Cambridge AS & A Level · Mathematics (9709) · Notes

Energy, work and power AS

Read online at pastopic.com/cambridge/mathematics-9709/notes/energy-work-and-power

Last checked 28 September 2026

Read first: Forces and equilibrium, Newton's laws of motion

What you need to know

By the end of this topic you should be able to:

1. **Work out the work done by a constant force**, $W = Fd\cos\theta$, including when the force is at an angle θ to the direction in which its point of application moves. You do not need the scalar product.
2. **Use kinetic energy and gravitational potential energy**: know what each one means and use $\mathrm{KE} = \frac{1}{2}mv^2$ and $\mathrm{PE} = mgh$.
3. **Link the change in energy to the work done by the forces.** The work done by the forces on a body changes its energy, and when only gravity does work, energy is conserved. This includes motion that is not in a straight line, such as a child on a smooth curved slide, where you only need the energy at the start and at the end.
4. **Use power**, the rate at which a force does work. Find an average power as $\dfrac{\text{work done}}{\text{time taken}}$, and use $P = Fv$ for a force acting in the direction of motion.
5. **Solve problems that put these together**, such as the acceleration of a car at the instant it has a given speed on a hill, against a resistance.

Take $g = 10\ \mathrm{m\ s^{-2}}$, as every Paper 4 question does. Elastic energy and variable forces are not in this syllabus.

Understand it

Work done by a force

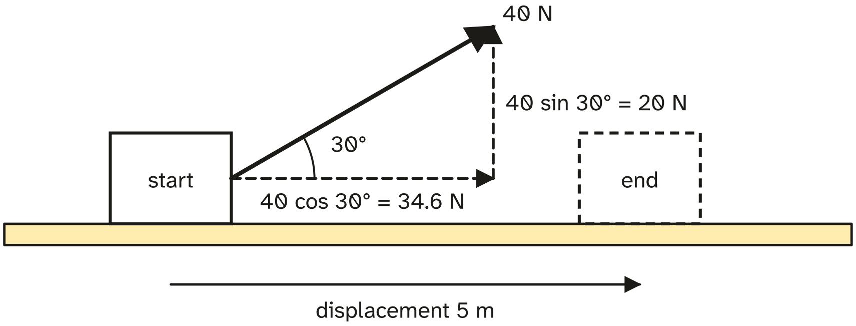
A force does **work** when the thing it pushes or pulls moves. If a constant force F newtons moves its point of application d metres in its own direction, the work done is Fd joules (J). One joule is the work done when a force of 1 N moves 1 m.

When the force is at an angle θ to the motion, only its component along the motion does work:

$$W = Fd\cos\theta.$$

The component at right angles to the motion does no work at all.

Only the part of the force along the motion does work.
The 20 N part is at right angles to the motion: it does none.



In the diagram the work done by the 40 N force is $40\cos 30^\circ \times 5 = 100\sqrt{3} = 173$ J (3 s.f.). The normal reaction and the weight are at right angles to the motion, so they do no work.

Two cases come up in almost every question:

- **Work done against a resistance or friction** F over a distance d is Fd. Friction always opposes the motion, so it takes energy away.
- **Work done by or against gravity** depends only on the **vertical** height. A 2 kg stone that falls 3 m has $2 \times 10 \times 3 = 60$ J of work done on it by its weight. On a slope at angle α, a distance d along the slope is a height of $d\sin\alpha$.

Kinetic and potential energy

Energy is the capacity to do work, and it is also measured in joules.

- **Kinetic energy** is the energy a body has because it moves: $\mathrm{KE} = \frac{1}{2}mv^2$. A 1200 kg car at $20\ \mathrm{m\ s^{-1}}$ has $\frac{1}{2} \times 1200 \times 20^2 = 240\,000$ J. KE is never negative and does not depend on direction.
- **Gravitational potential energy** is the energy a body has because of its height: $\mathrm{PE} = mgh$, where h is the height above a level you choose. Only **changes** in PE matter, so pick whichever level is easiest (usually the lowest point).

The work–energy principle

The total work done by all the forces on a body equals its gain in kinetic energy. In practice it is easiest to treat gravity as potential energy and write the energy "account" for the motion:

$$\mathrm{KE_{start}} + \mathrm{PE_{start}} + \text{work done by driving forces} = \mathrm{KE_{end}} + \mathrm{PE_{end}} + \text{work done against resistances.}$$

The left side is the energy put in, the right side is where it all went. A **driving force** (an engine, a pull, a push along the motion) adds energy; friction and air resistance remove it. The normal reaction, being at right angles to the motion, does nothing.

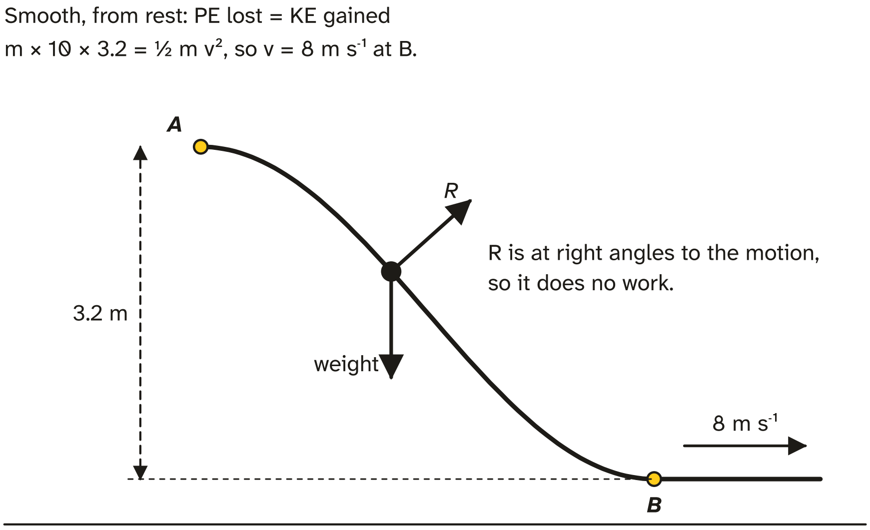
Use gravity **once**: either as a change in PE or as work done by the weight, never both.

Conservation of energy

If no driving force acts and there is no friction or resistance (a **smooth** surface), the only force doing work is gravity, and

$$\text{KE} + \text{PE stays the same.}$$

This is the **principle of conservation of energy**, and it works whatever the path, even along a curve where you could not use $F = ma$ and the constant acceleration formulae.



For the slide: PE lost $=$ KE gained, so $m \times 10 \times 3.2 = \frac{1}{2}mv^2$. The mass cancels: $v^2 = 64$ and $v = 8\ \text{m s}^{-1}$. If the slide were rough and the question told you the work done against the resistance, you would subtract that from the PE lost before finding the KE.

Power

Power is the rate at which a force does work, measured in watts (W): 1 W is 1 J per second, and 1 kW $= 1000$ W.

- **Average power** $= \dfrac{\text{work done}}{\text{time taken}}$. A crane that lifts 400 kg through 15 m at a steady speed in 20 s does $400 \times 10 \times 15 = 60\,000$ J of work, so its average power is $\frac{60\,000}{20} = 3000$ W.
- **Power at an instant.** In a very short time a force F moves the body a distance $v\times$ time, so the work done per second is Fv:

$$P = Fv.$$

For a vehicle, F is the **driving force** of the engine. So if you know the power and the speed, the driving force is $F = \dfrac{P}{v}$. The same engine power gives a **large** force at low speed and a **small** force at high speed.

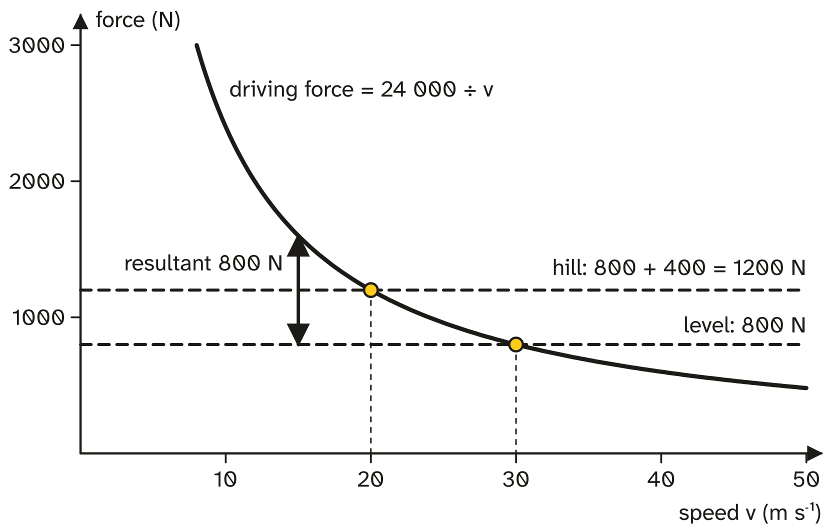
Cars on hills

Combine $P = Fv$ with Newton's second law (from "Newton's laws of motion"). For a car of mass m with power P at speed v going up a slope at angle α against a resistance R:

$$\frac{P}{v} - R - mg\sin\alpha = ma.$$

On level ground drop the $mg\sin\alpha$ term; downhill it becomes $+mg\sin\alpha$. At the **top (greatest, steady) speed** the car can no longer accelerate, so $a = 0$ and the driving force exactly balances the resistance and the weight component.

Top speed is where the curve meets the resistance line:
30 m s^{-1} on the level, 20 m s^{-1} up the hill.



The graph is for a car of mass 1000 kg working at 24 kW against a resistance of 800 N. At $15\ \mathrm{m\ s^{-1}}$ on the level, the driving force is $\frac{24\,000}{15} = 1600$ N, so $1600 - 800 = 1000a$ and $a = 0.8\ \mathrm{m\ s^{-2}}$. As the car speeds up the gap closes; it reaches its top speed where $\frac{24\,000}{v} = 800$, at $v = 30\ \mathrm{m\ s^{-1}}$. Up a hill with $\sin\alpha = 0.04$, the weight adds $1000 \times 10 \times 0.04 = 400$ N, so the top speed falls to $\frac{24\,000}{1200} = 20\ \mathrm{m\ s^{-1}}$.

Which method?

- **Energy** needs no acceleration and works when a force is not constant (a varying resistance, an engine at constant power, a curved track). It links **speeds and distances**.
- **Newton's second law** gives an **acceleration** at an instant, or a force. Use it when the question asks for an acceleration or a time.
- If a question says **"use an energy method"**, a constant-acceleration solution earns very little, even if the answer is right.

Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. "Learn it" means you must remember it. MF19 has no energy formulas at all, so learn every one of these.

Formula	Status
Work done by a constant force: $W = Fd\cos\theta$, in J	Learn it
Work done against a resistance F over a distance d: Fd	Learn it
Kinetic energy: $\mathrm{KE} = \frac{1}{2}mv^2$	Learn it
Gravitational potential energy: $\mathrm{PE} = mgh$ (h vertical); along a slope, $h = d\sin\alpha$	Learn it
Work–energy: $\mathrm{KE}_1 + \mathrm{PE}_1 + \mathrm{WD\ by\ driving\ forces} = \mathrm{KE}_2 + \mathrm{PE}_2 + \mathrm{WD\ against\ resistances}$	Learn it
Smooth, gravity only: $\mathrm{KE} + \mathrm{PE}$ is constant	Learn it
Average power $= \dfrac{\text{work done}}{\text{time}}$; work done by an engine at constant power $= Pt$	Learn it
$P = Fv$, so driving force $= \dfrac{P}{v}$	Learn it
Vehicle uphill: $\dfrac{P}{v} - R - mg\sin\alpha = ma$; top speed when $a = 0$	Learn it
Friction when sliding: $F = \mu R$	Learn it
$v = u + at$, $s = \frac{1}{2}(u+v)t$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$	Given

Units: mass in kg, speed in $\mathrm{m\ s^{-1}}$, force in N, energy and work in J, power in W. Change kW to W and kJ to J before you start.

How to do it

In the Paper 4 questions we have tagged for this topic (72 questions, 2021–2025), the types came up this often. 17 of the 72 questions mix two or three types, so the counts add up to more than 72.

Question type	Questions
Power, driving force and acceleration ($P = Fv$)	35
Work–energy equation for one body	29
Work done by an engine over a time; average power	11
Energy with an impact or a bounce	6
Connected particles by an energy method	5

Question type	Questions
Work done by a force at an angle	4

1. Power, driving force and acceleration (35 questions)

1. **Convert** kW to W.
2. **Driving force** $= \dfrac{P}{v}$ at the speed in the question.
3. **Newton's second law along the motion:** driving force $-$ resistance $- mg\sin\alpha = ma$ uphill; $+mg\sin\alpha$ downhill; no weight term on the level.
4. **Steady, constant or greatest speed** means $a = 0$. Solve for the unknown.

For example, a van of mass 2000 kg works at 40 kW against a resistance of 1000 N. On the level at $16\ \mathrm{m\,s^{-1}}$: $\frac{40\,000}{16} - 1000 = 2000a$, so $a = 0.75\ \mathrm{m\,s^{-2}}$. Its greatest speed on the level is $\frac{40\,000}{1000} = 40\ \mathrm{m\,s^{-1}}$.

Variations you will meet:

- **Find the power** from a steady speed: power $=$ (resistance $+$ weight component) $\times$ speed.
- **Find the angle of a hill** from a steady speed: $\frac{P}{v} - R = mg\sin\alpha$, then $\alpha = \sin^{-1}(\ldots)$.
- **Find the resistance or the mass** from an acceleration at a known speed: one Newton's law equation, one unknown.
- **Same power and resistance in two situations** (for example a steady speed on the level and another up a hill): write one equation for each and solve them together. With power P and resistance R: $\frac{P}{25} = R$ and $\frac{P}{15} = R + 1000$ give $P = 37\,500$, $R = 1500$.
- **Two accelerations at two speeds** (for example "the acceleration at B is half that at A"): write both equations with the unknown power and solve.
- **Resistance that depends on speed**, such as kv or $(800 + 40v)$ N. At a steady speed $\frac{P}{v} = kv$, so $P = kv^2$; with a sum you get a quadratic in v. Reject the negative root.
- **Power changed suddenly** ("increased **by** 9 kW" means add 9000 W; "increased **to** 40 kW" means the new power is $40\,000$ W; "decreased by 15%" means multiply by 0.85). The speed has not had time to change, so use the old speed in $\frac{P}{v}$ to get the "instantaneous" acceleration.
- **A car towing a trailer**: the engine's driving force pulls the whole system, so use the total mass and the total resistance, then Newton's law on the trailer alone for the tension.
- **A winch or crane** pulling at constant speed: the rope's force equals the resistances (and weight component) it balances, and power $=$ force $\times$ speed. From the force you can find a friction force, then $\mu = \frac{F}{R}$.

2. Work–energy equation for one body (29 questions)

1. **List the energies** at the start and end: KE from the speeds, PE from the **vertical** heights ($d\sin\alpha$ on a slope).
2. **List the work done** by each driving force (force $\times$ distance, or Pt) and against each resistance (friction $= \mu R$ with R from resolving at right angles to the surface; or a given number of joules).

3. **Write the energy equation**: energy at start $+$ work in $=$ energy at end $+$ work against resistances.
4. **Solve** for the unknown: a speed, a distance, a work done, a force, a mass or μ.

For example, a 2 kg block is projected at $6 \mathrm{~m~s}^{-1}$ down a slope of angle 30°, and 10 J of work is done against resistance in the first 3 m. PE lost $= 2 \times 10 \times 3 \sin 30^{\circ} = 30$ J, so $\frac{1}{2}(2)(6^2) + 30 = \frac{1}{2}(2)v^2 + 10$, which gives $v^2 = 56$ and $v = 7.48 \mathrm{~m~s}^{-1}$.

Variations you will meet:

- **Smooth** surfaces: no work against resistance, so KE $+$ PE is constant. For a body **dropped** or sliding from rest, $v^2 = 2gh$.
- **"Find the work done against the resistance"** (air resistance on a falling particle, a rough slide, a car): it is whatever energy is missing: (energy at start $+$ work in) $-$ energy at end. It must come out positive.
- **Rough plane with** μ: resolve at right angles to the plane, $R = mg\cos\alpha$ (less any component of a pulling force at an angle), and friction $= \mu R$ does work $\mu R \times d$.
- **Projected up a rough slope, comes to rest, slides back**: first the trip up ($\frac{1}{2}mu^2 = mgd\sin\alpha + Fd$ gives d), then the trip down ($mgd\sin\alpha - Fd = \frac{1}{2}mv^2$). Friction acts against the motion both ways, so the total loss over the round trip is $2Fd$. It only slides back if $mg\sin\alpha$ is more than the greatest friction, that is $\mu < \tan\alpha$.
- **Several sections** (a smooth curve then a rough straight, or a slope then a horizontal run): take the speed at the end of one section as the start of the next, or write one equation for the whole path when that is quicker.
- **Find** μ from the distance a body slides before stopping: the energy lost equals $\mu R \times d$, and solve for μ. Give it as a fraction if asked.
- **Work done by a person or engine** (a cyclist, a winch, an athlete) over a distance: it is the unknown "work in". At constant speed the KE terms cancel, so work done $=$ PE gained $+$ work against resistance.
- **Work done per metre** (for example "250 J per metre") is just a resistance force of 250 N.

3. Work done by an engine over a time; average power (11 questions)

When an engine works at a **constant power** for a time t, the force it produces changes as the speed changes, so do **not** use $\frac{P}{v}$ with one speed. Instead:

1. **Work done by the engine** $= P \times t$.
2. **Energy equation**: $Pt + \mathrm{KE}_1 + \mathrm{PE}_1 = \mathrm{KE}_2 + \mathrm{PE}_2 + \mathrm{WD~against~resistance.}$
3. **Solve** for the unknown, often the work against a resistance that is "not constant", or the final speed.

For example, a car of mass 1000 kg works at 20 kW for 10 s on a level road and its speed rises from 10 to $16 \mathrm{~m~s}^{-1}$. Work against resistance $= 200\,000 - \frac{1}{2}(1000)(16^2 - 10^2) = 200\,000 - 78\,000 = 122\,000$ J.

Variations you will meet:

- **Average power** over a motion: find the total work done by the engine (from the energy equation), then divide by the time. If the time isn't given but the acceleration is uniform, find it from $s = \frac{1}{2}(u+v)t$.

- **Work done by a force divided by the time** gives the power at which it works: a 25 N force doing 300 J of work in 8 s works at 37.5 W.
- **"Show that the distance is ..."**: the energy equation has the distance inside the PE or resistance term; solve for it and give more figures than the answer shown.
- A later part often returns to $P = Fv$ (type 1), for example the steady speed at the same power.

4. Energy with an impact or a bounce (6 questions)

1. **Before the impact**, use energy to find the speed (for a drop, $v^2 = 2gh$).
2. **At the impact**, use conservation of momentum for a collision or coalescence (see "Momentum"), or subtract the energy lost at a bounce from the KE.
3. **After the impact**, use energy again: for example a body driven into the ground against a constant resistance F over a distance s gives $\frac{1}{2}Mv^2 + Mgs = Fs$ (the weight still does work as it sinks).

For example, a 90 kg object hits a 30 kg post at 6 m s^{-1} and they move together: $90 \times 6 = 120v$, so $v = 4.5 \text{ m s}^{-1}$. Against a resistance of 5000 N: $\frac{1}{2}(120)(4.5^2) + 120 \times 10 \times s = 5000s$, so $1215 = 3800s$ and $s = 0.320$ m. For a bounce, a 0.5 kg ball dropped from 5 m has 25 J of KE at the ground; if it loses 9 J it leaves with 16 J and rises $h = \frac{16}{0.5 \times 10} = 3.2$ m.

5. Connected particles by an energy method (5 questions)

1. **Treat the system as one**: the tension is internal, so it does no net work and leaves the equation.
2. **PE**: one particle may go down (loses PE) while the other goes up or along a slope (gains $mgd \sin \alpha$). Both move the **same distance**.
3. **KE**: both particles have the same speed: $\frac{1}{2}(m_1 + m_2)v^2$.
4. **Work against friction** on the rough surface, $\mu R \times d$.
5. **Solve** $\text{PE lost} = \text{PE gained} + \text{KE gained} + \text{work against friction}$.

If you write an energy equation for **one** particle, the work done by the tension must be in it, and the tension is not the one from an earlier part if the situation has changed.

6. Work done by a force at an angle (4 questions)

1. **Resolve** the force along the motion: $F \cos \theta$.
2. **Work done** $= F \cos \theta \times d$.
3. If friction is involved, find R first, remembering that the force's other component $F \sin \theta$ changes R (it lifts a body when it pulls upwards). Then work against friction $= \mu R \times d$.

For example, 50 N at 15° above the horizontal moving a crate 10 m does $500 \cos 15^\circ = 483$ J of work. Divide by the time if the question then asks for the power.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **DM1** method mark that needs the M mark before it.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

A block of mass 5 kg is projected with speed $8.4\ \mathrm{m\ s^{-1}}$ up a line of greatest slope of a rough plane inclined at angle α to the horizontal, where $\sin\alpha = 0.6$. The coefficient of friction between the block and the plane is 0.3.

(a) Use an energy method to find the distance the block moves up the plane before coming to instantaneous rest. **[4]**

(b) Show that the block then slides back down, and find its speed when it returns to its starting point. **[4]**

(a) $\cos\alpha = 0.8$. Resolving at right angles to the plane, $R = 5 \times 10 \times 0.8 = 40$ N, so friction $= 0.3 \times 40 = 12$ N. **B1**

Going up a distance d: PE gained $= 5 \times 10 \times d \times 0.6 = 30d$, work against friction $= 12d$. **B1** for both terms.

$$\tfrac{1}{2} \times 5 \times 8.4^2 = 30d + 12d \ \Rightarrow\ 176.4 = 42d.$$

M1 for an energy equation with KE, PE and friction terms.

$d = 4.2$ m. **A1**

(b) Down the slope the weight component is $5 \times 10 \times 0.6 = 30$ N, more than the greatest friction of 12 N (equivalently $\mu = 0.3 < \tan\alpha = 0.75$), so the block slides back. **B1**

Coming down 4.2 m from rest:

$$\text{PE lost} - \text{work against friction} = \text{KE gained}: \quad 30 \times 4.2 - 12 \times 4.2 = \tfrac{1}{2} \times 5 \times v^2.$$

M1 for the energy equation for the trip down.

$126 - 50.4 = 75.6 = 2.5v^2$, so $v^2 = 30.24$ **A1** and $v = 5.50\ \mathrm{m\ s^{-1}}$. **A1**

(Check: the total friction loss over the round trip is $2 \times 12 \times 4.2 = 100.8$ J, and $176.4 - 100.8 = 75.6$ J.)

Example 2

A car of mass 1200 kg has an engine that works at a constant power of 30 kW. The car's greatest speed on a straight level road is $40\ \mathrm{m\ s^{-1}}$. The resistance to motion is constant.

(a) Find the resistance. **[2]**

(b) Find the acceleration of the car when its speed is $25\ \mathrm{m\ s^{-1}}$ on the level road. **[3]**

(c) The car goes up a straight hill inclined at α to the horizontal, where $\sin\alpha = 0.05$. The power is increased to 36 kW and the resistance is unchanged. Find the greatest steady speed of the car up the hill. **[3]**

(a) At the greatest speed $a = 0$, so the driving force equals the resistance: $R = \dfrac{30\,000}{40} = 750$ N. **M1 A1**

(b) Driving force $= \dfrac{30\,000}{25} = 1200$ N. **B1**

$1200 - 750 = 1200a$ **M1**, so $a = 0.375\ \mathrm{m\ s^{-2}}$. **A1**

(c) Weight component $= 1200 \times 10 \times 0.05 = 600$ N. At a steady speed:

$$\frac{36\,000}{v} - 750 - 600 = 0.$$

M1 for three terms with the weight component, **A1** for the correct equation.

$v = \dfrac{36\,000}{1350} = 26.7\ \mathrm{m\ s^{-1}}$ (3 s.f.). **A1**

Example 3

A lorry of mass 8000 kg moves up a straight hill inclined at α to the horizontal, where $\sin\alpha = 0.03$. The engine works at a constant 60 kW. In 20 s the lorry travels 280 m up the hill and its speed increases from $12\ \mathrm{m\ s^{-1}}$ to $15\ \mathrm{m\ s^{-1}}$. The resistance to motion is not constant.

(a) Use an energy method to find the work done against the resistance in the 20 s. **[5]**

The lorry then travels on a straight level road with the engine still working at 60 kW. The resistance is now $(800 + 40v)$ N when the speed is $v\ \mathrm{m\ s^{-1}}$.

(b) Find the steady speed of the lorry on the level road. **[3]**

(c) Find the acceleration of the lorry at an instant when its speed on the level road is $20\ \mathrm{m\ s^{-1}}$. **[2]**

(a) Work done by the engine $= 60\,000 \times 20 = 1\,200\,000$ J. **B1**

KE gained $= \frac{1}{2} \times 8000 \times (15^2 - 12^2) = 324\,000$ J. **B1**

PE gained $= 8000 \times 10 \times 280 \times 0.03 = 672\,000$ J. **B1**

$$1\,200\,000 = 324\,000 + 672\,000 + W_R.$$

M1 for an energy equation with all four terms.

$W_R = 204\,000$ J $= 204$ kJ. **A1**

The resistance is not constant and the engine's force changes, so a Newton's law method cannot be used here.

(b) At a steady speed, $\dfrac{60\,000}{v} = 800 + 40v$. **M1**

$40v^2 + 800v - 60\,000 = 0 \Rightarrow v^2 + 20v - 1500 = 0 \Rightarrow (v+50)(v-30) = 0$. **DM1**

$v = 30 \text{ m s}^{-1}$ (the root -50 is rejected). **A1**

(c) Driving force $= \dfrac{60\,000}{20} = 3000$ N and resistance $= 800 + 40 \times 20 = 1600$ N, so $3000 - 1600 = 8000a$ **M1** and $a = 0.175 \text{ m s}^{-2}$. **A1**

Example 4

Particle A, of mass 4 kg, lies on a rough plane inclined at α to the horizontal, where $\sin\alpha = 0.28$ and $\cos\alpha = 0.96$. A light inextensible string is attached to A, runs up a line of greatest slope and passes over a small smooth pulley at the top of the plane. Particle B, of mass 3 kg, hangs from the other end, 2 m above the ground. The coefficient of friction between A and the plane is 0.25. The system is released from rest with the string taut, and B moves down.

(a) Use an energy method to find the speed of the particles at the instant B reaches the ground. **[5]**

(b) B does not bounce and the string goes slack. A does not reach the pulley. Use an energy method to find how much further A moves up the plane. **[3]**

(a) Each particle moves 2 m.

PE lost by $B = 3 \times 10 \times 2 = 60$ J; PE gained by $A = 4 \times 10 \times 2 \times 0.28 = 22.4$ J. **B1**

$R = 4 \times 10 \times 0.96 = 38.4$ N, friction $= 0.25 \times 38.4 = 9.6$ N, work against friction $= 9.6 \times 2 = 19.2$ J. **B1**

KE gained $= \frac{1}{2}(4+3)v^2 = 3.5v^2$. **B1**

$$60 = 22.4 + 19.2 + 3.5v^2.$$

M1 for a system energy equation with all four terms (the tension does not appear).

$3.5v^2 = 18.4$, $v^2 = 5.257\ldots$, so $v = 2.29 \text{ m s}^{-1}$. **A1**

(b) Now only A moves. Its KE $= \frac{1}{2} \times 4 \times 5.257\ldots = 10.51\ldots$ J. Going a further distance x it gains PE $4 \times 10 \times 0.28 \times x = 11.2x$ and does work $9.6x$ against friction. **M1** for an energy equation for A alone.

$10.51\ldots = 20.8x$ **A1**, so $x = 0.505$ m (3 s.f.). **A1**

Keeping $v^2 = \frac{184}{35}$ exact gives $x = \frac{46}{91}$; using $v = 2.29$ in (b) would give 0.504, which is why you carry full accuracy.

Common mistakes

- **Using the distance along a slope as the height.** PE needs the vertical height, $d \sin\alpha$. This is the most common single error in energy questions.
- **Counting gravity twice.** Including both a PE change and "work done by the weight" uses the same energy twice. Pick one.
- **Writing the KE change as $\frac{1}{2}m(v-u)^2$.** It must be $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(v^2-u^2)$. Square each speed first.
- **Putting forces, not work, into an energy equation.** A 30 N driving force over 200 m contributes $30 \times 200 = 6000$ J, not 30. Every term must be in joules.
- **Using $F = ma$ when the resistance is "not constant"** or the engine works at constant power over a time. The acceleration is then not constant, so only the energy method works; work done by the engine is Pt.
- **Working out the engine's work as $\frac{P}{v} \times$ distance** with one of the speeds. The driving force changes as the speed changes, so this is wrong.
- **Mixing up "increased by" and "increased to"** a power, or forgetting to change kW to W and kJ to J.
- **Leaving out part of the system.** A car towing a caravan must provide a driving force that beats both resistances, and in a pulley system every particle's PE and KE must be counted.
- **Wrong signs.** Decide first what goes in (starting energy, driving work) and what comes out (end energy, work against resistance). A work done against resistance should come out positive.
- **Including a KE term when the speed does not change** (constant speed), or leaving one out when it does.
- **Ignoring "use an energy method".** A constant-acceleration method then earns at most a special-case mark or two.
- **Rounding too early.** Keep full accuracy in intermediate values such as v^2 or the work against friction, and round only the final answer to 3 s.f.

How to get the marks

- **Show each energy term separately** before combining them (KE change, PE change, work done against resistance, work done by the engine). Each can earn a mark on its own, and the method mark needs an equation with the **right number of terms**, each dimensionally correct.
- **Components**: a PE term on a slope must use $\sin\alpha$ (or the height), and friction on a slope uses $R = mg\cos\alpha$. A $\sin/\cos$ mix may keep the method mark but loses the accuracy mark.
- **"Use an energy method"** means your working must be a work–energy equation. Newton's second law and $v^2 = u^2 + 2as$ get only a special-case mark.
- **Power questions**: write $\frac{P}{v}$ as the driving force with the numbers in, then Newton's law with all the forces (driving force, resistance, weight component).
- **"Show that"** means the answer is given: write the equation you used and a step or two of solving, and if the given answer is a rounded decimal, show that your value rounds to it.

- **"Hence"** means use the result you just found, such as the power from part (a) in part (b).
- **Accuracy**: non-exact answers to 3 significant figures; angles to 1 decimal place. Use $g = 10$.
- **Units**: give power in W, or in kW **with** "kW" written. "Find, in kW, ..." means your final answer must be in kW.
- **Reject** a negative speed from a quadratic, and say you are rejecting it.
- **Method marks follow through**: a wrong value from part (a) used correctly in part (b) still earns the method marks there.

Quick check

1. A sledge is pulled 8 m along horizontal ground in 12 s by a force of 60 N inclined at 35° above the horizontal. Find the work done by the force and the average power at which it works.
2. A particle of mass 0.5 kg is dropped from rest at a height of 20 m above the ground. It hits the ground with speed $18 \mathrm{~m~s}^{-1}$. Find the work done against air resistance.
3. A cyclist and bicycle have total mass 80 kg. The cyclist works at 240 W against a constant resistance of 20 N. Find (a) the acceleration on a level road when the speed is $4 \mathrm{~m~s}^{-1}$, and (b) the steady speed up a hill inclined at α to the horizontal, where $\sin\alpha = 0.02$.
4. A crane raises a load of mass 500 kg vertically through 12 m at a constant speed of $0.8 \mathrm{~m~s}^{-1}$. Ignoring resistance, find the work done by the crane and its power.
5. A child of mass 30 kg slides from rest down a smooth curved slide whose top is 2.45 m above its horizontal bottom. She then moves along a rough horizontal surface against a resistance of 60 N. Find her speed at the bottom of the slide and how far she travels along the horizontal surface before stopping.

Answers

1. Work done $= 60\cos 35^\circ \times 8 = 393$ J (3 s.f.). Average power $= \dfrac{393.19\ldots}{12} = 32.8$ W.
2. PE lost $= 0.5 \times 10 \times 20 = 100$ J; KE gained $= \frac{1}{2} \times 0.5 \times 18^2 = 81$ J. Work done against air resistance $= 100 - 81 = 19$ J.
3. (a) Driving force $= \frac{240}{4} = 60$ N, so $60 - 20 = 80a$ and $a = 0.5\ \mathrm{m\,s^{-2}}$. (b) Weight component $= 80 \times 10 \times 0.02 = 16$ N, so $\frac{240}{v} = 20 + 16 = 36$ and $v = \frac{20}{3} = 6.67\ \mathrm{m\,s^{-1}}$.
4. At constant speed the work done is the PE gained: $500 \times 10 \times 12 = 60\,000$ J. The lift takes $\frac{12}{0.8} = 15$ s, so the power is $\frac{60\,000}{15} = 4000$ W $= 4$ kW (check: $P = Fv = 5000 \times 0.8 = 4000$ W).
5. Smooth, so $30 \times 10 \times 2.45 = \frac{1}{2} \times 30 \times v^2$, $v^2 = 49$ and $v = 7\ \mathrm{m\,s^{-1}}$. Her KE is then $\frac{1}{2} \times 30 \times 49 = 735$ J, all used against the resistance: $60d = 735$, so $d = 12.25$ m.

Past questions to try

- **9709/42/M/J/23 Q1**: work done against air resistance on a falling particle.
- **9709/42/O/N/24 Q2**: the work–energy equation down a slope with a given resistance.
- **9709/41/M/J/25 Q4**: power, acceleration and the angle of a hill.
- **9709/41/M/J/23 Q7**: power and steady speed, then an engine working for a time up a hill.