Cambridge AS & A Level · Mathematics (9709) · Notes

Coordinate geometry AS

Read online at pastopic.com/cambridge/mathematics-9709/notes/coordinate-geometry

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Read first: Quadratics

What you need to know

By the end of this topic you should be able to:

1. **Find a straight line's equation** from two points on it, or from one point and its gradient (the gradient may come from a line it is parallel or perpendicular to).
2. **Use whichever form of a line's equation helps**: $y = mx + c$, $y - y_1 = m(x - x_1)$ or $ax + by + c = 0$. From coordinates, calculate how steep a segment is, how long it is, its middle point, and where two lines cross. Parallel lines share a gradient; perpendicular lines have gradients that multiply to -1.
3. **Know the circle equation** $(x - a)^2 + (y - b)^2 = r^2$, centre (a, b) and radius r, and its multiplied-out version $x^2 + y^2 + 2gx + 2fy + c = 0$, and switch between them.
4. **Solve line-and-circle problems with algebra and basic circle geometry**: a radius meets the tangent at its end at 90°, a triangle standing on a diameter has a 90° angle at the circle, and a circle is symmetrical about each diameter. Tangent gradients come from the radius; differentiating the circle's equation (implicit differentiation) is not in this course.
5. **Link graphs and equations**: where two graphs meet, both equations are true, so solving them together gives the meeting points, and a discriminant counts them. That is how you find the constant that makes a line a tangent to a circle or curve, or makes it miss.

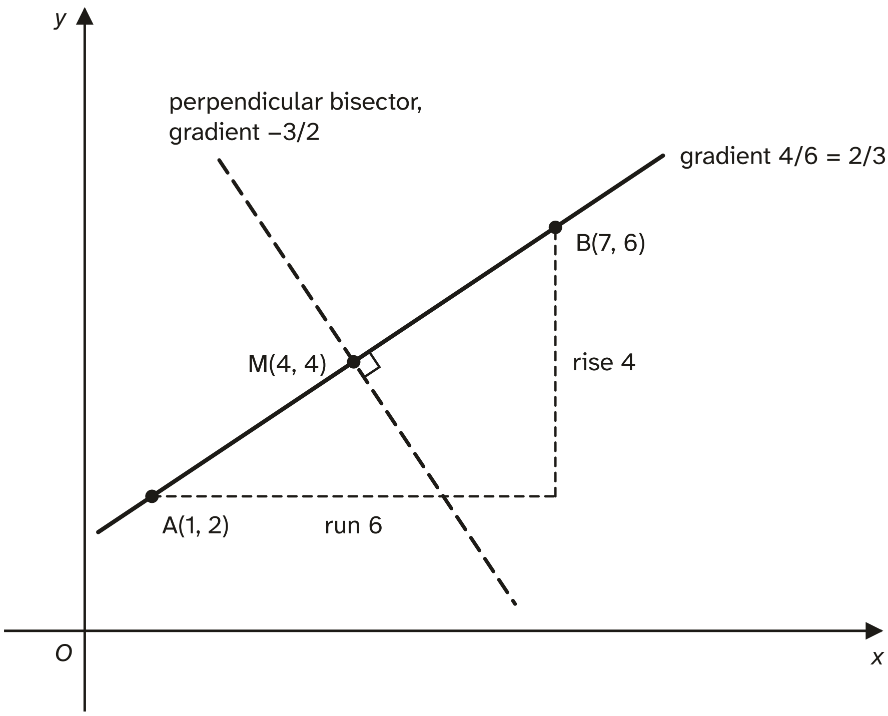
Solving quadratics and the discriminant are in Quadratics. Arcs and sectors that finish some circle questions are in Circular measure.

Understand it

Points, gradients and midpoints

Three measurements between $A(x_1, y_1)$ and $B(x_2, y_2)$ underlie everything:

- **Gradient**, "rise over run": $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Positive gradients go up to the right.
- **Midpoint**, the average of the coordinates: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.
- **Length**: the run and rise are two sides of a right-angled triangle, so $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



For $A(1,2)$ and $B(7,6)$: gradient $\frac{4}{6}=\frac{2}{3}$, midpoint $(4,4)$, length $\sqrt{36+16}=\sqrt{52}=2\sqrt{13}$.

Writing a line

A line's equation is the rule its points obey. It can be written as:

- $y=mx+c$: gradient m, crossing the y-axis at $(0,c)$;
- $y-y_1=m(x-x_1)$: gradient m through (x_1,y_1). This is the one to **write** with, since you usually know a point and a gradient;
- $ax+by+c=0$: all terms on one side, often asked for with whole numbers.

To read the gradient of $2x-3y+6=0$, rearrange: $y=\frac{2}{3}x+2$, gradient $\frac{2}{3}$.

Parallel, perpendicular, and where lines meet

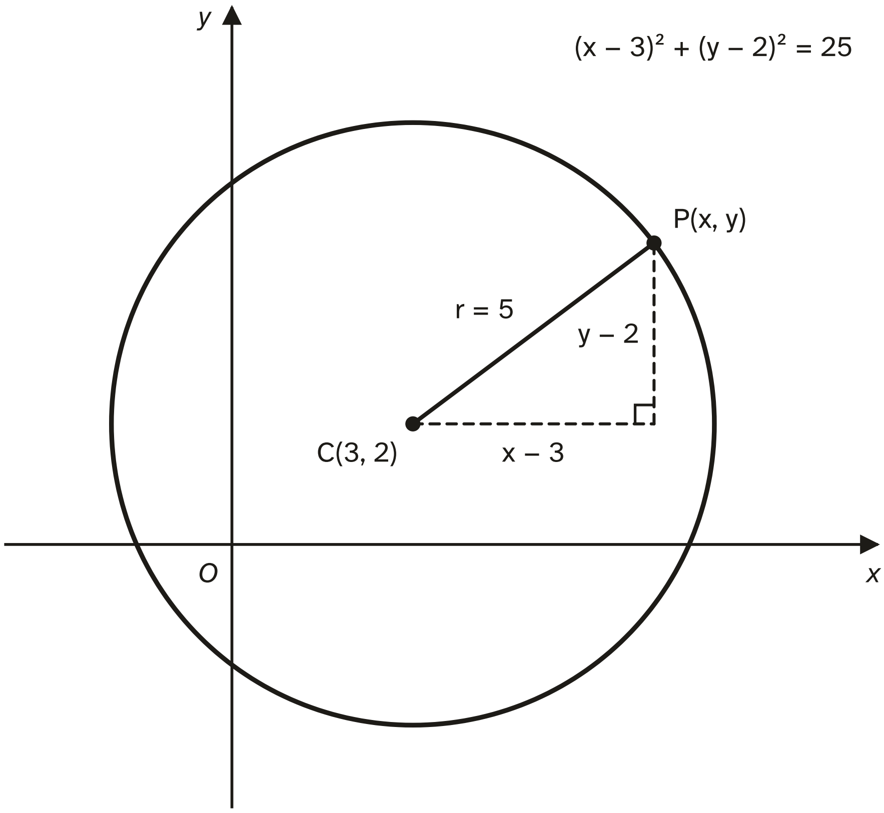
Parallel lines have the **same gradient**. Perpendicular gradients **multiply to** -1: if one is m, the other is $-\frac{1}{m}$ (flip and change the sign), so $\frac{2}{3}$ pairs with $-\frac{3}{2}$. Horizontal and vertical lines are also perpendicular.

The **perpendicular bisector** of AB goes through its midpoint at right angles to it; every point on it is equally far from A and B. In the diagram it is $y-4=-\frac{3}{2}(x-4)$, or $3x+2y-20=0$.

Two lines meet where both equations hold, so solve them simultaneously: $y=2x-1$ and $x+3y=11$ give $x+3(2x-1)=11$, so $x=2$, $y=3$. **Meeting points are solutions of simultaneous equations**: the key idea of the topic.

The equation of a circle

A circle is every point at distance r from its centre. With centre $C(3, 2)$ and radius 5, a point $P(x, y)$ is on the circle when $CP = 5$. The run from C to P is $x - 3$ and the rise is $y - 2$, so Pythagoras gives:

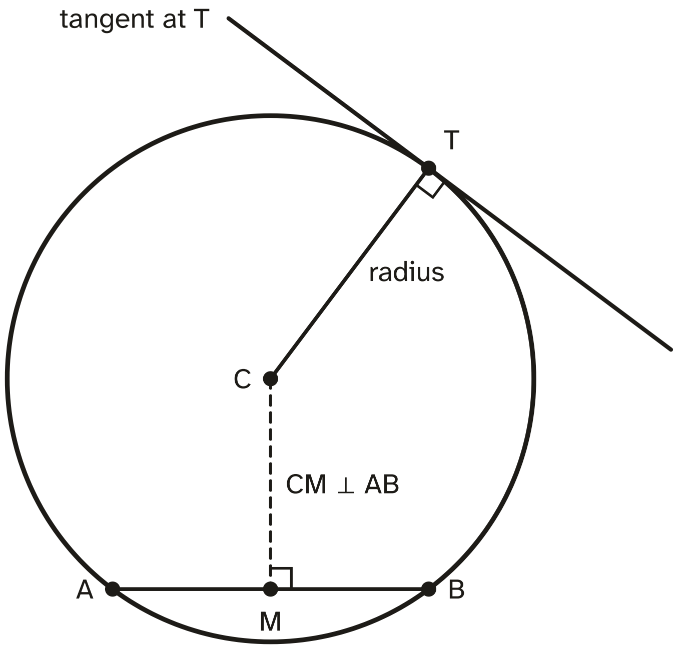


$$(x - 3)^2 + (y - 2)^2 = 25.$$

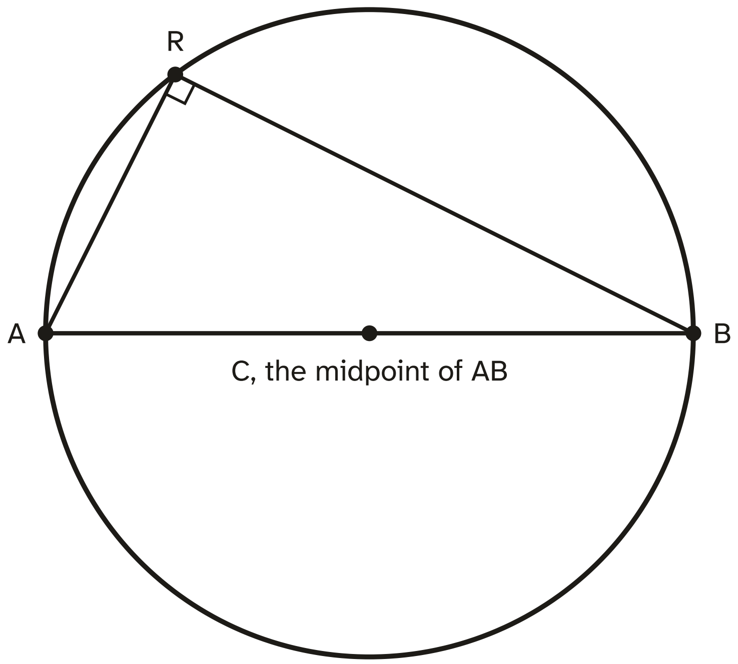
In general $(x - a)^2 + (y - b)^2 = r^2$ has centre (a, b) and radius r. Watch the signs ($(x + 4)^2$ means $a = -4$), and remember the right-hand side is r^2.

Multiplying out gives $x^2 + y^2 - 6x - 4y - 12 = 0$. To go back, **complete the square** on the x terms and on the y terms. In general $x^2 + y^2 + 2gx + 2fy + c = 0$ has centre $(-g, -f)$, **minus half** of each coefficient, and radius $\sqrt{g^2 + f^2 - c}$. It is a circle only if x^2 and y^2 have equal coefficients and there is no xy term.

Three facts about circles



1. **A tangent is perpendicular to the radius where it touches**, so its gradient at T is $-\frac{1}{\text{gradient of } CT}$. The line CT is the **normal** at T.
2. **The line from the centre to the midpoint of a chord is perpendicular to the chord.** So the perpendicular bisector of every chord passes through the centre, and two of them meet at the centre.
3. **The angle in a semicircle is a right angle.** If AB is a diameter and R is another point on the circle, angle $ARB = 90^\circ$.



AB is a diameter, so angle ARB = 90°

Fact 3 is usually used backwards: **if a triangle has a right angle, the side opposite it is a diameter** of the circle through its corners, and the midpoint of that side is the centre.

A circle is also symmetrical about every line through its centre: its highest point is $(a, b+r)$ and its lowest $(a, b-r)$.

Where a line meets a circle

Substituting the line into the circle gives a **quadratic**; each real root is a meeting point, so its discriminant decides:

- $b^2 - 4ac > 0$: the line crosses twice (it contains a **chord**);
- $b^2 - 4ac = 0$: a repeated root, so the line **touches**: it is a **tangent**;
- $b^2 - 4ac < 0$: the line **misses** the circle.

The same works for a line and a curve such as a parabola.

Key facts and formulas

MF19 has no coordinate geometry formulas, so everything in this topic is "learn it".

Formula	Status
Gradient of AB: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$	Learn it
Midpoint of AB: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$	Learn it
Length of AB: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Learn it
Line through (x_1, y_1) with gradient m: $y - y_1 = m(x - x_1)$	Learn it
Parallel: $m_1 = m_2$. Perpendicular: $m_1 m_2 = -1$, so $m_2 = -\dfrac{1}{m_1}$	Learn it
Circle, centre (a, b), radius r: $(x-a)^2 + (y-b)^2 = r^2$	Learn it
$x^2 + y^2 + 2gx + 2fy + c = 0$: centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$	Learn it
Tangent $\perp$ radius at the point of contact; the normal passes through the centre	Learn it
The perpendicular from the centre to a chord bisects the chord	Learn it
Angle in a semicircle $= 90^\circ$; a right-angled triangle's hypotenuse is a diameter of its circle	Learn it
Line and circle: substitute, then $b^2 - 4ac > 0$ two points, $= 0$ tangent, < 0 no meeting	Learn it

How to do it

In the Paper 1 questions we have tagged for this topic (43 questions, 2021–2025), the types came up this often. Most questions mix two or three types.

Question type	Questions
Find or use the equation of a circle	21
Find where a line meets a circle or an axis	21
Find a tangent or normal to a circle	20
Straight lines: gradients, midpoints, lengths, perpendicular bisectors	11
Areas, distances and special points in circle diagrams	11
Use the discriminant: tangents with an unknown, lines that meet or miss	7
Use a right angle: perpendicular gradients, angle in a semicircle	5

Sketch first, every time: examiners repeatedly note that candidates who drew a diagram chose better methods.

1. Find or use the equation of a circle (21 questions)

Centre and radius from an equation. Complete the square on x and on y, then move the numbers across:

$$x^2 + y^2 - 4x + 10y + 20 = 0 \;\Rightarrow\; (x-2)^2 - 4 + (y+5)^2 - 25 + 20 = 0 \;\Rightarrow\; (x-2)^2 + (y+5)^2 = 9,$$

so the centre is $(2,-5)$ and the radius 3. Or use the general form: $x^2+y^2+10x-6y+9=0$ has $g=5$, $f=-3$, $c=9$, centre $(-5,3)$ and $r=\sqrt{25+9-9}=5$.

Writing a circle's equation: find the centre and r^2, then write $(x-a)^2+(y-b)^2=r^2$. What you are told decides how:

- **Centre and a point:** r^2 is the squared distance between them.
- **Ends of a diameter:** the centre is their midpoint; r is **half** their distance apart. Ends $(2,-1)$ and $(5,3)$ give centre $\left(\frac{7}{2},1\right)$ and $r^2=\frac{25}{4}$.
- **Three points with a right angle** (show it first, type 7): the side opposite the right angle is a diameter.
- **Three points in general:** the centre is where the perpendicular bisectors of two chords meet (Example 2). Or substitute the points into $x^2+y^2+2gx+2fy+c=0$ and solve.
- **Some coefficients given**, such as $x^2+y^2+ax+by-20=0$ through two known points: substitute each point, solve the two equations for a and b, then the centre is $\left(-\frac{a}{2},-\frac{b}{2}\right)$.
- **Two points and the radius:** the centre is on their perpendicular bisector, so write it as, say, $(t,\ 2t+1)$ using that line, set its distance to one point equal to r, and solve the quadratic in t. There are **two** circles.
- **Centre known, and a line is a tangent:** r is the distance from the centre to the foot of its perpendicular on the line (if the line cuts a circle with that centre at A and B, that foot is the midpoint of AB).
- **A second circle, same radius, same tangent at the same point A:** reflect the centre in A: $2A-C$. Reflecting $(-3,2)$ in $(1,4)$ gives $(5,6)$.
- **Distances from the axes:** a circle of radius r whose nearest approach to the x-axis is 3 (above it) has its centre at height $r+3$.
- **Letters in the equation:** treat them as numbers. $x^2+y^2+px-6y+q=0$ has centre $\left(-\frac{p}{2},3\right)$ and $r^2=\frac{p^2}{4}+9-q$. Other facts (a point on the circle, the centre on a known normal) give equations for p and q.
- **A point on the circle:** if $(4,k)$ is on it, substitute and solve for k; usually there are two values.

2. Find where a line meets a circle or an axis (21 questions)

1. **Make y (or x) the subject of the line**, whichever avoids fractions.
2. **Substitute into the circle**, squaring each whole bracket.
3. **Collect into a three-term quadratic** $=0$ and solve it by factorising, the formula or completing the square, **showing the method**.
4. **Find the other coordinate for each root** from the line, and pair them up.

Variations you will meet:

- **An axis or a vertical/horizontal line:** put $x=0$, $y=0$ or, say, $x=5$ straight into the circle.
- **"Surd form" or "exact":** use completing the square or the formula and leave surds, such as $x=3\pm\sqrt{2}$.
- **One meeting point already known:** its x-value is a root, so one factor is known.
- **Only one point wanted** (such as the one between B and C): solve fully, then keep the right one and say why.

- **Length of a chord:** find both ends, then use the distance formula.
- **Two circles:** to check a given meeting point, substitute it into both. To find the meeting points, subtract the equations: the squared terms cancel, leaving a line to substitute back.

3. Find a tangent or normal to a circle (20 questions)

1. Find the centre C (type 1).
2. Find the gradient of the radius CP.
3. The tangent's gradient is $-\dfrac{1}{\text{gradient of } CP}$.
4. Write $y - y_P = m(x - x_P)$ and rearrange into the form asked for.

The normal at P is the line CP, through the centre. So if you know the tangent at A, the centre lies on the normal at A.

Variations you will meet:

- **"Show that the line is the tangent at A":** show **both** that A is on the circle **and** that the line is perpendicular to CA.
- **Tangents at two points, and where they meet:** find both, then solve simultaneously. If the two points are mirror images in a line through the centre (for example both on $x = k$), the tangents meet on that mirror line.
- **The other end of a diameter:** $2C - P$. Tangents at the two ends are parallel.
- **Tangents with a given gradient m:** they touch at the ends of the diameter with gradient $-\frac{1}{m}$. Substitute that diameter into the circle to find the points (Example 3 shows this and the discriminant way).
- **The point furthest from (or nearest to) an outside point**, such as the origin: it is on the line through that point and the centre, which is also the radius there.
- **Where a tangent meets the axes:** put $x = 0$ and $y = 0$ into it.

Mark schemes sometimes show differentiating the circle's equation instead; that isn't in this syllabus, and the radius method is quicker.

4. Straight lines: gradients, midpoints, lengths, perpendicular bisectors (11 questions)

Perpendicular bisector of AB: midpoint M, gradient of AB flipped with its sign changed, then $y - y_M = m(x - x_M)$. Use M, never A or B.

Variations you will meet:

- **An unknown coordinate:** "the gradient of AB is $\frac{3}{4}$" gives an equation from the gradient formula; "$AC = BC$" gives $AC^2 = BC^2$, where the squared unknowns usually cancel.
- **B from A and the perpendicular bisector of AB:** the line through A perpendicular to the bisector meets it at the midpoint M; then $B = 2M - A$. B is **not** on the bisector.
- **Fourth corner of a parallelogram $ABCD$:** the diagonals share a midpoint, so $C = B + D - A$.

- **A line and the axes make a triangle:** find both intercepts; the area is $\frac{1}{2} \times |x\text{-intercept}| \times |y\text{-intercept}|$.
- **Form $ax + by + c = 0$ with integers:** multiply out the fractions and collect every term on one side.

The same methods apply to lines inside calculus questions, such as the perpendicular bisector of two points on a curve.

5. Areas, distances and special points in circle diagrams (11 questions)

- **Area of a triangle:** use a horizontal or vertical side as the base, so the height is a straight distance. $(1, 1)$, $(7, 1)$, $(4, 5)$: base 6, height 4, area 12. For a chord AB and the centre C, use base AB and height CM, with M the midpoint of AB.
- **Highest, lowest, leftmost, rightmost points:** go r straight up, down, left or right from the centre. $(x - 3)^2 + (y - 2)^2 = 25$ has highest point $(3, 7)$ and lowest $(3, -3)$.
- **Two separate circles:** with d the distance between centres, the least distance between points on them is $d - r_1 - r_2$ and the greatest is $d + r_1 + r_2$. Centres $(1, 2)$ and $(7, 10)$, radii 3 and 4: $d = 10$, least 3, greatest 17.
- **Angles and curved pieces:** the radius–tangent right angle and the perpendicular to a chord's midpoint make right-angled triangles for trigonometry (or use the cosine rule). Arcs, sectors and segments then use Circular measure; volumes use Integration.

6. Use the discriminant: tangents with an unknown, lines that meet or miss (7 questions)

1. **Substitute the line into the circle**, keeping the unknown (k, m or a) as a letter.
2. **Collect into $Ax^2 + Bx + C = 0$**, with A, B, C in terms of the unknown.
3. **Apply the condition to $B^2 - 4AC$:** $= 0$ tangent, > 0 two points, < 0 no meeting.
4. **Simplify and solve** in the unknown, as in Quadratics.

Variations you will meet:

- **A line through a fixed point with gradient m**, such as $y - 2 = m(x - 6)$, is a tangent: the m^4 terms usually cancel (Example 4).
- **The unknown is in the circle**, such as $(x - 2)^2 + (y - a)^2 = 8$: same steps.
- **Points of contact:** put each value back; the quadratic is a perfect square with root $x = -\frac{B}{2A}$. Then y from the line.
- **"Show that" an inequality** for a line that misses a circle: the discriminant is often $-4(\ldots) < 0$. Dividing by -4 **reverses** the sign to $(\ldots) > 0$; say so.
- **"Find the set of values of k"** for two meeting points: solve the quadratic inequality, as in Quadratics.

7. Use a right angle: perpendicular gradients, angle in a semicircle (5 questions)

- **"Angle ABC is a right angle", with an unknown in B:** write the gradients of BA and BC in terms of the unknown, set their product to -1, clear fractions and solve the quadratic, showing the method. Or use Pythagoras: $BA^2 + BC^2 = AC^2$.
- **"Show that angle $ABC = 90^\circ$":** show the gradients multiply to -1 and say what that means.
- **Then the circle through the three points** has the side opposite the right angle as a diameter, with its midpoint as the centre.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

The circle S has equation $x^2 + y^2 + 6x - 8y = 0$.

(a) Find the coordinates of the centre of S and its radius. **[3]**

(b) Show that the point $P(1, 7)$ lies on S. **[1]**

(c) Find the equation of the tangent to S at P, giving your answer in the form $ax + by + c = 0$. **[3]**

(a) Complete the square on each letter:

$$(x+3)^2 - 9 + (y-4)^2 - 16 = 0 \quad \Rightarrow \quad (x+3)^2 + (y-4)^2 = 25$$

M1 for completing both squares. The centre is $(-3, 4)$ **A1** and the radius is 5 **A1**.

(b) $1^2 + 7^2 + 6(1) - 8(7) = 1 + 49 + 6 - 56 = 0$, so P is on S. **B1**

(c) Gradient of the radius CP: $\dfrac{7-4}{1-(-3)} = \dfrac{3}{4}$. **B1**

The tangent is perpendicular to it, so its gradient is $-\dfrac{4}{3}$. **M1** for using $m_1 m_2 = -1$ and the point P:

$$y - 7 = -\frac{4}{3}(x-1) \quad \Rightarrow \quad 3y - 21 = -4x + 4 \quad \Rightarrow \quad 4x + 3y - 25 = 0$$

A1, in the form asked for.

Example 2

A triangle has vertices $A(-2, 2)$, $B(6, 2)$ and $C(2, -6)$. The circle S passes through A, B and C.

(a) Find the equation of the perpendicular bisector of BC. **[3]**

(b) Find the coordinates of the centre of S, and the equation of S. **[4]**

(c) The perpendicular bisector of BC meets S at the points P and Q. Find the exact coordinates of P and Q. **[3]**

(a) Midpoint of BC: $\left(\frac{6+2}{2}, \frac{2-6}{2}\right) = (4, -2)$. **B1**

Gradient of BC: $\dfrac{-6-2}{2-6} = 2$, so the perpendicular gradient is $-\dfrac{1}{2}$. **M1** for the midpoint with the perpendicular gradient.

$$y + 2 = -\frac{1}{2}(x - 4) \quad \Rightarrow \quad 2y + 4 = -x + 4 \quad \Rightarrow \quad x + 2y = 0$$

A1

(b) No two of the gradients 0, 2, -2 multiply to -1, so no side is a diameter. Instead use a second perpendicular bisector: AB is horizontal with midpoint $(2, 2)$, so its bisector is $x = 2$. **B1**

Put $x = 2$ into $x + 2y = 0$: $y = -1$. The centre is $(2, -1)$. **M1 A1**

$r^2 = (6-2)^2 + (2+1)^2 = 25$, so S is

$$(x-2)^2 + (y+1)^2 = 25.$$

B1. Check with C: $0^2 + (-5)^2 = 25$ ✓.

(c) From part (a), $x = -2y$. Substitute into S:

$$(-2y-2)^2 + (y+1)^2 = 25 \quad \Rightarrow \quad 4(y+1)^2 + (y+1)^2 = 25 \quad \Rightarrow \quad (y+1)^2 = 5$$

M1 for substituting, **A1**

So $y = -1 \pm \sqrt{5}$, and $x = -2y = 2 \mp 2\sqrt{5}$. The points are $\left(2 - 2\sqrt{5},\ -1 + \sqrt{5}\right)$ and $\left(2 + 2\sqrt{5},\ -1 - \sqrt{5}\right)$. **A1**, exact and correctly paired.

Example 3

The circle S has centre $(3, 1)$ and radius $\sqrt{10}$.

(a) Show that S passes through the origin, and find the coordinates of the other point where the line $y = 3x$ meets S. **[3]**

(b) Find the equations of the two tangents to S that are parallel to the line $y = 3x$, and the coordinates of the points where they touch S. **[7]**

(a) S is $(x-3)^2 + (y-1)^2 = 10$. At $(0, 0)$: $9 + 1 = 10$ ✓. **B1**

Substitute $y = 3x$: $(x-3)^2 + (3x-1)^2 = 10$, so $10x^2 - 12x = 0$. **M1**

$2x(5x-6) = 0$, so $x = 0$ or $x = \frac{6}{5}$. The other point is $\left(\frac{6}{5}, \frac{18}{5}\right)$. **A1**

(b) A parallel line is $y = 3x + c$. Substitute into S:

$$(x-3)^2 + (3x+c-1)^2 = 10 \quad \Rightarrow \quad 10x^2 + (6c-12)x + (c-1)^2 - 1 = 0$$

M1 for substituting, **A1** for a correct three-term quadratic.

For a tangent, $b^2 - 4ac = 0$:

$$(6c-12)^2 - 40\left[(c-1)^2 - 1\right] = 0 \quad \Rightarrow \quad -4c^2 - 64c + 144 = 0 \quad \Rightarrow \quad c^2 + 16c - 36 = 0$$

M1 for using the discriminant with these coefficients.

$(c+18)(c-2) = 0$, so $c = 2$ or $c = -18$. The tangents are $y = 3x + 2$ and $y = 3x - 18$. **A1**

Points of contact: with $c = 2$ the quadratic is $10x^2 = 0$, so $x = 0$ and the point is $(0, 2)$. With $c = -18$ it is $10x^2 - 120x + 360 = 0$, which is $10(x-6)^2 = 0$, so $x = 6$ and the point is $(6, 0)$. **M1 A1 A1**

Another way: the points of contact are the ends of the diameter $y - 1 = -\frac{1}{3}(x-3)$. Substituting gives $\frac{10}{9}(x-3)^2 = 10$, so $x = 0$ or 6: the points $(0, 2)$ and $(6, 0)$, and the tangents through them with gradient 3.

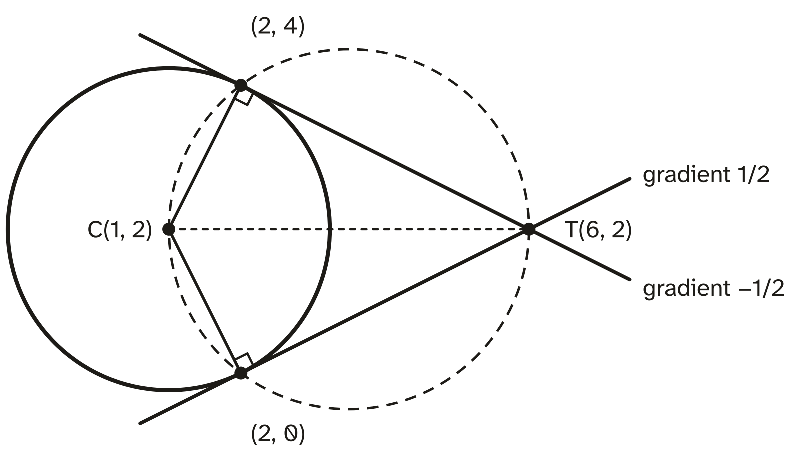
Example 4

The circle S has equation $(x-1)^2 + (y-2)^2 = 5$ and centre C. The point T is $(6, 2)$.

(a) Find the exact length of a tangent from T to S. **[2]**

(b) Each tangent from T to S has gradient m, where $4m^2 = 1$. Show this, and state the gradients of the two tangents. **[5]**

(c) The two tangents from T touch S at P and Q. Explain why C, T, P and Q all lie on one circle, and find its equation. **[3]**



(a) C is $(1,2)$, so $CT=5$. At a point of contact P, angle $CPT=90^\circ$, so by Pythagoras $TP^2=CT^2-r^2=25-5=20$. **M1**

$TP=\sqrt{20}=2\sqrt{5}$. **A1**

(b) The line is $y-2=m(x-6)$, so substitute $m(x-6)$ for $y-2$ in S:

$$(x-1)^2+m^2(x-6)^2=5 \quad \Rightarrow \quad (1+m^2)x^2-(2+12m^2)x+(36m^2-4)=0$$

M1 for substituting, **A1** for the three-term quadratic with every term on one side.

It touches, so the discriminant is 0:

$$(2+12m^2)^2-4(1+m^2)(36m^2-4)=0$$

M1. Expanding, $4+48m^2+144m^4-144m^4-128m^2+16=0$, so $20-80m^2=0$, which is $4m^2=1$. **A1**, the given result with the m^4 terms seen to cancel.

The gradients are $\frac{1}{2}$ and $-\frac{1}{2}$. **B1**

(Putting $m=\frac{1}{2}$ back gives $\frac{5}{4}x^2-5x+5=0$, that is $(x-2)^2=0$: the point of contact is $(2,0)$. By symmetry in $y=2$, the other is $(2,4)$.)

(c) Angles CPT and CQT are 90° (tangent perpendicular to radius), so, by the angle in a semicircle used backwards, P and Q both lie on the circle with diameter CT. **B1**

Its centre is the midpoint of CT, $\left(\frac{7}{2},2\right)$, and its radius is $\frac{5}{2}$. **M1**

$$\left(x-\tfrac{7}{2}\right)^2+(y-2)^2=\tfrac{25}{4}$$

A1. Check with $(2,0)$: $\frac{9}{4}+4=\frac{25}{4}$ ✓.

Common mistakes

- **Solving the quadratic on the calculator.** Examiners report this in many circle questions: roots with no factorisation, formula or completed square lose the method mark, even when correct.
- **Sign errors in the centre.** $(x+3)^2+(y-4)^2=25$ and $x^2+y^2+6x-8y=0$ both have centre $(-3,4)$.
- **Radius against radius squared.** The right-hand side is r^2. A circle of radius 10 is $\ldots=100$; one with $r^2=29$ is $\ldots=29$, not $\ldots=\sqrt{29}$. Write r^2 as a simplified number: mark schemes don't accept $\left(\sqrt{29}\right)^2$ or a rounded decimal as the final answer.
- **Using the diameter as the radius.** When two points are the ends of a diameter, the radius is half the distance between them.
- **Using the wrong centre**, such as the origin or a point on the circle. Find the centre first and label it on your sketch.
- **Assuming a line is parallel to a tangent.** A chord such as AC is not the tangent's direction unless you have shown it. Always get the tangent's gradient from the radius.

- **Getting the perpendicular gradient wrong.** It is $-\frac{1}{m}$: flip **and** change the sign. "AC is perpendicular to BC" means the gradients multiply to -1; setting them equal is wrong.
- **Using A or B instead of the midpoint** in a perpendicular bisector, or assuming that B lies on the perpendicular bisector of AB (it doesn't).
- **Showing only half of "show that the line is a tangent at A".** You need A on the circle **and** the line perpendicular to the radius CA.
- **Swapping x and y.** When the centre is described in words ("2 from the y-axis, 7 from the x-axis"), the distance from the y-axis is the x-coordinate.
- **Squaring terms one at a time.** Substituting $y = 3x - 1$ into $(y-1)^2$ gives $(3x-2)^2 = 9x^2 - 12x + 4$, not $9x^2 + 4$. Expansion slips cost the final marks in many discriminant questions.
- **Keeping both roots when only one fits.** If the diagram shows the point between two others, or r must be positive, reject the other root and say why.
- **Rounding surds.** "Exact" or "surd form" means keep $\sqrt{5}$; early decimals lose the accuracy mark.

How to get the marks

- **Sketch the diagram**, marking the centre, radius, points and lines. It shows the method and which root to keep.
- **Show how you solve every quadratic**, as the rubric says: factors, the formula with numbers in, or a completed square.
- **Circle equations** usually earn a mark for the centre, one for the radius and one for the equation. $(x-a)^2 + (y-b)^2 = r^2$ with r^2 as a number is the safe form; give the expanded form only when asked.
- **Tangents** earn marks for the radius gradient, the negative reciprocal used with the right point, and the equation. If a form such as $ax + by + c = 0$ with integers is asked for, the last mark needs that form.
- **Coordinates** go in pairs, $(2, 7)$ and $(6, -1)$, with each x matched to its own y.
- **"Show that"** means the answer is given: show every step, including any change of inequality sign, and end exactly on the given result.
- **"Hence"** means use the previous part, such as the centre or the points you have just found.
- **"State"** means write it down with little or no working.
- **"Exact"**, **"surd form"**: fractions and surds, not decimals. Otherwise give answers to 3 significant figures, and angles in degrees to 1 decimal place.
- **Method marks follow through**: a wrong centre used correctly later still earns method marks, so keep going.

Quick check

1. Find the equation of the line through $(2, -5)$ that is perpendicular to $4x - 3y + 7 = 0$, in the form $ax + by + c = 0$.
2. Find the centre and radius of the circle $x^2 + y^2 - 8x + 2y - 8 = 0$, and the points where it crosses the y-axis.

3. The line $y = x - 4$ cuts the circle in question 2 at A and B. Find A and B, and the equation of the perpendicular bisector of AB.
4. Find the equation of the tangent to the circle in question 2 at the point $(8, 2)$.
5. Find the exact values of k for which the line $y = x + k$ is a tangent to the circle in question 2.

Answers

1. $4x - 3y + 7 = 0$ has gradient $\frac{4}{3}$, so the perpendicular gradient is $-\frac{3}{4}$: $y + 5 = -\frac{3}{4}(x - 2)$, which is $3x + 4y + 14 = 0$.
2. $(x-4)^2 - 16 + (y+1)^2 - 1 - 8 = 0$, so $(x-4)^2 + (y+1)^2 = 25$: centre $(4, -1)$, radius 5. On the y-axis $x = 0$: $y^2 + 2y - 8 = 0$, $(y+4)(y-2) = 0$, so $(0, 2)$ and $(0, -4)$.
3. $x^2 + (x-4)^2 - 8x + 2(x-4) - 8 = 0$ gives $2x^2 - 14x = 0$, $2x(x-7) = 0$, so $x = 0$ or 7: $A(0, -4)$ and $B(7, 3)$. The perpendicular bisector of a chord passes through the centre, so it is the line through $(4, -1)$ with gradient -1: $y = -x + 3$. (Check: the midpoint $\left(\frac{7}{2}, -\frac{1}{2}\right)$ is on it.)
4. The radius from $(4, -1)$ to $(8, 2)$ has gradient $\frac{3}{4}$, so the tangent has gradient $-\frac{4}{3}$: $y - 2 = -\frac{4}{3}(x - 8)$, which is $4x + 3y - 38 = 0$.
5. Substituting gives $2x^2 + (2k-6)x + k^2 + 2k - 8 = 0$. A tangent needs $(2k-6)^2 - 8(k^2 + 2k - 8) = 0$, that is $-4k^2 - 40k + 100 = 0$, so $k^2 + 10k - 25 = 0$ and $(k+5)^2 = 50$: $k = -5 \pm 5\sqrt{2}$.

Past questions to try

- **9709/11/M/J/25 Q8**: tangents at the two points where a vertical line meets a circle, and the triangle they make.
- **9709/12/M/J/24 Q7**: a tangent with the same unknown in the line and the circle.
- **9709/13/O/N/24 Q10**: two circles of a given radius through the same two points.
- **9709/15/M/J/25 Q7**: gradients, a parallelogram and a perpendicular bisector.