Cambridge AS & A Level · Mathematics 9709

Mathematics 9709 formulas and definitions: Pure Mathematics 1

All 68 formulas, facts and definitions to learn, by syllabus topic. Formulas the exam gives you are left out.

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1 Pure Mathematics 1

1.1 Quadratics

Discriminant $b^2 - 4ac$: > 0 two distinct real roots, $= 0$ one repeated root, < 0 no real roots

Real roots (distinct or repeated): $b^2 - 4ac \geqslant 0$

$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

$y = a(x + p)^2 + q$ has vertex $(-p,\ q)$ and line of symmetry $x = -p$; a minimum if $a > 0$, a maximum if $a < 0$

Line of symmetry of $y = ax^2 + bx + c$: $x = -\dfrac{b}{2a}$

Line and curve: two points $\Leftrightarrow b^2 - 4ac > 0$; tangent $\Leftrightarrow b^2 - 4ac = 0$; do not meet $\Leftrightarrow b^2 - 4ac < 0$ (for the equation made by eliminating y)

For $\alpha < \beta$: $(x - \alpha)(x - \beta) < 0 \Leftrightarrow \alpha < x < \beta$, and $(x - \alpha)(x - \beta) > 0 \Leftrightarrow x < \alpha$ or $x > \beta$

1.2 Functions

$gf(x) = g\big(f(x)\big)$: f acts first

gf exists only if the range of f lies inside the domain of g

Only a one-one function has an inverse; $f^{-1}f(x) = ff^{-1}(x) = x$

Domain of f^{-1} = range of f; range of f^{-1} = domain of f

The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are reflections in $y = x$

$y = f(x) + a$: translation $\begin{pmatrix} 0 \\ a \end{pmatrix}$

$y = f(x + a)$: translation $\begin{pmatrix} -a \\ 0 \end{pmatrix}$

$y = af(x)$: stretch parallel to the y-axis, scale factor a

$y = f(ax)$: stretch parallel to the x-axis, scale factor $\frac{1}{a}$

$y = -f(x)$: reflection in the x-axis; $y = f(-x)$: reflection in the y-axis

$a(x+p)^2 + q$ has its vertex at $(-p, q)$: the least value is q if $a > 0$, the greatest value is q if $a < 0$

1.3 Coordinate geometry

Gradient of AB: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Midpoint of AB: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Length of AB: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Line through (x_1, y_1) with gradient m: $y - y_1 = m(x - x_1)$

Parallel: $m_1 = m_2$. Perpendicular: $m_1 m_2 = -1$, so $m_2 = -\dfrac{1}{m_1}$

Circle, centre (a, b), radius r: $(x-a)^2 + (y-b)^2 = r^2$

$x^2 + y^2 + 2gx + 2fy + c = 0$: centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$

Tangent $\perp$ radius at the point of contact; the normal passes through the centre

The perpendicular from the centre to a chord bisects the chord

Angle in a semicircle $= 90^\circ$; a right-angled triangle's hypotenuse is a diameter of its circle

Line and circle: substitute, then $b^2 - 4ac > 0$ two points, $= 0$ tangent, < 0 no meeting

1.4 Circular measure

π radians $= 180^\circ$; degrees $\times \frac{\pi}{180}$ = radians; radians $\times \frac{180}{\pi}$ = degrees

Perimeter of a sector $= 2r + r\theta$

Area of a triangle $= \frac{1}{2}ab\sin C$, so triangle $OAB = \frac{1}{2}r^2 \sin\theta$

Minor segment $= \frac{1}{2}r^2(\theta - \sin\theta)$

Chord $AB = 2r\sin\frac{\theta}{2}$

Reflex angle $= 2\pi - \theta$; major sector $= \frac{1}{2}r^2(2\pi - \theta)$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$; cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Exact values of $\sin$, $\cos$, $\tan$ at $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$

1.5 Trigonometry

Exact values of $\sin$, $\cos$, $\tan$ at $0, 30^\circ, 45^\circ, 60^\circ, 90^\circ$ (table above)

$\sin(180^\circ - \theta) = \sin\theta$, $\cos(360^\circ - \theta) = \cos\theta$, $\tan(180^\circ + \theta) = \tan\theta$

Signs: all positive (first quadrant), $\sin$ (second), $\tan$ (third), $\cos$ (fourth)

$\sin x$, $\cos x$ have period 360° (2π) and lie between -1 and 1; $\tan x$ has period 180° (π) and asymptotes at $x = 90^\circ, 270^\circ, \ldots$

$y = a\sin(bx) + c$: greatest $c + a$, least $c - a$ (for $a > 0$), period $\frac{360^\circ}{b}$ or $\frac{2\pi}{b}$

$180^\circ = \pi$ radians

1.6 Series

The term in b^r is $\binom{n}{r}a^{n-r}b^r$

$d = u_2 - u_1$ and $r = \dfrac{u_2}{u_1}$

a, b, c in AP: $2b = a + c$; a, b, c in GP: $b^2 = ac$

A GP converges (has a sum to infinity) only when $-1 < r < 1$

$u_n = S_n - S_{n-1}$; the sum of the terms after the kth is $S_\infty - S_k$

1.7 Differentiation

Constant multiples and sums: $\dfrac{d}{dx}\big(kf(x)\big) = kf'(x)$; differentiate term by term; a constant gives 0

Chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\dfrac{d}{dx}(ax + b)^n = an(ax + b)^{n-1}$

Tangent at (x_1, y_1) with gradient m: $y - y_1 = m(x - x_1)$

Normal gradient $= -\dfrac{1}{m}$

Stationary point: $\dfrac{dy}{dx} = 0$

$\dfrac{d^2y}{dx^2} < 0$: maximum; $\dfrac{d^2y}{dx^2} > 0$: minimum

Increasing: $\dfrac{dy}{dx} > 0$; decreasing: $\dfrac{dy}{dx} < 0$

Connected rates: $\dfrac{dy}{dt} = \dfrac{dy}{dx} \times \dfrac{dx}{dt}$, and $\dfrac{dx}{dt} = \dfrac{dy}{dt} \div \dfrac{dy}{dx}$

A line making angle θ with the positive x-axis has gradient $\tan\theta$

Cylinder: $V = \pi r^2 h$, curved surface $2\pi rh$; circle: $A = \pi r^2$, circumference $2\pi r$; cube: $V = x^3$

1.8 Integration

$\displaystyle\int (ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a(n + 1)} + c \quad (n \neq -1)$

$\displaystyle\int kf(x) \, dx = k\int f(x) \, dx$, and integrate sums term by term

$\int_a^b f(x)\,dx = F(b) - F(a)$, where F is an integral of f

Area under a curve: $\int_a^b y\,dx$; area next to the y-axis: $\int_c^d x\,dy$

Area between two graphs: $\int_a^b (y_{\text{top}} - y_{\text{bottom}})\,dx$

Volume about the x-axis: $V = \pi \int_a^b y^2\,dx$

Volume about the y-axis: $V = \pi \int_c^d x^2\,dy$

Washer: $V = \pi \int_a^b (R^2 - r^2)\,dx$

Volume of a cylinder $= \pi r^2 h$; area of a triangle $= \frac{1}{2}bh$; trapezium $= \frac{1}{2}(a+b)h$