{\pi}$. So $0.8$ rad $= 0.8 \times \dfrac{180}{\pi} \approx 45.8^\circ$.

Learn the common ones so you can read them straight off: $\frac{\pi}{6} = 30^\circ$, $\frac{\pi}{4} = 45^\circ$, $\frac{\pi}{3} = 60^\circ$, $\frac{\pi}{2} = 90^\circ$, $\frac{2\pi}{3} = 120^\circ$, $\frac{3\pi}{4} = 135^\circ$, $\pi = 180^\circ$, $\frac{3\pi}{2} = 270^\circ$.

An angle written with no degree sign, such as "angle $AOB = 1.2$", is in radians. It means $1.2$ radians, not $1.2\pi$.

## Arc length and sector area

A **sector** is a slice of a circle between two radii. If its angle is $\theta$ radians, it is $\dfrac{\theta}{2\pi}$ of the whole circle, so its arc is that fraction of the circumference and its area is that fraction of $\pi r^2$:

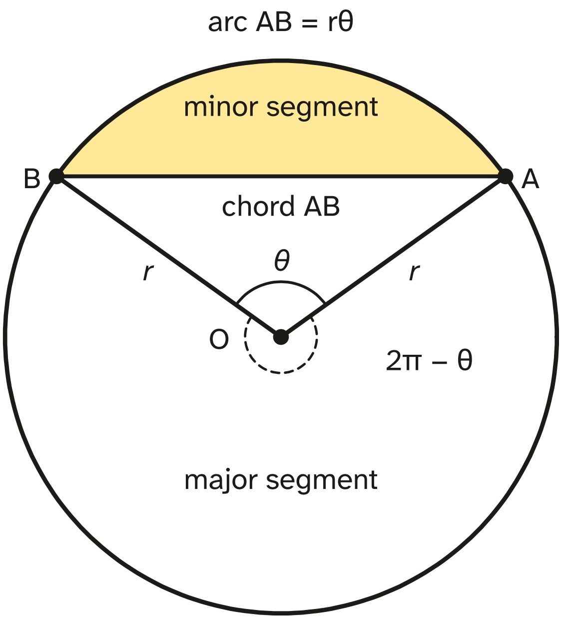
$$s = \frac{\theta}{2\pi} \times 2\pi r = r\theta, \qquad A = \frac{\theta}{2\pi} \times \pi r^2 = \tfrac{1}{2} r^2\theta.$$

This is why radians are used: the formulas lose the $\frac{\ldots}{360}$ clutter. A sector with radius $6$ cm and angle $0.5$ rad has arc $6 \times 0.5 = 3$ cm and area $\frac{1}{2} \times 36 \times 0.5 = 9$ cm². Its **perimeter** is the arc plus two radii: $3 + 12 = 15$ cm.

Both formulas only work with $\theta$ in **radians**. Your calculator must be in radian mode for $\sin$, $\cos$ and $\tan$ of these angles too.

## Chords, triangles and segments

Join the ends of the arc with a straight line, the **chord** $AB$. This splits the sector into the triangle $OAB$ and a **segment**, the piece between the chord and the arc.




- **Triangle** $OAB$ has two sides $r$ with angle $\theta$ between them, so its area is $\frac{1}{2}r^2 \sin\theta$.
- **The minor segment** is sector minus triangle: $\frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta = \frac{1}{2}r^2(\theta - \sin\theta)$.
- **The chord**: the line from $O$ to the midpoint of $AB$ is at right angles to $AB$ and halves the angle, so $AB = 2r\sin\frac{\theta}{2}$. The cosine rule gives the same length.
- **The major sector and major segment** use the reflex angle $2\pi - \theta$: the major sector is $\frac{1}{2}r^2(2\pi - \theta)$, and the major segment is the whole circle minus the minor segment.

For $r = 10$ and $\theta = 1.2$: the sector is $60$, the triangle $\frac{1}{2} \times 100 \times \sin 1.2 \approx 46.6$, so the minor segment is about $13.4$. The chord is $20\sin 0.6 \approx 11.3$. The reflex angle is $2\pi - 1.2 \approx 5.08$, and the major segment is $100\pi - 13.4 \approx 301$.

## Every shape is built from a few pieces

Exam diagrams look complicated, but every shaded region is a sum or difference of **sectors, triangles, rectangles and segments**. The plan is always:

1. **Find every angle at a centre**, in radians, and every radius. These usually come from a triangle.
2. **Perimeter:** add the arcs and the straight edges that are actually on the boundary.
3. **Area:** add and subtract the pieces, writing the plan in words first ("area = sector $OAB$ – triangle $OAB$").

The triangle tools come from earlier work:

- **Right-angled triangles:** $\sin$, $\cos$, $\tan$ and Pythagoras. A radius meets a tangent at $90^\circ$, and the line from the centre to a chord's midpoint meets the chord at $90^\circ$.
- **Any triangle:** the sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$, the cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$, and area $\frac{1}{2}ab\sin C$.
- **Special triangles:** two radii and a chord all of length $r$ make an **equilateral** triangle, so the angle is $\frac{\pi}{3}$. Two equal radii make an **isosceles** triangle: with apex angle $\theta$, each base angle is $\frac{\pi-\theta}{2}$. Angles round a point add to $2\pi$, and angles in a triangle to $\pi$.
- **Exact values** for exact answers: $\sin\frac{\pi}{6} = \frac{1}{2}$, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{\pi}{3} = \frac{1}{2}$, $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, $\tan\frac{\pi}{4} = 1$, $\tan\frac{\pi}{3} = \sqrt{3}$, $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$.

## Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. "Learn it" means you must remember it. $\theta$ is always in radians.

| Formula | Status |
|---|---|
| Arc length $s = r\theta$ | Given |
| Sector area $A = \frac{1}{2}r^2\theta$ | Given |
| $\pi$ radians $= 180^\circ$; degrees $\times\frac{\pi}{180}$ = radians; radians $\times\frac{180}{\pi}$ = degrees | Learn it |
| Perimeter of a sector $= 2r + r\theta$ | Learn it |
| Area of a triangle $= \frac{1}{2}ab\sin C$, so triangle $OAB = \frac{1}{2}r^2\sin\theta$ | Learn it |
| Minor segment $= \frac{1}{2}r^2(\theta - \sin\theta)$ | Learn it |
| Chord $AB = 2r\sin\frac{\theta}{2}$ | Learn it |
| Reflex angle $= 2\pi - \theta$; major sector $= \frac{1}{2}r^2(2\pi - \theta)$ | Learn it |
| Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$; cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$ | Learn it |
| Exact values of $\sin$, $\cos$, $\tan$ at $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$ | Learn it |

## How to do it

In the Paper 1 questions we have tagged for this topic (37 questions, 2021–2025), the types came up this often. Almost every question mixes several, and nearly all of them hinge on finding an angle first.

| Question type | Questions |
|---|---|
| Area of a region made of sectors, segments and triangles | 33 |
| Perimeter of a region: arcs plus straight edges | 26 |
| Find or show an angle or length from the geometry | 22 |
| Exact answers, or answers in terms of $r$, $\pi$ and $\sqrt{3}$ | 17 |

| Question type | Questions |
| --- | --- |
| Work backwards from a given arc, area or perimeter | 12 |
| With other topics: circles in coordinate geometry, rates of change | 3 |

**Mark up the diagram first**: every radius, every angle at a centre, every right angle. Students who mark up the diagram first tend to find the route more easily.

## 1. Area of a region made of sectors, segments and triangles (33 questions)

1. **Find the angle at each centre in radians**, and each radius (type 3).
2. **Write the plan in words**: which pieces add, which subtract.
3. **Use** $\frac{1}{2}r^2\theta$ for sectors, $\frac{1}{2}ab\sin C$ (or $\frac{1}{2}\times$ base $\times$ height) for triangles, $\frac{1}{2}r^2(\theta - \sin\theta)$ for segments.
4. **Keep 4 or more significant figures** until the final answer.

**Variations you will meet:**

- **Segment:** sector minus the triangle on the same two radii. The **major segment** is the circle minus the minor segment, or the major sector plus the triangle.
- **Triangle minus sector:** a sector cut out of a corner of a triangle leaves triangle minus sector. With a tangent from an outside point, the region between the tangents and the arc is the kite minus the sector (Example 3).
- **Sector minus a triangle that isn't $OAB$:** when a perpendicular drops from the arc to a radius, subtract the right-angled triangle it makes, using $\sin$ and $\cos$ for its sides.
- **A polygon with pieces removed**, such as a square minus two triangles and a sector: find each piece, then subtract.
- **Two overlapping circles:** the shared region is two segments, one from each circle. With equal radii, it is twice one segment.
- **A crescent between two arcs:** one segment minus another, or a semicircle minus a segment (Example 4). Each piece has its own radius and its own angle.
- **Composite plates**: several sectors (sometimes different radii) plus triangles, rectangles or a rhombus, or semicircles ($\frac{1}{2}\pi r^2$) on chords. A rhombus with side $r$ and angle $\alpha$ has area $r^2\sin\alpha$ (two triangles).
- **A curved shape of constant width** (arcs centred at the corners of an equilateral triangle): triangle plus three segments, or three sectors minus two triangles.
- **"Difference of the areas of two triangles"**: find both, then subtract the smaller from the larger.

## 2. Perimeter of a region: arcs plus straight edges (26 questions)

1. **Trace the boundary** of the region with your finger, listing each piece: which arcs, which straight edges.
2. **Arcs:** $r\theta$, each with its own radius and angle. A **major arc** uses $2\pi - \theta$; a semicircle is $\pi r$.
3. **Straight edges:** given lengths, a chord ($2r\sin\frac{\theta}{2}$ or the cosine rule), a side from right-angled trigonometry, or a difference of two lengths.

4. **Add them**, and give the form asked for.

**Variations you will meet:**

- **Perimeter of a segment:** arc plus chord.
- **Straight edges found by subtraction:** if $AB$ is made of overlapping radii, $AB =$ first radius $+$ second radius $-$ overlap. If $C$ is on $AD$ extended, $CD = AC - AD$.
- **Arcs from different circles**: an arc of radius $3$ with angle $\frac{5\pi}{6}$ plus an arc of radius $2$ with angle $\frac{\pi}{2}$ is $\frac{5\pi}{2} + \pi = \frac{7\pi}{2}$. Never add angles from different circles.
- **Several equal arcs:** a shape made of four equal arcs has perimeter $4r\theta$. A rope wrapped around equal circles is straight pieces plus arcs, and the arc angles add to one full turn, $2\pi$.
- **Perimeter in terms of $r$**, such as $\frac{7}{4}\pi r + 2r$: collect like terms.

## 3. Find or show an angle or length from the geometry (22 questions)

- **Equilateral triangles:** radii and a chord all equal, or two circles of radius $r$ whose centres are $r$ apart. Say why it is equilateral, then use $\frac{\pi}{3}$. Equal angles round a point share $2\pi$: six equal ones are each $\frac{\pi}{3}$, three are each $\frac{2\pi}{3}$ (for example at the centre of the circle through the corners of an equilateral triangle).
- **Isosceles triangles** (two radii): base angles are $\frac{\pi-\theta}{2}$. With apex $1.4$, each base angle is $\frac{\pi-1.4}{2} \approx 0.871$. Half the chord is $r\sin\frac{\theta}{2}$.
- **Angle from a chord:** with radius $r$ and chord $c$, $\sin\frac{\theta}{2} = \frac{c}{2r}$, or use the cosine rule $\cos\theta = \frac{2r^2-c^2}{2r^2}$.
- **Right angles:** radius and tangent, or the perpendicular to a chord, or a square's corner. Then $\tan$, $\sin$ or $\cos$. In a square, a corner angle minus two smaller angles gives the one you want: $\frac{\pi}{2} - 2\tan^{-1}(\ldots)$.
- **Parallel lines:** a perpendicular between them gives a right-angled triangle, so $\cos\alpha = \frac{\text{distance}}{\text{radius}}$ or similar.
- **Reflex and supplementary angles:** the angle on the other side of a centre is $2\pi - \theta$; angles on a straight line make $\pi$. The reflex angle for $0.9$ is $2\pi - 0.9 \approx 5.38$.
- **"Show that angle $= 0.685$" (or a length):** work it out and write the value to **more** figures than given, such as $0.6847\ldots$, then say it rounds to the given value. For an exact result, show the exact trig value before the final line.
- **"State"** an angle, such as the angle at the centre of a regular shape: it should follow from one fact, so say which.

## 4. Exact answers, or answers in terms of $r$, $\pi$ and $\sqrt{3}$ (17 questions)

- **Use exact angles** ($\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{2\pi}{3}$ and so on) and exact trig values, never decimals.
- **Keep letters as letters.** A sector of radius $r$ and angle $\frac{\pi}{3}$ has area $\frac{\pi r^2}{6}$; the triangle is $\frac{1}{2}r^2\sin\frac{\pi}{3} = \frac{\sqrt{3}}{4}r^2$.
- **Collect into the form asked for**, such as $\left(a\pi + b\sqrt{3}\right)r^2$ or one single fraction. For $\frac{\pi r^2}{6} - \frac{\sqrt{3}r^2}{4}$, that is $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)r^2$ or $\frac{(2\pi-3\sqrt{3})r^2}{12}$.
- **Lengths in terms of another letter:** in an isosceles triangle with equal sides $x$ and base angles $\theta$, the base is $2x\cos\theta$ (drop the perpendicular from the apex), and that length can then be a radius or a side elsewhere.

- **A general angle $\theta$:** when two areas are equal, set up the equation with $\theta$ as a letter and simplify. If you then use an inequality you are told, remember a length ratio must be positive: reject the negative square root.

## 5. Work backwards from a given arc, area or perimeter (12 questions)

1. **Write each fact as an equation** using $r\theta$, $\frac{1}{2}r^2\theta$ or $\frac{1}{2}ab\sin C$.
2. **Solve for the unknowns**: one fact gives one unknown; two facts give $r$ and $\theta$ together.
3. **Carry on** with the rest of the question using your $r$ and $\theta$.

**Variations you will meet:**

- **Arc and area given:** divide them. $\frac{\frac{1}{2}r^2\theta}{r\theta} = \frac{r}{2}$, which gives $r$ at once, then $\theta$.
- **Arc and perimeter given:** perimeter $= 2r+$ arc gives $r$, then $\theta = \frac{\text{arc}}{r}$.
- **Perimeter and area given:** use the perimeter to write $\theta$ in terms of $r$, substitute into the area and solve the quadratic (Example 2). Check each answer: $\theta$ must be between $0$ and $2\pi$, and any stated range (such as acute) rules one out.
- **Only the angle or the radius is unknown:** a sector of area $15$ with radius $5$ has $12.5\theta = 15$, so $\theta = 1.2$.
- **The angle is in a letter:** a sector area of $18\alpha$ with angle $\alpha$ gives $\frac{1}{2}r^2\alpha = 18\alpha$, so $r = 6$ (the $\alpha$ cancels). Then a triangle area gives $\sin\alpha$.
- **A length leads to the angle:** a chord or semicircle area gives a radius or a side, and then the angle through type 3.
- **A shape's perimeter is a number:** write the perimeter in terms of $r$ (type 4), set it equal and solve the linear equation.
- **Unknown inner radius** of a ring-shaped sector (between two arcs with the same centre): area $= \frac{1}{2}R^2\theta - \frac{1}{2}x^2\theta$; solve for $x$.

## 6. With other topics (3 questions)

- **Coordinate geometry:** find the centre and the ends of the chord, then the angle at the centre. For the circle with centre $(2, 1)$ and radius $13$, the line $y = -4$ meets it where $(x-2)^2 + 25 = 169$, at $(-10, -4)$ and $(14, -4)$. The centre is $5$ above the chord and each half-chord is $12$, so the angle at the centre is $2\tan^{-1}\frac{12}{5} \approx 2.35$ rad. Then segment areas and perimeters are as above. Circle equations are in Coordinate geometry.
- **Rates of change:** if the radius grows with time, the arc and area are functions of $r$. For a fixed angle $0.8$, $A = 0.4r^2$, so $\frac{dA}{dr} = 0.8r$. With $\frac{dr}{dt} = 0.5$ cm s$^{-1}$ at $r = 10$, $\frac{dA}{dt} = 8 \times 0.5 = 4$ cm² s$^{-1}$. The arc $s = 0.8r$ grows at $0.8 \times 0.5 = 0.4$ cm s$^{-1}$. The chain rule is in Differentiation.

## Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

### Example 1

The sector $OPQ$ of a circle has centre $O$ and radius 12 cm. The angle $POQ$ is $105^\circ$.

(a) Express angle $POQ$ in radians as a multiple of $\pi$. **[1]**

(b) Find the exact length of the arc $PQ$ and the exact area of the sector $OPQ$. **[2]**

(c) Find the area of the region between the chord $PQ$ and the arc $PQ$. **[3]**

**(a)** $105 \times \frac{\pi}{180} = \frac{7\pi}{12}$. **B1**

**(b)** Arc $= 12 \times \frac{7\pi}{12} = 7\pi$ cm. **B1**

Sector $= \frac{1}{2} \times 12^2 \times \frac{7\pi}{12} = 42\pi$ cm². **B1**

**(c)** The region is a segment: sector minus triangle $OPQ$.

Triangle $OPQ = \frac{1}{2} \times 12 \times 12 \times \sin\frac{7\pi}{12} = 72\sin\frac{7\pi}{12} = 69.55\ldots$ **M1** (calculator in radian mode, or use $\sin 105^\circ$).

Segment $= 42\pi - 72\sin\frac{7\pi}{12} = 131.9\ldots - 69.55\ldots$ **M1**

$= 62.4$ cm² (3 s.f.). **A1**

### Example 2

A piece of wire $40$ cm long is bent to form the boundary of a sector $OAB$: the two radii $OA$ and $OB$, each $r$ cm, and the arc $AB$. The angle $AOB$ is $\theta$ radians.

(a) Show that $\theta = \frac{40 - 2r}{r}$ and that the area of the sector is $A = 20r - r^2$. **[3]**

(b) The area of the sector is $96$ cm². Find the two possible values of $r$ and the value of $\theta$ for each. **[3]**

(c) By completing the square, find the largest possible area of the sector and the angle $\theta$ that gives it. **[2]**

**(a)** The wire is the perimeter: $2r + r\theta = 40$, so $r\theta = 40 - 2r$ and $\theta = \dfrac{40-2r}{r}$. **B1**

$A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}r^2 \times \dfrac{40-2r}{r}$ **M1** $= \dfrac{1}{2}r(40-2r) = 20r - r^2$. **A1** (given answer, every step shown)

**(b)** $20r - r^2 = 96$, so $r^2 - 20r + 96 = 0$, which is $(r-8)(r-12) = 0$. **M1**

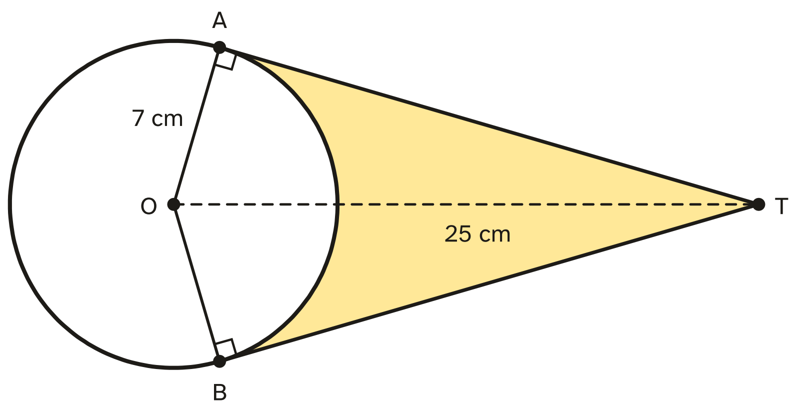
$r = 8$ or $r = 12$. **A1**

$r = 8$: $\theta = \dfrac{24}{8} = 3$. $r = 12$: $\theta = \dfrac{16}{12} = \dfrac{4}{3}$. Both lie between $0$ and $2\pi$, so both are possible. **A1**

**(c)** $20r - r^2 = 100 - (r-10)^2$. **M1**

The largest area is $100$ cm², when $r = 10$, and then $\theta = \dfrac{20}{10} = 2$ radians. **A1**

## Example 3




The circle has centre $O$ and radius $7$ cm. The point $T$ is $25$ cm from $O$, and the two tangents from $T$ touch the circle at $A$ and $B$.

(a) Find the length $TA$, and show that angle $AOB = 2.574$ radians, correct to 4 significant figures. **[3]**

(b) Find the area of the region bounded by $TA$, $TB$ and the minor arc $AB$. **[3]**

(c) A tight loop of string runs from $T$ to $A$, round the major arc $AB$ and back from $B$ to $T$. Find the length of the string. **[3]**

**(a)** A tangent is perpendicular to the radius, so angle $OAT = \frac{\pi}{2}$ and $TA = \sqrt{25^2 - 7^2} = \sqrt{576} = 24$ cm. **B1**

$\cos AOT = \dfrac{7}{25}$, so $AOT = 1.287\,00\ldots$ **M1**. By symmetry $AOB = 2 \times 1.287\,00\ldots = 2.574\,00\ldots = 2.574$ (4 s.f.). **A1**

**(b)** Region $=$ kite $OATB$ $-$ sector $OAB$.

Kite $= 2 \times \dfrac{1}{2} \times 7 \times 24 = 168$. **B1**

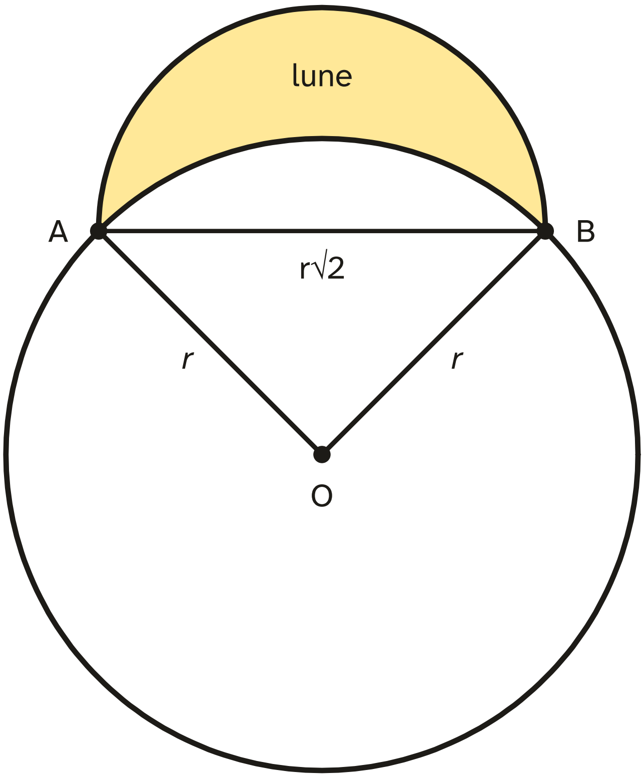
Sector $= \frac{1}{2} \times 7^2 \times 2.5740 = 63.06\ldots$ **M1**

Area $= 168 - 63.06 = 104.9\ldots = 105$ cm² (3 s.f.). **A1**

**(c)** The major arc has angle $2\pi - 2.5740 = 3.709\ldots$ **M1**, so its length is $7 \times 3.709\ldots = 25.96\ldots$ **M1**

String $= 24 + 24 + 25.96 = 74.0$ cm (3 s.f.). **A1**

## Example 4




A circle has centre $O$ and radius $r$. The chord $AB$ has length $r\sqrt{2}$. A semicircle is drawn outside the circle with $AB$ as its diameter. The crescent-shaped region between the arc of the semicircle and the minor arc $AB$ of the circle is called a lune.

(a) Show that angle $AOB = \frac{\pi}{2}$. **[2]**

(b) Show that the area of the lune is $\frac{1}{2}r^2$. **[4]**

(c) The perimeter of the lune is $10$ cm. Find $r$. **[3]**

**(a)** Cosine rule in triangle $OAB$: $\cos AOB = \dfrac{r^2 + r^2 - 2r^2}{2r^2} = 0$ **M1**, so $AOB = \frac{\pi}{2}$. **A1**

(Or: $OA^2 + OB^2 = 2r^2 = AB^2$, so by Pythagoras the angle at $O$ is a right angle.)

**(b)** The semicircle has radius $\dfrac{r\sqrt{2}}{2} = \dfrac{r}{\sqrt{2}}$, so its area is $\dfrac{1}{2}\pi \times \dfrac{r^2}{2} = \dfrac{\pi r^2}{4}$. **B1**

The minor segment cut off by $AB$ is $\dfrac{1}{2}r^2\left(\dfrac{\pi}{2} - \sin\dfrac{\pi}{2}\right) = \dfrac{\pi r^2}{4} - \dfrac{r^2}{2}$. **M1 A1**

The lune is the half-disc minus the segment it covers:

$$\frac{\pi r^2}{4} - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right) = \frac{1}{2}r^2.$$

**A1** (given answer). The $\pi$ terms cancel: a curved region with an area free of $\pi$.

**(c)** Semicircle arc $= \pi \times \dfrac{r}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}\pi r$. Minor arc $= r \times \dfrac{\pi}{2}$. **B1**

Perimeter $= \dfrac{\sqrt{2}}{2}\pi r + \dfrac{1}{2}\pi r = \dfrac{\pi r\left(\sqrt{2}+1\right)}{2}$. **M1** for adding the two arcs.

$\dfrac{\pi r(\sqrt{2}+1)}{2} = 10$, so $r = \dfrac{20}{\pi(\sqrt{2}+1)} = 2.64$ cm (3 s.f.). **A1**

## Common mistakes

- **Using degrees in $r\theta$ or $\frac{1}{2}r^2\theta$.** These need radians. Working in degrees is allowed only with the fraction-of-a-circle version ($\frac{\theta}{360} \times 2\pi r$), and mixing the two in one formula loses the marks. Switching to degrees part-way is a common source of lost accuracy, so stay in radians. Check your calculator is in radian mode before any $\sin$ or $\cos$.
- **Reading "angle $= \frac{4}{3}$" as $\frac{4}{3}\pi$.** An angle with no $\pi$ is a plain number of radians.
- **Rounding too early.** Angles such as $1.8546$ used as $1.85$ make the final answer wrong. Examiners report this in many sector questions. Keep at least 4 significant figures (or the calculator's memory) until the end.
- **Guessing an angle from the picture.** A triangle that looks equilateral or right-angled may not be. Examiners report candidates assuming a chord equals the radius, or a right angle that wasn't there (a side written as $r\tan\theta$). Work out every angle from the information, and give the reason when a triangle really is special.
- **Using the wrong radius.** The radius of an arc is the distance from **that arc's centre** to it. In a square with an arc centred at a corner, the radius is the distance to where the arc meets the side, not the side of the square.
- **Half a triangle instead of the whole**, or the half-angle $\frac{\theta}{2}$ in $\frac{1}{2}r^2\sin\theta$. Use the full angle with the two radii, or double the half-triangle.
- **The wrong angle for a major arc.** The major arc uses $2\pi - \theta$; taking $\frac{2\pi}{3}$ for an arc that really needs $\frac{4\pi}{3}$ is a frequent slip.
- **Adding angles from different circles** into one arc. Each arc has its own radius and angle.
- **Leaving out** $\pi$ in a semicircle's arc ($\pi r$, not $r$) or leaving an answer as $\cos^{-1}\frac{2}{3}$. Work out $\cos^{-1}$ and similar to a number; an unevaluated one doesn't count as an answer.

- **"Exact" answers given as decimals**, or left as separate unsimplified terms. Collect like terms into a single expression such as $14 + \frac{9\pi}{4}$.
- **Keeping impossible solutions.** When you solve for $r$ or $k$, reject negative values and angles outside the range; a ratio of lengths can't be negative, so drop the negative root.

## How to get the marks

- **Sketch or annotate the diagram** with every radius and angle you find. It shows the method and stops you using the wrong radius.
- **Write the plan for an area in words** ("sector – triangle") before the numbers. Method marks come for a correct formula with your angle and radius, even if those were wrong earlier: marks follow through.
- **Typical mark pattern:** one mark for the angle, one for each arc or area formula used correctly, one for combining them, one for the answer.
- **"Show that" angles and lengths:** the answer is given, so show the working and a value to **more** figures than the given one ($2.0344\ldots$ then "$= 2.03$ to 3 s.f."). For an exact result such as $s = \sqrt{3}r$, show the exact trig value ($\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$) before the last line.
- **"State"** means write it down with little or no working, such as an angle that follows from a symmetry or a straight line.
- **"Hence"** means use the part before, such as the angle you have just shown.
- **"Exact"** or **"in terms of $\pi$"**: keep $\pi$ and surds, simplify fractions and collect like terms into one expression.
- **Accuracy:** give non-exact answers to 3 significant figures unless the question says otherwise (such as 1 decimal place); angles in radians to 3 significant figures. A correct method with the answer given to the wrong number of figures still drops the last mark.
- **Units:** cm for lengths, cm² for areas, radians for angles.

## Quick check

1. Convert $2.5$ radians to degrees, correct to 1 decimal place, and $150^\circ$ to radians as a multiple of $\pi$.
2. A sector has radius $5$ cm and arc length $7$ cm. Find the angle of the sector and its area.
3. A sector has area $24$ cm² and angle $0.75$ radians. Find its radius and its perimeter.
4. A straight line is $12$ cm from the centre $O$ of a circle of radius $13$ cm, and cuts the circle at $A$ and $B$. Find the length $AB$, the angle $AOB$ in radians, and the length of the minor arc $AB$.
5. A sector has radius $r$ cm and angle $\theta$ radians. Its perimeter is twice its arc length, and its area is $25$ cm². Find $\theta$ and $r$.

## Answers

1. $2.5 \times \frac{180}{\pi} = 143.2^\circ$. $150 \times \frac{\pi}{180} = \frac{5\pi}{6}$.
2. $5\theta = 7$, so $\theta = 1.4$ rad. Area $= \frac{1}{2} \times 25 \times 1.4 = 17.5$ cm².
3. $\frac{1}{2}r^2 \times 0.75 = 24$, so $r^2 = 64$ and $r = 8$ cm. Perimeter $= 2 \times 8 + 8 \times 0.75 = 22$ cm.
4. The perpendicular from $O$ to the line bisects $AB$, so half of $AB$ is $\sqrt{13^2 - 12^2} = 5$ and $AB = 10$ cm. $\sin\frac{\theta}{2} = \frac{5}{13}$, so $\theta = 2\sin^{-1}\frac{5}{13} = 0.790$ rad (3 s.f.). Arc $= 13 \times 0.7896 = 10.3$ cm.
5. $2r + r\theta = 2r\theta$, and dividing by $r$ gives $2 + \theta = 2\theta$, so $\theta = 2$. Then $\frac{1}{2}r^2 \times 2 = 25$, so $r = 5$ cm.

## Past questions to try

- **9709/13/O/N/22 Q8**: two equal circles overlapping, with the answers in terms of $r$.
- **9709/12/M/J/21 Q12**: a rope wrapped around seven pipes.
- **9709/11/M/J/24 Q7**: arcs between two parallel lines, and a shape's perimeter and area.
- **9709/13/O/N/23 Q10**: a region between arcs of two different circles, in terms of $r$.