Cambridge AS & A Level · Mathematics 9709 · Notes

Discrete random variables AS

Read online at **pastopic.com/cambridge/mathematics-9709/notes/discrete-random-variables**

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Read first: **Permutations and combinations**

What you need to know

By the end of this topic you should be able to:

1. **Draw up a probability distribution table** for a discrete random variable X from a situation you are given (dice, spinners, coins, choosing objects, a formula such as $P(X = x) = kx^2$), and **work out** $E(X)$ **and** $\mathrm{Var}(X)$ from the table.
2. **Use the binomial distribution** $B(n, p)$ and the **geometric distribution** $\mathrm{Geo}(p)$: find probabilities with their formulas, and recognise the real situations each one models (and when neither does).
3. **Use the mean and variance of** $B(n, p)$, np and $np(1 - p)$, and **the mean of** $\mathrm{Geo}(p)$, $\frac{1}{p}$. You don't need to prove these formulas.

Here $\mathrm{Geo}(p)$ means the number of trials up to and including the first success, so it takes the values $1, 2, 3, \ldots$. The normal approximation to the binomial is part of the next topic, the normal distribution.

Understand it

Random variables and their distributions

A **random variable** is a number that depends on chance: the score on a spinner, the number of heads in five throws, the number of attempts until you pass. It is **discrete** when it can take only separate values (usually whole numbers) that you can list. Capital X names the variable; small x stands for one of its values.

Its **probability distribution** lists every value x with its probability $P(X = x)$. The probabilities are all between 0 and 1, and they **add up to exactly 1**, because X must take one of the values. That fact, $\sum P(X = x) = 1$, is the first equation in almost every question with an unknown in the table.

Drawing up the table from a situation

Take two fair spinners: A is numbered 1, 2, 3 and B is numbered 1, 2, 3, 4. Both are spun, and X is the **smaller** of the two scores (the common score if they are equal). There are $3 \times 4 = 12$ equally likely pairs, so a grid of outcomes (a **sample-space diagram**) is the quickest way to count.

Spinner B

Spinner A	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	2	3	3

shaded: X = 1 in 6 of the 12 cells

x	1	2	3
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Check: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$.

When the outcomes are **not** equally likely (biased coins, choosing without replacement), list the ways each value can happen, multiply along each way and add the ways. Three coins, for instance, give $X = 1$ head in three ways: HTT, THT, TTH.

Expectation: the long-run mean

If you spun the two spinners thousands of times, half the results would be 1, a third 2 and a sixth 3, so the average would settle at

$$E(X) = \sum x\, P(X = x) = 1 \times \tfrac{1}{2} + 2 \times \tfrac{1}{3} + 3 \times \tfrac{1}{6} = \tfrac{5}{3}.$$

$E(X)$, the **expectation** or **mean** (also written μ), is a weighted average: each value counts in proportion to its probability. It need not be a value X can take.

Variance: the spread

The **variance** measures how far the values spread around the mean. The working form is

$$\mathrm{Var}(X) = \sum x^2\, P(X = x) - \{E(X)\}^2,$$

that is, "the mean of the squares minus the square of the mean". For the spinners, $\sum x^2 P(X = x) = 1 \times \frac{1}{2} + 4 \times \frac{1}{3} + 9 \times \frac{1}{6} = \frac{10}{3}$, so $\mathrm{Var}(X) = \frac{10}{3} - \left(\frac{5}{3}\right)^2 = \frac{5}{9}$. The **standard deviation** is $\sqrt{\mathrm{Var}(X)}$. A variance can never be negative; a negative answer means you squared the probabilities instead of the values, or forgot to square the mean.

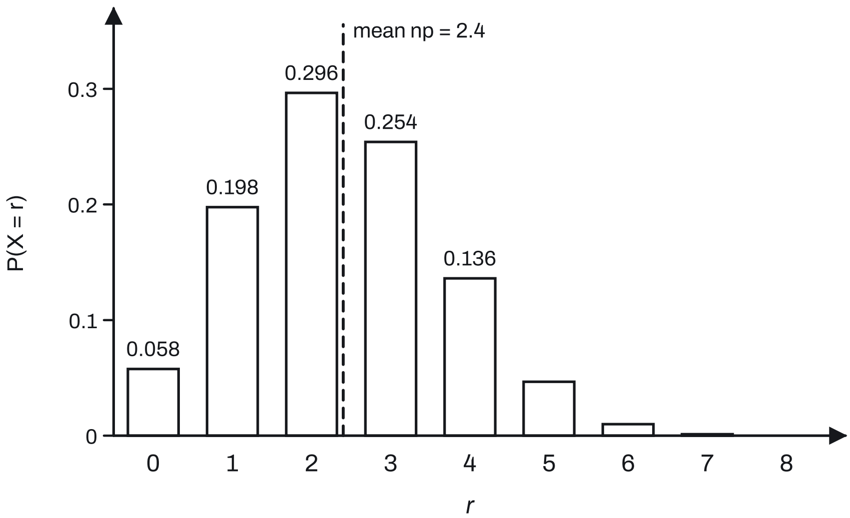
The binomial distribution

Many situations have the same shape. A trial with two outcomes, "success" or "failure", is repeated. X counts the successes. X has the **binomial distribution** $B(n, p)$ when:

- there is a **fixed number** n of trials;
- each trial has just **two outcomes** (success or failure);
- the probability of success p is the **same** on every trial;
- the trials are **independent**.

To get exactly r successes you need r successes and $n - r$ failures, with probability $p^r(1-p)^{n-r}$ for any one order. The number of different orders is $\binom{n}{r}$ (choosing which r of the n trials are the successes), so

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}, \qquad r = 0, 1, \ldots, n.$$



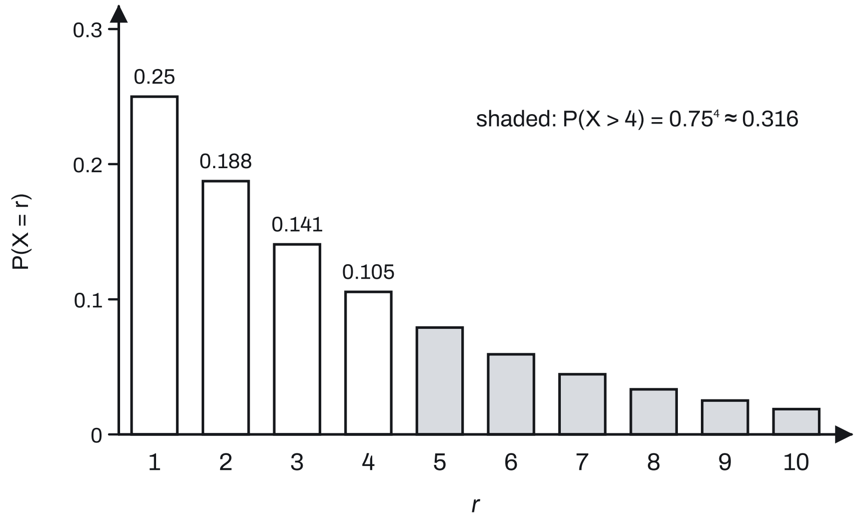
"Success" is whatever you are counting, even if it is bad news (a faulty item, a wet day). When there are three or more categories, group them into the two you need: if 15% of commuters cycle, 50% take the train and the rest drive, then "cycles or takes the train" has $p = 0.65$.

Choosing **without** replacement from a small bag is **not** binomial, because the probability changes from one pick to the next. Work those out as a list of cases.

The geometric distribution

Now keep going **until the first success**, with the same p on every independent trial. X = the number of trials needed, including the success. There is no fixed n: X can be $1, 2, 3, \ldots$ with no upper limit. This is the **geometric distribution** $\mathrm{Geo}(p)$. The first success on trial r means $r - 1$ failures and then a success:

$$P(X = r) = (1-p)^{r-1} p.$$



Each bar is $(1-p)$ times the one before, so the most likely value is always $X=1$. Two shortcuts save a long sum, and both are true for $\mathrm{Geo}(p)$ and every whole number $n \geqslant 0$:

- $P(X>n)=(1-p)^n$, because "more than n trials needed" means exactly "the first n trials all fail";
- $P(X \leqslant n)=1-(1-p)^n$.

Write $q=1-p$. Then "before the 8th" means $X \leqslant 7$, so $1-q^7$; "after the 5th" means $X>5$, so q^5; and $P(3 \leqslant X \leqslant 6)=P(X>2)-P(X>6)=q^2-q^6$.

Mean and variance of these distributions

You don't need to work through a table for these:

- $X \sim B(n,p)$: $E(X)=np$ and $\mathrm{Var}(X)=np(1-p)$. With 8 trials and $p=0.3$ you expect 2.4 successes on average, with variance $8 \times 0.3 \times 0.7 = 1.68$.
- $X \sim \mathrm{Geo}(p)$: $E(X)=\dfrac{1}{p}$. With $p=0.25$ it takes 4 trials on average to get the first success. That fits: if 1 trial in 4 succeeds, successes come about every 4th trial.

Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. "Learn it" means you must remember it.

Formula	Status
$\sum P(X=x)=1$	Learn it
$E(X)=\sum x\,p$	Given
$\mathrm{Var}(X)=\sum x^2p-\{E(X)\}^2$	Given
Standard deviation $=\sqrt{\mathrm{Var}(X)}$	Learn it

Formula	Status
$B(n, p)$: $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$	Given
$B(n, p)$: $\mu = np$, $\sigma^2 = np(1-p)$	Given
The four conditions for a binomial model (fixed n, two outcomes, constant p, independent trials)	Learn it
$\text{Geo}(p)$: $P(X = r) = (1-p)^{r-1} p$ for $r = 1, 2, 3, \ldots$	Given
$\text{Geo}(p)$: $\mu = \frac{1}{p}$	Given
$\text{Geo}(p)$: $P(X > n) = (1-p)^n$ and $P(X \leqslant n) = 1 - (1-p)^n$	Learn it
rth success on trial n: $\binom{n-1}{r-1} p^r (1-p)^{n-r}$	Learn it
$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$	Given

How to do it

In the Paper 5 questions we have tagged for this topic (87 questions, 2021–2025), the types came up this often. 59 of the 87 questions mix two or more types, so the counts add up to more than 87.

Question type	Questions
Binomial probabilities	37
Geometric probabilities	32
$E(X)$ and $\text{Var}(X)$ from a distribution	27
Probability distribution table from a situation	23
Mean and variance of the binomial and geometric distributions	22
Unknown constants in a distribution	14

1. Binomial probabilities (37 questions)

1. **Recognise the model.** A fixed number of independent trials, each a success or not, with the same p: $X \sim B(n, p)$. Write down n and p. If the question gives several categories, add the ones that count as a success.
2. **Turn the words into values of X.** "Fewer than 3" is $X \leqslant 2$; "more than 7" is $X \geqslant 8$; "at least 2" is $X \geqslant 2$; "no more than 2" is $X \leqslant 2$; "more than 2 but fewer than 6" is $X = 3, 4, 5$.
3. **Choose the shorter list.** Add the terms directly, or use $1 - (\text{the other values})$. For $n = 12$, $P(X \geqslant 2) = 1 - P(0) - P(1)$ needs two terms instead of eleven.
4. **Write every term** as $\binom{n}{r} p^r (1-p)^{n-r}$, then evaluate on your calculator. Give the answer to 3 significant figures.

For example, with $X \sim B(12, 0.2)$:

$$P(X \geqslant 2) = 1 - \left\{0.8^{12} + \binom{12}{1}(0.2)(0.8)^{11}\right\} = 0.725.$$

Variations you will meet:

- **"At least one"**: $1 - P(X = 0) = 1 - (1-p)^n$. **"None"** or **"all"**: just $(1-p)^n$ or p^n.
- **Success is the rarer outcome, or the other one.** Read carefully which outcome you are counting. "Fewer than 3 are faulty" uses $p = P(\text{faulty})$; "more than 7 are red or green" uses $p = P(\text{red}) + P(\text{green})$.
- **The success probability comes from an earlier part.** It may be a probability you worked out from dice (for example "a total of 10 or more with two dice", $p = \frac{6}{36} = \frac{1}{6}$), from a normal distribution, or from your own table. Keep it exact or to at least 4 significant figures.
- **Two stages (a binomial of a binomial).** First find the probability that one group (a week, a box, a turn) has the property. Then that group-level probability is the p of a new binomial over several groups. For example, if a box is "good" with probability 0.3, the chance that exactly 2 of 6 boxes are good is $\binom{6}{2}(0.3)^2(0.7)^4 = 0.324$.
- **Unknown p.** An equation such as $P(X = 0) = k\,P(X = n)$ becomes $(1-p)^n = k\,p^n$; take nth roots to get $\frac{1-p}{p} = \sqrt[n]{k}$.

2. Geometric probabilities (32 questions)

1. **Recognise the model.** Independent trials with the same p, repeated **until the first success**: $X \sim \text{Geo}(p)$, the number of trials needed. Write down p (for "a 6 on a fair dice", $p = \frac{1}{6}$; for "a double with two dice", $p = \frac{1}{6}$ too).
2. **Match the wording to a formula** (with $q = 1 - p$):

The question says	Value	Probability
first success **on** the nth trial	$X = n$	$q^{n-1}p$
first success **before** the nth trial / **fewer than** n trials	$X \leqslant n-1$	$1 - q^{n-1}$
first success **on or before** the nth / **no more than** n trials	$X \leqslant n$	$1 - q^n$
first success **after** the nth / **more than** n trials	$X > n$	q^n
between the ath and bth trials inclusive	$a \leqslant X \leqslant b$	$q^{a-1} - q^b$

3. Evaluate, keeping exact fractions where they are simple.

For example, with $p = 0.2$: $P(X = 5) = 0.8^4 \times 0.2 = 0.0819$; $P(X \leqslant 6) = 1 - 0.8^6 = 0.738$; $P(X > 6) = 0.8^6 = 0.262$; $P(3 \leqslant X \leqslant 7) = 0.8^2 - 0.8^7 = 0.430$.

Variations you will meet:

- **The rth success on the nth trial** (for example "a 6 for the third time on the 9th throw"). The nth trial must be a success, and the first $n-1$ trials must contain exactly $r-1$ successes in any order. So the probability is $\binom{n-1}{r-1}p^{r-1}q^{n-r} \times p$. With $p = 0.2$, the third success on the 9th trial has probability $\binom{8}{2}(0.2)^3(0.8)^6 = 0.0587$. Don't leave out the $\binom{n-1}{r-1}$, and don't use $\binom{n}{r}$: the last trial is fixed.

- **"The rth success on or before the nth trial."** Add the cases "rth success on trial r, $r+1$, ..., n", each worked as above. For the second success on or before trial 4: trials 2, 3 and 4 give $p^2 + \binom{2}{1}p^2q + \binom{3}{1}p^2q^2$.
- **The success comes from an earlier part**, such as "a total of 5 or less with three dice" ($1+3+6=10$ of the 216 outcomes, so $p = \frac{10}{216}$). Work out that p first by listing outcomes, then use it in the geometric formula.
- **Choosing between models in one question.** "How many successes in 10 trials" is binomial; "how many trials until the first success" is geometric. Many questions switch between them from one part to the next.

3. $E(X)$ and $\mathrm{Var}(X)$ from a distribution (27 questions)

1. $E(X)$: multiply each value by its probability and add: $\sum x\,p$.
2. $\sum x^2p$: multiply each **squared value** by its probability and add. Don't square the probabilities.
3. $\mathrm{Var}(X) = \sum x^2p - \{E(X)\}^2$**.** Subtract the square of the mean.
4. Give exact fractions or 3 significant figures, and say which answer is which ($E(X) = \ldots$, $\mathrm{Var}(X) = \ldots$).

For example, for the table with values $0, 1, 4$ and probabilities $0.5, 0.3, 0.2$: $E(X) = 0 + 0.3 + 0.8 = 1.1$, $\sum x^2p = 0 + 0.3 + 3.2 = 3.5$, so $\mathrm{Var}(X) = 3.5 - 1.21 = 2.29$.

Variations you will meet:

- $E(X)$ **is given** ("Given that $E(X) = 2.1$, find $\mathrm{Var}(X)$"): use the given value and don't recalculate it.
- **Negative values of** x: x^2 is positive, so $(-2)^2p = 4p$. Put negative values in brackets when you type them.
- **A probability from the table**, such as $P(X \geqslant 2)$ or $P(X > E(X))$: add the right entries of the table.
- **A conditional probability from the table**: $P(A \mid B) = \dfrac{P(A \text{ and } B)}{P(B)}$. For example, if X takes $1, 2, 3, 4$ with probabilities $0.1, 0.2, 0.3, 0.4$, then $P(X = 4 \mid X > 1) = \dfrac{P(X=4)}{P(X>1)} = \dfrac{0.4}{0.9} = \dfrac{4}{9}$.

4. Probability distribution table from a situation (23 questions)

1. **List the possible values of X.** Watch for 0 (no heads, or a rule such as "if the scores are equal, $X = 0$"), which is easy to leave out.
2. **Find each probability.**
   - **Two dice or spinners**: draw the sample-space grid, write the value of X in each cell and count.
   - **Coins with different biases, or several trials**: list every way to get each value and multiply along each way, then add the ways. For three coins with head probabilities a, b, c: $P(X=0) = (1-a)(1-b)(1-c)$, and $P(X=1) = a(1-b)(1-c) + (1-a)b(1-c) + (1-a)(1-b)c$.
   - **Choosing without replacement**: the probabilities change after each pick. For "the number of blue in 3 chosen", use $\dfrac{\binom{\text{blue}}{r}\binom{\text{other}}{3-r}}{\binom{\text{total}}{3}}$. For "the number of picks until the first blue", multiply fractions along the sequence, for example $P(X=3) = \frac{\text{not blue}}{\text{total}} \times \frac{\text{not blue}}{\text{total}-1} \times \frac{\text{blue}}{\text{total}-2}$.
3. **Put them in a table**, x in the top row and $P(X = x)$ below, as fractions (no need to cancel) or exact decimals.

4. **Check that the probabilities add up to 1.** If they don't, look for a missing value or a missing arrangement.

Variations you will meet:

- **"Show that $P(X = 2) = \ldots$"** comes first. Show every case you add, including the number of arrangements (for example "$\times 3$" or $\frac{4!}{2!\,2!}$), and reach the printed value.
- **"Complete the table"**: some probabilities are given. Work out one more from the situation, then get the last from $\sum p = 1$.
- **A stopping rule** (stop at the first success, or after at most k tries) gives a table that ends at a largest value. The last value takes all the remaining probability.
- **A table followed by $E(X)$ and $\mathrm{Var}(X)$** (type 3) is the most common pairing.

5. Mean and variance of the binomial and geometric distributions (22 questions)

1. Identify the distribution and its parameters.
2. $B(n, p)$: $E(X) = np$, $\mathrm{Var}(X) = np(1 - p)$. $\mathrm{Geo}(p)$: $E(X) = \frac{1}{p}$.
3. **"State"** or **"write down"** the mean means no working is needed: a fair 8-sided dice rolled until it shows 8 has $E(X) = 8$.

Variations you will meet:

- **Mean and variance for a normal approximation.** In a question such as "use an approximation to find the probability that more than 45 of 125 people ...", the first mark is for np and $np(1 - p)$. The rest of the method is in the normal distribution topic.
- **Finding n and p from the mean and variance**: divide, $\dfrac{np(1-p)}{np} = 1 - p$. With mean 6 and variance 4.2, $1 - p = 0.7$, so $p = 0.3$ and $n = 20$.
- **A table that is really binomial.** If X is the number of heads when five fair coins are thrown, $X \sim B(5, \frac{1}{2})$, so $E(X) = 2.5$ with no table sum needed.

6. Unknown constants in a distribution (14 questions)

1. **Write each probability** in terms of the unknowns. For a formula such as $P(X = x) = k(x + 2)$, substitute each value of x.
2. **Form equations.** One always comes from $\sum P(X = x) = 1$. The other comes from the extra fact: a given $E(X)$ (so $\sum x\,p =$ that value), a given $\mathrm{Var}(X)$, or a link such as $P(X = 4) = 3P(X = 2)$, or $P(X \geqslant 0) = 3P(X < 0)$.
3. **Solve** (simultaneous equations for two unknowns), then write the table with **numerical** probabilities if asked.
4. **Check** that every probability is between 0 and 1.

For example, X takes the values $0, 1, 2$ with $P(X = 0) = P(X = 2) = a$, so $P(X = 1) = 1 - 2a$. If $\mathrm{Var}(X) = 0.6$: $E(X) = (1 - 2a) + 2a = 1$ and $\sum x^2 p = (1 - 2a) + 4a = 1 + 2a$, so $\mathrm{Var}(X) = 1 + 2a - 1 = 2a = 0.6$, giving $a = 0.3$.

Variations you will meet:

- **"Draw up the table in terms of k"** means leave k in; find k only in the part that asks.
- **"Show that $\text{Var}(X) = \ldots$"**: find k, then show both $E(X)$ and $\sum x^2 p$ before the final subtraction.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

In a game, Tom rolls a fair six-sided dice at most three times. He stops as soon as he rolls a 6, and he stops after the third roll whatever happens. The random variable X is the number of times Tom rolls the dice.

(a) Explain why X does not have a geometric distribution. **[1]**

(b) Draw up the probability distribution table for X. **[3]**

(c) Find $E(X)$ and $\text{Var}(X)$. **[3]**

(a) A geometric variable can take any value $1, 2, 3, \ldots$ with no upper limit, but Tom never rolls more than 3 times, so X can only be 1, 2 or 3. **B1**

(b) $X = 1$: a 6 first time, $\frac{1}{6}$. $X = 2$: not a 6, then a 6, $\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$. **B1** for these two.

$X = 3$ happens whenever the first two rolls are not 6s, whatever the third roll is: $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$. **B1**

x	**1**	**2**	**3**
$P(X = x)$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{25}{36}$

B1 for the complete table. Check: $6 + 5 + 25 = 36$.

(c) $E(X) = 1 \times \frac{6}{36} + 2 \times \frac{5}{36} + 3 \times \frac{25}{36} = \frac{91}{36} = 2.53$ (3 s.f.). **B1**

$\sum x^2 p = 1 \times \frac{6}{36} + 4 \times \frac{5}{36} + 9 \times \frac{25}{36} = \frac{251}{36}$. **M1** for the variance formula with their $E(X)$:

$$\text{Var}(X) = \frac{251}{36} - \left(\frac{91}{36}\right)^2 = \frac{755}{1296} = 0.583 \text{ (3 s.f.)}$$

A1

Example 2

The random variable X takes the values $1, 2, 3, 4$ only, and $P(X = x) = a + bx$, where a and b are constants. It is given that $E(X) = 2.8$.

(a) Find the values of a and b. **[4]**

(b) Find $\mathrm{Var}(X)$. **[2]**

(c) Find $P(X > E(X))$. **[1]**

(a) The probabilities are $a + b$, $a + 2b$, $a + 3b$, $a + 4b$.

Sum is 1: $4a + 10b = 1$. **B1**

$E(X)$: $1(a + b) + 2(a + 2b) + 3(a + 3b) + 4(a + 4b) = 10a + 30b = 2.8$. **M1** for $\sum x\,p$ set equal to 2.8.

Multiply the first equation by 3: $12a + 30b = 3$. Subtract: $2a = 0.2$, so $a = 0.1$, and then $10b = 0.6$, so $b = 0.06$. **M1 A1**

The probabilities are $0.16, 0.22, 0.28, 0.34$, all between 0 and 1.

(b) $\sum x^2 p = 1(0.16) + 4(0.22) + 9(0.28) + 16(0.34) = 9$. **M1**

$\mathrm{Var}(X) = 9 - 2.8^2 = 9 - 7.84 = 1.16$. **A1**

(c) $P(X > 2.8) = P(X = 3) + P(X = 4) = 0.28 + 0.34 = 0.62$. **B1**

Example 3

A quiz has 10 questions, each with 4 possible answers, only one of which is correct. Rana guesses the answer to every question, independently. The random variable X is the number of questions she answers correctly.

(a) State the distribution of X, and find $E(X)$ and $\mathrm{Var}(X)$. **[2]**

(b) Find $P(2 \leqslant X \leqslant 4)$. **[3]**

(c) Each correct answer scores 3 points and each wrong answer loses 1 point. Find the probability that Rana's total score is positive. **[3]**

(a) $X \sim B(10, 0.25)$. **B1** $E(X) = 10 \times 0.25 = 2.5$ and $\mathrm{Var}(X) = 10 \times 0.25 \times 0.75 = 1.875$. **B1**

(b)

$$P(2 \leqslant X \leqslant 4) = \binom{10}{2}(0.25)^2(0.75)^8 + \binom{10}{3}(0.25)^3(0.75)^7 + \binom{10}{4}(0.25)^4(0.75)^6$$

M1 for one term of the form $\binom{10}{r}(0.25)^r(0.75)^{10-r}$, **A1** for the correct sum with no terms missing.

$= 0.2816 + 0.2503 + 0.1460 = 0.678$ (3 s.f.). **B1**

(c) With X correct and $10 - X$ wrong, the score is $3X - (10 - X) = 4X - 10$. This is positive when $X > 2.5$, that is $X \geqslant 3$. **B1**

$$P(X \geqslant 3) = 1 - \left\{0.75^{10} + \binom{10}{1}(0.25)(0.75)^9 + \binom{10}{2}(0.25)^2(0.75)^8\right\}$$

M1 for $1 - P(0, 1, 2)$ with binomial terms.

$= 1 - 0.5256 = 0.474$ (3 s.f.). **A1**

Example 4

Each computer chip made in a factory passes a quality test with probability 0.9, independently of all other chips. A technician tests chips one at a time until she finds a chip that fails. The random variable X is the number of chips she tests.

(a) State the value of $E(X)$. **[1]**

(b) Find $P(X = 4)$. **[1]**

(c) Find $P(X > E(X))$. **[2]**

(d) She carries on testing after the first chip that fails. Find the probability that the 12th chip she tests is the second chip to fail. **[2]**

(a) A chip fails with probability 0.1, so $X \sim \text{Geo}(0.1)$ and $E(X) = \dfrac{1}{0.1} = 10$. **B1**

(b) $P(X = 4) = 0.9^3 \times 0.1 = 0.0729$. **B1**

(c) $P(X > 10)$ means the first 10 chips all pass: $0.9^{10} = 0.349$ (3 s.f.). **M1 A1**

(d) The 12th chip fails, and exactly 1 of the first 11 fails, in any of $\binom{11}{1} = 11$ positions:

$$\binom{11}{1}(0.1)(0.9)^{10} \times 0.1 = 0.0384 \text{ (3 s.f.)}$$

M1 for $\binom{11}{1}p^2(1-p)^{10}$, **A1**.

Common mistakes

- **Misreading the success condition.** "More than 7" does not include 7; "at least 7" does. "Before the 6th throw" means $X \leqslant 5$, not $X \leqslant 6$. Examiners report this every year, in both binomial and geometric questions. Write the list of values of X before you calculate.
- **Leaving out $\binom{n}{r}$.** Each binomial term needs its number of arrangements, and so does the "rth success on the nth trial" calculation, where it is $\binom{n-1}{r-1}$ (the last trial is fixed as a success).
- **Using the geometric formula for a binomial situation, or the other way round.** Ask: is the number of trials fixed (binomial), or do we stop at the first success (geometric)? Parts of one question often switch between them.

- **Using the wrong p.** When there are three categories, check which ones count as a success, and whether the question asks about the success or the failure.
- **Leaving $X = 0$ out of a table.** Zero heads, or a rule such as "if equal, $X = 0$", is a value too. Check that the probabilities add up to 1.
- **Forgetting to subtract $\{E(X)\}^2$**, or squaring the probabilities instead of the values, in $\mathrm{Var}(X)$.
- **Recalculating a given $E(X)$.** If the question gives $E(X)$, use it; working it out again wastes time and can bring in an error.
- **Rounding too early.** Using 0.167 for $\frac{1}{6}$ in a long product, or a rounded probability from an earlier part, can push the final answer outside the accepted range. Keep exact fractions or at least 4 significant figures until the end; there is no need to change an exact fraction to a decimal.
- **Summing a long geometric list.** Adding $P(X = 1) + \cdots + P(X = 7)$ term by term invites slips; $1 - q^7$ does it in one step.

How to get the marks

- **Write the expression before the answer.** For binomial questions the method mark is for at least one term in the form $\binom{n}{r}p^r(1-p)^{n-r}$, and the accuracy mark needs every term present, unsimplified. Then give the final value. A bare calculator answer risks losing the method marks.
- **Distribution tables:** the values of x in one row and their probabilities in the next. Marks come for the right values and then for groups of correct probabilities, so a table with some errors still earns something.
- **$E(X)$ and $\mathrm{Var}(X)$:** show the sum $\sum x\,p$ and the sum $\sum x^2 p$ with the numbers in, then subtract $\{E(X)\}^2$. Method marks follow through from your own table, so a slip in the table costs little here.
- **"Show that"** means the answer is printed: show every case and every multiplication (including the number of arrangements), and end with the exact printed value.
- **"State"** or **"write down"** (for example the mean of a geometric distribution) needs only the answer.
- **Accuracy:** give probabilities exactly (a fraction) or to 3 significant figures. Keep more figures in the working.
- **Name what you find.** "$E(X) = \ldots$", "$\mathrm{Var}(X) = \ldots$": if you find both, label them.

Quick check

1. The random variable X takes the values $1, 2, 3, 6$, and $P(X = x) = \dfrac{c}{x}$, where c is a constant. Find c, $E(X)$ and $\mathrm{Var}(X)$.
2. $X \sim B(8, 0.35)$. Find $P(X \leqslant 1)$.
3. $Y \sim \mathrm{Geo}(0.25)$. Find $P(Y \geqslant 4)$ and $E(Y)$.
4. $X \sim B(n, p)$ has mean 4.8 and variance 3.84. Find n and p.
5. A fair coin is thrown 3 times. X is the number of times the result changes from one throw to the next (for example, HTT has one change). Draw up the probability distribution table for X and find $\mathrm{Var}(X)$.

Answers

1. $c\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{6}\right)=2c=1$, so $c=\frac{1}{2}$. $E(X)=\sum x\cdot\frac{c}{x}=4c=2$. $\sum x^2p=c(1+2+3+6)=6$, so $\mathrm{Var}(X)=6-2^2=2$.
2. $P(X\leqslant 1)=0.65^8+\binom{8}{1}(0.35)(0.65)^7=0.0319+0.1373=0.169$ (3 s.f.).
3. $P(Y\geqslant 4)=P(Y>3)=0.75^3=\frac{27}{64}=0.422$ (3 s.f.): the first three trials all fail. $E(Y)=\frac{1}{0.25}=4$.
4. $1-p=\frac{3.84}{4.8}=0.8$, so $p=0.2$ and $n=\frac{4.8}{0.2}=24$.
5. Of the 8 equally likely outcomes, HHH and TTT have no change, HTH and THT have two changes, and the other 4 have one change.

x	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$E(X)=\frac{1}{2}+\frac{1}{2}=1$, $\sum x^2p=\frac{1}{2}+1=\frac{3}{2}$, so $\mathrm{Var}(X)=\frac{3}{2}-1=\frac{1}{2}$.

Past questions to try

- **9709/51/M/J/25 Q6**: a distribution table from choosing without replacement, then a conditional probability.
- **9709/51/O/N/25 Q1**: a distribution given by a formula in k, then a conditional probability.
- **9709/53/M/J/25 Q3**: geometric probabilities, including the second success before a given throw.
- **9709/52/M/J/25 Q4**: three categories, then the first and the second success in a geometric setting.